CURVE-FITTING AND ECONOMETRIC OIL PRODUCTION MODELS IN BRAZIL

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Orientador(es): Alexandre Salem Szklo
Amaro Olímpio Pereira Júnior

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A todos (as) que conciliaram dias de
trabalho com noites de estudo.

“There is a driving force more powerful than steam, electricity and nuclear power: the will.” (Albert Einstein)

“I learned that courage was not the absence of fear, but the triumph over it. The brave man is not he who does not feel afraid, but he who conquers that fear.” (Nelson Mandela)

“If you can't fly then run, if you can't run then walk, if you can't walk then crawl, but whatever you do you have to keep moving forward.” (Martin Luther King Jr.)

“Say no to protectionism. It is like locking yourself in a dark room. Wind and rain are kept out, but so are light and air.” (Xi Jinping, Chinese President at the World Economic Forum in Davos)

“When the wind of change blows, some build walls, while others build windmills.” (Chinese Proverb)

“If you do not push the boundaries, you will never know where they are.” (T. S. Eliot)

“Time flies over us, but leaves its shadow behind.” (Nathaniel Hawthorne)

“The place I love, Brazil, is a three-legged dog. Everyone who's ever loved a three-legged dog knows you can love that dog more than one with a handsome pedigree.” (Adapted from the book *The Hard Way on Purpose*)

“Serenity now. Insanity later.” (159th episode of Seinfeld)

“Já que se há de escrever... que ao menos não se esmaguem com palavras as entrelinhas.” (Clarice Lispector)

“Eu sei que não sou nada e que talvez nunca tenha tudo. Aparte isso, eu tenho em mim todos os sonhos do mundo.” (Fernando Pessoa)

“The key to realizing a dream is to focus not on success but on significance — and then even the small steps and little victories along your path will take on greater meaning.” (Oprah Winfrey)

“Dream big, work hard, stay humble.” (Brad Meltzer)

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MODELOS DE AJUSTE DE CURVA E ECONOMÉTRICOS PARA PRODUÇÃO DE PETRÓLEO NO BRASIL

Larissa Nogueira Hallack

Março/2019

Orientadores: Alexandre Salem Szklo
Amaro Olímpio Pereira Júnior

Programa: Planejamento Energético

Este trabalho desenvolve ciclos de produção de petróleo multi-Hubbert e assimétricos para analisar a produção de petróleo no pós-sal offshore brasileiro. Um modelo híbrido emprega a análise de regressão para explicar o desvio da produção de petróleo em relação à curva de Hubbert. A partir da teoria da curva *creaming*, esta tese avalia o potencial para descobertas de petróleo no Brasil. Uma variante da curva *creaming* emprega o tamanho das descobertas dos campos considerando um mesmo fator de recuperação para campos de uma bacia sedimentar. Essa suposição é necessária devido à confidencialidade da informação de reservas de um campo. Além disso, esta tese analisa o número de poços de desenvolvimento no pré-sal aplicando técnicas econométricas de cointegração e modelagem de correção de erros para séries temporais. Após um aumento no risco-país e o colapso do preço do petróleo em 2014, o número de poços de desenvolvimento completados nas zonas do pré-sal é analisado considerando o efeito do preço do petróleo, da volatilidade dos preços, da produtividade e do risco-país. Os resultados mostram os efeitos negativo e positivo de se elevar, respectivamente, o risco-país e a produtividade para o número de poços de desenvolvimento no pré-sal.

Abstract of Thesis presented to COPPE/UFRJ as a partial fulfillment of the requirements for the degree of Doctor of Science (D.Sc.)

CURVE-FITTING AND ECONOMETRIC OIL PRODUCTION MODELS IN BRAZIL

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March/2019

Advisors: Alexandre Salem Szklo
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This work develops multi-Hubbert and asymmetrical oil production cycles to analyze the Brazilian post-salt oil production offshore. A hybrid model employs regression analysis to explain the deviation of crude oil production rate from the Hubbert curve. By looking at the creaming phenomenon, this thesis evaluates the potential for future oil discoveries in Brazil. A creaming curve variant employs the fields' size of discoveries based on the assumption that the recovery factor is the same for all fields within a sedimentary basin. This assumption is necessary because the size of the fields' reserves is confidential data. In addition, this thesis analyzes the number of development wells completed in the pre-salt zones by using time series econometric techniques of cointegration and error correction modeling. Following an increase in the country risk, and the 2014 oil price collapse, this thesis analyzes the number of development wells drilled in the pre-salt zones by identifying the effect of oil price, price volatility, productivity, and country risk. Results show the negative and positive effect of raising, respectively, the country risk and productivity for the number of pre-salt development wells.

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1. Introduction

The oil sector has significant relevance for national and worldwide economies, with a privileged position in the political and economic agenda of countries, especially concerning energy security policies. The issue of energy security frequently leads to assessments of oil and gas reserves and resources (Pickering, 2008; Sorrell et al., 2010). Such reserves evaluation can support the elaboration of development scenarios for countries and companies.

The nations' dependence on fossil fuels, together with the reliance on oil exports revenue, and the attempt to reduce the vulnerability of net-import oil countries are among the traditional issues of geopolitics (Greene, 2010; Greene e Liu, 2015; Sovacool, 2007; Vivoda, 2009). Today, the geopolitics of energy face new realities in assessing the energy system, such as: (i) the changing patterns of economic growth, (ii) the development of new methods to extract oil resources (particularly from tight and deepwater formations), (iii) environmental pressures, (iv) emerging technologies that will enable the commercial use of renewable energy sources, (v) actions to address climate change, and (vi) the efficiency (or lack thereof) of national oil companies (NOCs) (Harvard, 2018).

The hypothesis that peak oil has already occurred is inconsistent with observations especially in the United States (US). After seeming to peak in the early 1970s, oil production in the US rises from 5.4 million barrels per day (mbd) in January 2010 to 11.9 mbd in November 2018 mostly due to tight formations' production (EIA, 2018a). In this same period, Brazilian oil[1] production from the pre-salt (deep-lying oil underneath an

[1] Information disclosed by the Brazilian petroleum regulatory agency, ANP, adopts the petroleum term as the sum of oil and condensate (e.g. the Monthly Oil and Natural Gas Production Bulletin (ANP, 2018a) and the data of petroleum imports (ANP, 2018b), not including Natural Gas Liquid (NGL). This work as well

extremely thick layer of salt) increases by 1.4 mbd (ANP, 2018a). This rise occurs despite a decline in oil prices, which dropped from \$100 per barrel in June 2014 to \$30 per barrel at the beginning of 2016.

Greater output seems to shift the paradigm from the age of (perceived) scarcity to a period of abundance. Such abundance leads the OPEC+ group – Organization of the Petroleum Exporting Countries and country allies, as Russia – to choose between an ‘accommodation' strategy and a 'squeeze strategy.' In the accommodation strategy, the OPEC + group reduce production to defend higher prices, whereas in a ‘squeeze' strategy they maintain production to reduce non-OPEC output and increase consumption (Manescu and Nuño, 2015; Ansari, 2017; Behar and Ritz, 2017) and/or dampen the development of alternatives, such as renewables.

The new energy bonanza is characterized by comparatively low oil prices, but price spikes still occur given that many factors affect oil prices (O’Sullivan, 2017). After the 2014 price collapse, the price for Brent increases to 81 dollars per barrel in October 2018 (highest level since 2014). At the 4th OPEC and non-OPEC Ministerial Meeting in June 2018, the OPEC+ group decided to increase output. This decision occurs during a period of growing demand, supply disruptions in Venezuela and Libya, US sanctions against Iran, infrastructure bottlenecks in the US tight oil production and new regulations on ship sulfur emissions. All these episodes have the potential to increase oil prices (Reuters, 2018a, 2018b; Smith, 2018; Cunningham, 2018).

does not include NGL in its petroleum term. Considering the small amount of condensate produced in Brazil compared to crude oil (ANP, 2018a), this work assumes there are no major differences between the amount of crude oil and petroleum for the case of Brazil and uses the term oil as a general definition. EIA (2013) defines NGL and lease condensate.

The chance of oil and gas companies shrinking their investment budget in the wake of the 2014 price drop makes a price spike in the coming years possible. However, such a spike would likely mild in the short-term if the production of tight oil responds quickly to higher oil prices (O'Sullivan, 2017). Furthermore, oil price fluctuations likely represent a more profound long-run structural transformation in oil demand, that reduces the likelihood that price will recover to previous levels (Waterworth and Bradshaw, 2018).

To develop pre-salt, Petrobras' investments in the upstream sector increased substantially as well as Petrobras' total debt (Petrobras, 2018a, 2018b). For the last five years, production and development have had a share of more than 70% from investments in the upstream sector (with a focus on the development of pre-salt), according to the Business and Management Plans[2] of Petrobras. In light of the oil price drop, budget constraint and the high productivity from the pre-salt layer, Petrobras[3] focus on the production and development of pre-salt and place the revitalization of post-salt offshore production at a disadvantage. Even though the output from post-salt offshore zone declines since 2010, it still represents 41% of the total oil production in Brazil (1.05 mbd in 2018).

Despite an increasing tendency in the output from the pre-salt layer, the 2014 price collapse jeopardized the oil production landscape, as well as the financial situation of Petrobras and the Brazilian economy. In the wake of an economic recession, Brazil and Petrobras lose its investment-grade rating in 2015. In this same year, the Organization for

[2] It was considered the CAPEX 2019-2023, CAPEX 2018-2022, CAPEX 2017-2021, CAPEX 2015-2019 and CAPEX 2014-2018 of Petrobras' Business and Management Plans.

[3] Petrobras is a mixed capital company: partially state-owned enterprise and partially private. Petrobras' shares have been traded in the New York Stock Exchange (US) and BOVESPA Stock Market (Brazil).

Economic Co-operation and Development (OECD) increases the score for Brazil's risk from 3 to 4 (followed by another increase from 4 to 5 in 2016).

With that in mind, this thesis zooms in on details of the Brazilian oil production and oil discovery forecasts and develops a curve-fitting methodology to assess production and discovery cycles in order to overcome some modeling issues. Such analysis took on board:

1) the adaptation of an asymmetric Hubbert model based on a Gaussian curve, developed by Brandt (2007)[4], to the original Hubbert model. This study assesses the influence of techno-economic parameters to the post-salt oil production in Brazil by using a regression model to explain the differences between the Hubbert model and observed production data, inspired by Kaufmann (1991)[5]. After that, a logistic curve is fitted to pre-salt historical oil production using different scenarios of ultimately recoverable resources (URR) as a preliminary (and plain) effort to estimate the peak of oil production in the pre-salt province.
2) the use of a creaming curve variant model (plotted as the cumulative discovery against the number of new field wildcats) to assess the potential for increasing reserves from known fields and future discoveries[6]. To estimate the fields' size of discoveries, this variant assumes the recovery factor is the same for all fields within a sedimentary basin. Such an assumption is necessary because the size of

[4] This asymmetric Hubbert model is based on the paper "Curve-fitting variants to model Brazil's crude oil offshore post-salt production" published in 2017 by the Journal of Petroleum Science and Engineering co-authored with Alexandre Salem Szklo, Amaro Olímpio Pereira Júnior and Johannes Schmidt.
[5] This regression model is based on the paper "Curve-fitting variants to model Brazil's crude oil offshore post-salt production" published in 2017 by the Journal of Petroleum Science and Engineering co-authored with Alexandre Salem Szklo, Amaro Olímpio Pereira Júnior and Johannes Schmidt.
[6] The creaming curve analysis is based upon the paper "Assessing the exploratory potential in Brazil by applying a creaming curve variant" published in 2019 by *Energy Policy*, co-authored with Alexandre Szklo.

the fields' reserves is confidential data. Moreover, in light of reduced investments for exploratory activities in previous Petrobras' Business and Management Plans, high oil prices volatility and the greater time interval between discovery and production for offshore projects, assessing oil discovery cycles becomes of great importance.

Proceeding on this track, this thesis estimates a series of models to quantify how crude oil prices, price volatility, productivity, and country risk affect the number of development wells that are completed in the pre-salt zones of Brazil[7]. This thesis extends the work of Ansari and Kaufmann (2019) by including the country risk as a variable into the models. Specifically, this thesis extends previous efforts in several ways:

3) The identification of the break-even price (BEP) for petroleum production in pre-salt formations by specifying a range of values and determining the BEP that generates the 'most accurate' model. The BEP also is used to modify the way firms perceive volatility. This work postulates that volatility has its greatest effect when prices are near the BEP (Ansari and Kaufmann, 2019). As prices move below or above the BEP, price volatility has a smaller effect because prices are too low to justify exploration and development (E&D) or so high that volatility has little effect on economic returns. These effects are represented by calculating a variable termed 'perceived volatility.'
4) This work explores the price used to drill development wells by estimating models that specify spot prices and futures contracts with a range of maturities for two

[7] This analysis follows the paper "The Effect of Productivity and Country Risk on Development in the Brazilian Pre-salt Province" published in 2019 by the journal Energy Sources, Part B: Economics, Planning, and Policy, co-authored with Robert K. Kaufmann and Alexandre Szklo.

benchmark crude oils: West Texas Intermediate (WTI) and Brent. This thesis explicitly represents the effect of productivity in oil-producing wells, both as a stand-alone measure and how it interacts with the price to affect net revenue.

5) The models include country risk, which represents the willingness of foreign investors to fund E&D in the pre-salt zone. According to this thesis' literature review, none econometric technique has been applied before to analyze the development of the pre-salt province in Brazil.

The higher output from post-salt and pre-salt zones occurred in December 2016, when the oil production reached 2.7 mbd. Since then, the oil output remains stable at about 2.6 mbd. As a consequence of the large pre-salt discoveries, previous scenarios proposed the production of oil in Brazil would reach in 2018: (i) 3.2 mbd, according to Petrobras' 2014-2018 Business and Management Plan; (ii) 3.5 mbd, according to IEA (2013, p. 369); and (iii) between 3.3 mbd and 5.5 mbd, respectively, from the Ten-Year Energy Expansion Plan – PDE 2015-2024 and the Ten-Year Energy Expansion Plan – PDE 2011-2020, according to the Energy Research Office (EPE in its Portuguese acronym). Remarkably, all these noticeable institutions overestimated the oil production in Brazil, which underscores how complicated oil production forecast can be.

This complexity lies in the fact that all procedures to model supply curves for oil production have strengths and weaknesses. The principal methodologies are summarized by Sorrell et al. (2010) and Brandt (2010). According to them, curve-fitting methods are simple to implement and therefore broadly used. However, they require a suitable theoretical basis and neglect essential variables. Simulation models do not predefine the form of the production curve but generate it by simulating the interaction of physical and, sometimes, economic factors. This approach requires many data, which are not always publicly available. Bottom-up models are promising for short to medium term projections, but restricted by their dependence on proprietary data, lack of transparency, uncertainty over significant variables and the need to consider various premises. Economic models focus on investments, optimal extraction paths and effects of oil prices rather than focusing on physical or technological aspects. This approach can be unsatisfactory because it does not account for critical geological conditions that become important in the long-term.

This thesis describes the development of curve-fitting and econometric methods to model oil production in Brazil due to the viability of data for developing these approaches. Through these analyses, this thesis sets out to address the following overarching questions:

1) How much does the pre-salt production have to increase in order to offset the declines in post-salt production?
2) How much time does it take to the post-salt oil production in Brazil adjust to changing price levels?
3) What is the potential for increasing the recovery factor and discoveries in Brazil?
4) How do oil prices, oil prices' volatility, productivity, and country risk affect oil development in the pre-salt zone in Brazil?

However, before moving on to the description of the methodology and the results, it is essential to provide some background in the form of a literature review.

2. Historical and technical background

2.1. Historical background

2.1.1. Historical development of oil production in Brazil

Globally, offshore oil production accounted for about 30% of total oil production over the past decade (EIA, 2016) and accounted for an estimated 20% of the world's oil reserves (Total, 2015).

Offshore production is subdivided into three categories: shallow-water, deep-water and ultra-deepwater. EIA (2016) defines the three categories as follows: water-depth up to 125m refers to shallow-water production; water-depth between 125m and 1500m refers to deep-water production, and water-depth above 1500m refers to ultra-deep-water production.

Morais (2013) and Sallh et al. (2015) consider 300m of water-depth as the threshold between shallow water and deep water, but they also consider 1500m of water-depth as the limit for ultra-deepwater.

The majority of deep-water or ultra-deep-water production occurs in four countries: Brazil, the US, Angola, and Norway, being Brazil and the US in charge of more than 90% of global ultra-deep-water production (EIA, 2016).

In Brazil, the first discoveries of offshore oil fields occurred in the coastal waters of the Northeast region in 1968-1973, followed by shallow waters' discoveries in Campos Basin in 1974 (Morais, 2013). In the following decades, the advancement of exploration activities and innovations in maritime production systems enabled the exploitation of hydrocarbon deposits at longer distances from the coast and in deeper waters. These advancements led to discoveries of giant and super-giant fields in Santos and Campos basins – 500 million barrels and 5 billion barrels of recoverable oil (or equivalent gas) are the cutoffs, respectively, for giant and supergiant oil fields. The oil production progress to deeper waters can be observed in Figure 1.

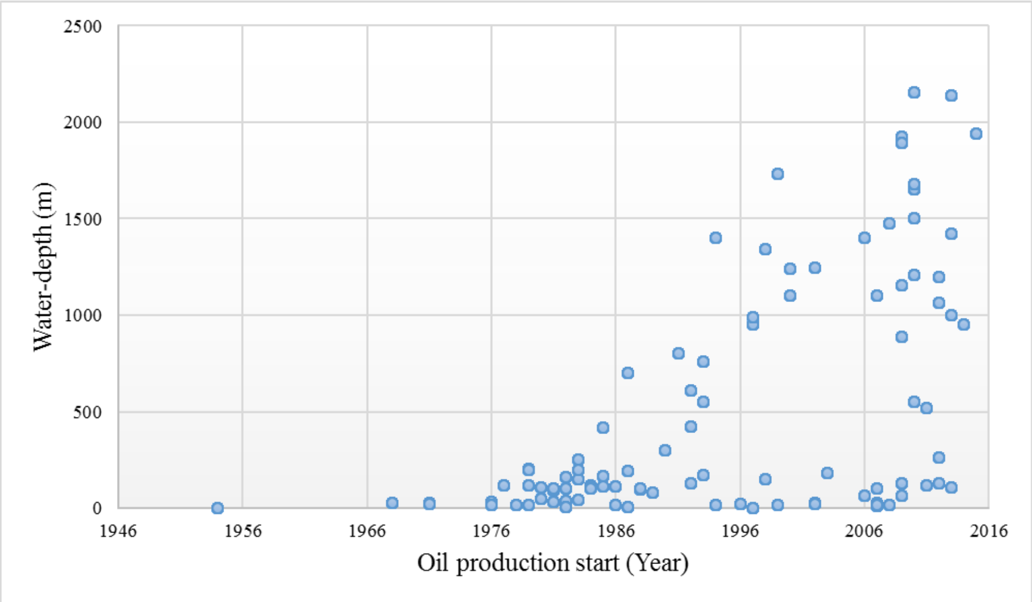

Figure 1: Oil production water-depth versus the year of production start in Brazil
Source: Based on ANP (2016)

As Brazilian oil production evolved into deeper water depths, the national outlook of oil reserves altered substantially (Figure 2).

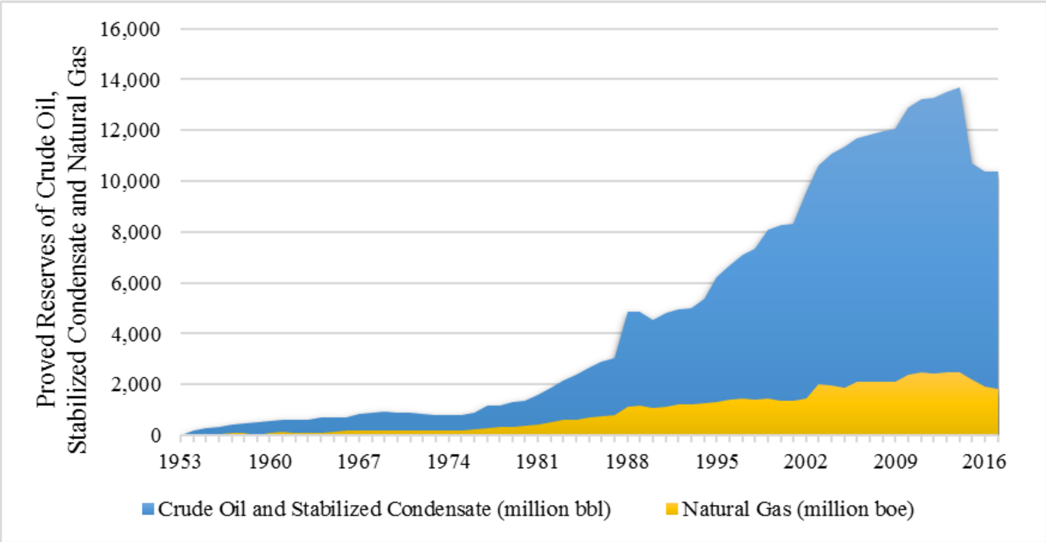

Figure 2: National Historical Proved Reserves - ANP/SPE Criteria
Source: Based on Petrobras (2018c)

Besides the increase of reserves, the progress of technologies into deep and ultra-deep waters allowed a significant reduction in the dependence of oil imports.

The Brazilian oil industry dates from the first half of the 20th century. Exploration of oil fields began onshore in Brazil, where the first oil field was discovered in 1941 at Recôncavo basin. Afterward, a period targeting the reduction of the dependence on oil imports began. Although significant oil fields were discovered onshore, Brazil still imported two-thirds of its consumption in the mid-1960s (Morais, 2013). At that time, the offshore oil discoveries were insufficient to change the landscape of high dependence on imported oil in Brazil. Offshore exploration started evolving in the 1960s. The first discoveries of oil fields off the coast occurred in the Northeast region in 1968-1973, followed by shallow waters' discoveries in Campos Basin in 1974 (Morais, 2013). In the following decades, significant oil fields were discovered (e.g., Albacora, Marlim, Albacora Leste, Marlim Leste and Marlim Sul in the 1980s, and Barracuda and Roncador in the 1990s). These discoveries were carried out by Petrobras; the Brazilian state-controlled oil company. The advancement of exploration activities and innovations in maritime production systems enabled the exploitation of hydrocarbon deposits at longer distances from the coast and in deeper waters. In the last three decades, these advancements led to discoveries of giant and super-giant fields in Santos and Campos basins in the so-called post-salt layers.

The first oil discovery in the Campos Basin in the mid-1970s started a decisive phase of discoveries that reduced the country’s import dependence gradually. Brazil achieved the self-sufficiency regarding crude oil volume for the first time in 2006 when the oil production of 100 million cubic meters exceeded the oil consumption of 99 million cubic meters (EPE, 2015, p. 44). Despite the self-sufficiency in volume terms occurred in 2006, the oil export revenue still accounted for 75% of import expenditures this year. The self-sufficiency in monetary terms was achieved in 2009 (ANP, 2018b), although Brazil still relies on imported oil to achieve a higher-quality blend for oil refining processes (Saraiva et al., 2014).

Currently, Brazil holds the 16th position in the rank of crude oil exporting countries (CIA, 2018; Workman, 2018). Brazil’s gross oil exports increased from 631 thousand barrels per day (kbd) in 2010 (when Lula field’s commercial oil production began) to 997 kbd in 2017. In this period, the net positive oil exports almost tripled: from 292.7 kbd to 847.3 kbd.

Brazil´s oil production is meaningful on a global scale – around 2.69 mbd of oil in December 2018 (ANP, 2018a). Oil and oil products still account for the leading share of domestic energy supply, although their share may reduce from 38% in 2015 to 35% in 2024, because of gasoline replacement by ethanol and fuel oil by natural gas (EPE, 2015a, p. 437).

Figure 3 shows the significant share of the offshore oil production in Brazil, the new oil production cycle in the pre-salt and the decline of oil production in the Brazilian post-salt offshore.

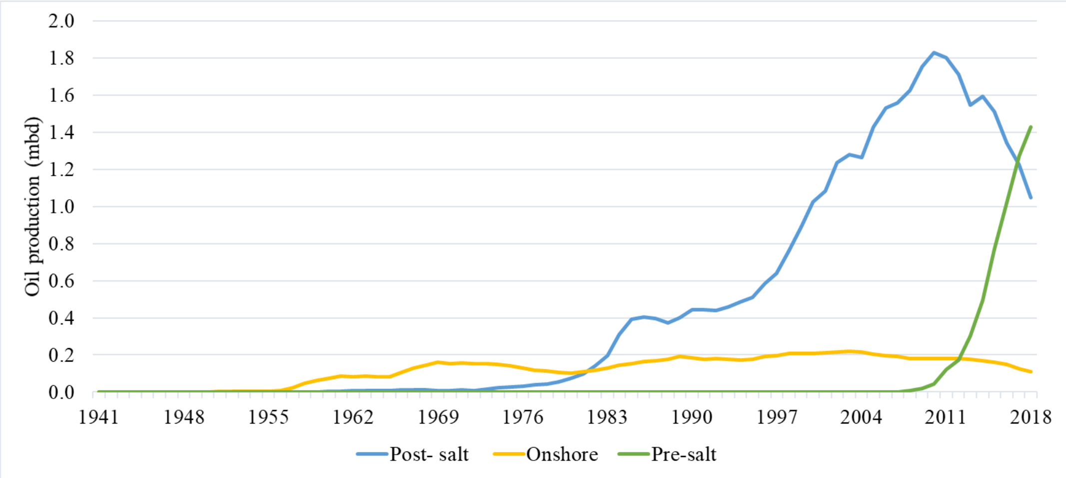

Figure 3: Onshore and offshore oil production curves
Source: Based on ANP (2016, 2018a)

Brazil´s post-salt oil production offshore history can be divided into two periods: one that reached its peak in the mid-1980s and another one that began to decline around 2010. The first cycle includes the shallow water producing fields (water-depth up to 250m) and the second cycle includes the deep-water producing fields (water-depth deeper than 250m)[8].

The post-salt oil production offshore in shallow water is characterized by a slowly declining curve for more than three decades. Figure 4 shows that after 1986 (the peak production year) some fields increased their oil production expressively, thus contributing to an asymmetric cycle profile.

[8] Differently from all the above definitions, this thesis considers 250m of water-depth as the threshold between shallow water and deep water, without differentiating between deep and ultra-deep water.

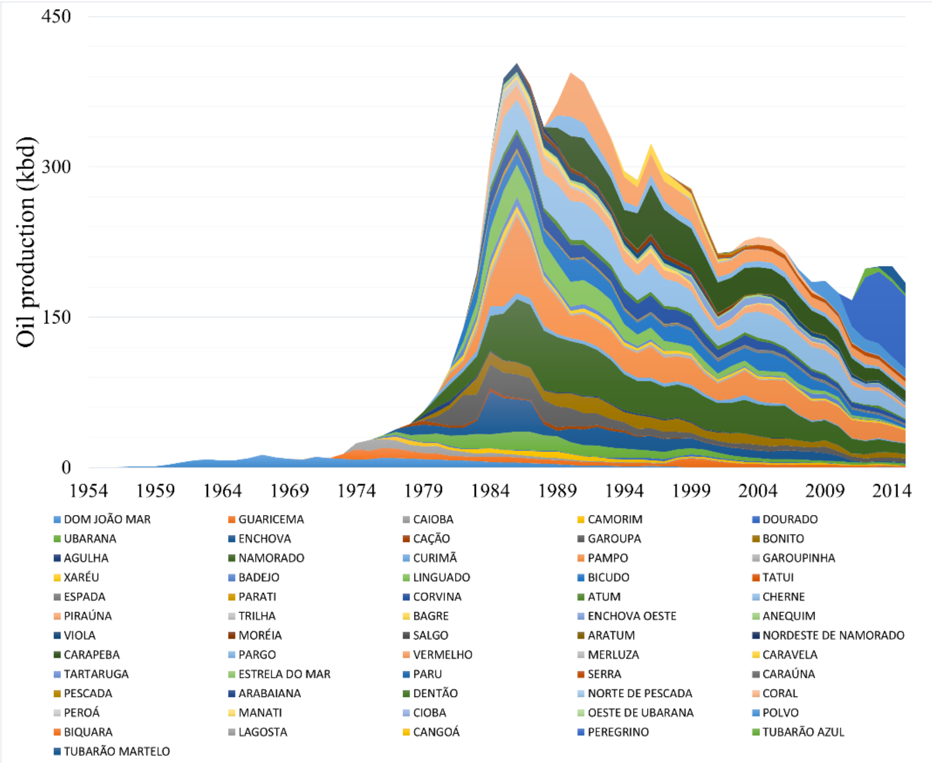

Figure 4: Post-salt oil production offshore in shallow waters ordered by fields date of start-up production
Source: Based on ANP (2016)

Deep-water post-salt fields started to be discovered in the mid-1980s and accounted for almost 90% of the Brazilian post-salt production in 2015. In the deep-water post-salt cycle, a few giant fields[9] in ultra-deep waters have been discovered. However, Figure 5 shows that no post-salt field with representative production started producing after 2010 when the offshore deep-water post-salt peak production occurred.

[9] Giants fields include Albacora, Marlim and Barracuda (Morais, 2013), whose oil production started, respectively, in 1987, 1991 and 1997 (ANP, 2016).

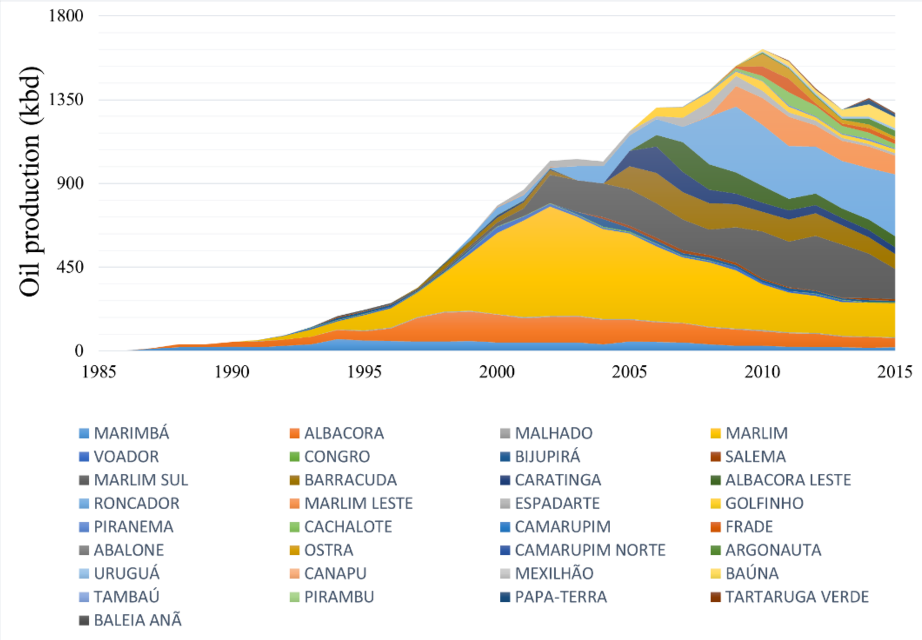

Figure 5: Post-salt oil production offshore in deep waters ordered by fields´ date of start-up production
Source: Based on ANP (2016)
Observation: There is a small error in the production numbers related to fields mainly producing in post-salt but also producing in pre-salt to some extent.

Post-salt declining oil production off the coast still represents a representative share of the Brazilian production (41% in 2018). However, in times of budget constraint, revitalizing it may be a second-order strategy, as pre-salt production is more promising (Ferreira, 2016). The decline in output from post-salt fields has been offset by the increasing pre-salt production, which accounts for 55% of Brazil's oil production in 2018.

2.1.2. Recent changes in the Brazilian petroleum institutional arrangements

Brazil has continental-size, but less than 5% of the sedimentary areas are contracted for the exploration and extraction of oil (Oddone, 2018). There are still frontier areas completely unexplored, such as Pernambuco-Paraíba offshore basin and Madre de Deus onshore basin, where no wildcat well has been drilled yet, and only two basins (Santos and Campos basins) produce 95% of Brazil's oil production in December 2018 (ANP, 2018a). As such, Brazil's exploration intensity indicator, which consists of the ratio between the sedimentary area and the number of exploratory wells drilled, varies from around eight km^2/ExpW (square kilometers/exploratory well) in the Recôncavo onshore basin to 6,602 km^2/ExpW in the Parnaíba onshore basin (Table 1).

Table 1: Exploration intensity indicator for selected sedimentary basins in Brazil

Sedimentary Basin	**Area (km^2)**	**Exploratory Wells**	**km^2/ExpW**
	Onshore		
Recôncavo	9,082	1,194	8
Espírito Santo	10,875	561	19
Potiguar	26,725	1,075	25
Tucano Sul	7,700	104	74
Camamu-Almada	8,101	37	219
Rio do Peixe	1,500	5	300
Solimões	1,210,000	206	5874
Parnaíba	673,400	102	6602
	Offshore		
Campos	174,000	1,241	140
Espírito Santo	109,599	224	489
Sergipe-Alagoas	163,273	318	513
Santos	307,800	443	695
Potiguar	196,000	243	807
Ceará	170,000	139	1223

Source: Based on Almeida and Arruda (2017)

Pre-salt resources were first discovered in Brazil’s offshore Santos basin in 2005, and further exploration revealed an estimated five to eight billion barrels of oil equivalent in this layer (EIA, 2017). Pre-salt oil deposits are located off the Brazilian coast under deep, thick layers of rock and salt. To develop such important discoveries, the Brazilian state-controlled oil company, Petrobras increased its investments substantially in the upstream sector (Petrobras, 2018b), what led to a steep rise in its debt (Petrobras, 2018a). In the wake of the oil price drops (first in 2008 and again in 2014), the company diminished its investments in exploratory activities to focus on production and development activities. For the last five years, production and development have had a share of more than 70% from investments in the upstream sector, according to the Business and Management

Plans[10] of Petrobras. In this context, the number of exploratory wells in Brazil declined from 239 in 2011 to 26 in 2017 (ANP, 2018c, 2018d). In contrast, the US has drilled hundreds to thousands of exploratory wells yearly since 1949 (EIA, 2018c).

However, budget contraction is not limited to Petrobras. The oil industry reduced investment in the wake of the 2014 price collapse, making additional investment fundamental to spur supply growth after 2020 (IEA, 2018a). In a market environment of moderate oil prices, capital constraints and declining access to 'easy oil' (i.e. conventional and cheap to produce), international oil companies (IOCs) are increasingly careful to invest in oil ventures, especially for the exploration and development in extreme environments (e.g. Brazilian pre-salt and Arctic oil) (Waterworth and Bradshaw, 2018). This scenario of constrained capital and need for foreign investment can make fiercer the competition for capital between different prospective development (e.g., deep-water offshore in Brazil, onshore tight oil and shale gas in the US and Argentina).

Also, since 2010, exploration cycles in Brazil follow an institutional arrangement based on three simultaneous fiscal systems, depending on the type of area to be explored. Besides, bid rounds base the exploratory activities in Brazil– i.e., areas with similar geological factors are organized in exploratory blocks in bid rounds to promote competition between companies. As detailed by Braga and Szklo (2014), the Petroleum Law (Federal Law No.9,478/1997) opened the upstream petroleum activity in Brazil to private companies under the concession (royalty/tax) regime[11]. Later, the production

[10] It was considered the CAPEX 2019-2023, CAPEX 2018-2022, CAPEX 2017-2021, CAPEX 2015-2019 and CAPEX 2014-2018 of Petrobras' Business and Management Plans.

[11] Under the concession agreement, the concessionaire takes on all risks and investments in exploration and production. In case a commercial discovery happens, the concessionaire shall pay the Union, in cash, taxes levied on income, plus the applicable government take. After the Union receives the payments, the concessionaire is entitled to the exclusive property of the oil and natural gas production lifted from a block.

sharing agreement (PSA) was also established by the Pre-salt Law (Federal Law No.12,351/2010) within pre-salt and strategic areas, considered of low exploratory risk[12]. Besides, the Law of Onerous Assignment (Federal Law No. 12,276/2010) established the onerous transfer of rights: a unique petroleum agreement in the world. This last fiscal regime allowed the Government to onerously assign Petrobras, without bidding round, to produce five billion barrels of oil equivalent in pre-salt formation for forty years. This Law authorized the Government to sign for Petrobras' joint-stock shares and pay for them with federal public debt securities[13].

These laws introduced a mixed regulatory regime in Brazil, coexisting PSA and onerous assignment agreements, in addition to the before existing concession agreements[14]. The fields discovered in pre-salt before the PSA establishment in 2010 are being developed by consortiums under the concession agreement (e.g., Lula, Sapinhoá, Lapa, Berbigão, Carcará, and Sururu). Lula field is responsible for 60% of the oil production in the pre-salt layer and one-third of the entire Brazilian oil production in December 2018 (ANP, 2018a). The beginning of Lula's commercial oil production in Santos Basin was in 2010. Afterward, other important pre-salt fields started to produce in the Santos Basin. For this reason, pre-salt oil productivity jumps from the average 370 barrel/day/well between

[12] Under the production sharing agreement, if there is a commercial discovery, the company or consortium receives in kind, as compensation, the production volumes corresponding to their exploration cost (the so-called cost-oil). Besides the oil cost, it also gets production volumes corresponding to the royalties due and profit oil. In the bidding process, whoever offers the Union the largest profit oil share is the winner.

[13] The onerous transfer of rights agreement allowed the government to capitalize Petrobras by granting the company 5 billion barrels of unlicensed pre-salt oil reserves in exchange for a larger ownership share. Wells drilled into the pre-salt fields, under the onerous transfer of rights agreement, are developed by Petrobras so far.

[14] Under the concession agreement, the concessionaire takes on all risks and investments in exploration and production. In case a commercial discovery happens, the concessionaire shall pay the Union, in cash, taxes levied on income, plus the applicable government take. After the Union receives the payments, the concessionaire is entitled to the exclusive property of the oil and natural gas production lifted from a block.

January 2007 and May 2009 (period before the Extended Well Test starts in Lula) to 3,400 barrel/day/well in 2010 (the beginning of Lula's production commercially) and 16,500 barrel/day/well in 2017 (after other pre-salt fields start to produce in Santos Basin).

After the PSA establishment in 2010, the 1st Production Sharing Bidding Round took place in 2013, offering blocks of Mero field (the former Libra prospect). Four years after, the 2nd PSA and 3rd Production Sharing Bidding Rounds took place in 2017, followed by the 4^{th,} and 5st Production Sharing Bidding Rounds in 2018 (Tables 2 and 3). The 6th, 7th and 8th Production Sharing Bidding Rounds are planned to happen, respectively, in the triennium 2019-2021, according to the Brazilian petroleum regulatory agency, ANP.

Table 2: Summary PSA Bidding Rounds

Year	**2013**	**2017**				**2017**			
Round	**1st PSA**	**2nd PSA**				**3rd PSA**			
Sedimentary Basin	Santos	Campos	Santos			Santos			Campos
Offered blocks	Libra	Sudoeste Tartaruga Verde	Sul de Gato do Mato	Entorno de Sapinhoá	Norte de Carcará	Pau Brasil	Peroba	Alto de Cabo Frio - Oeste	Alto de Cabo Frio - Central
# Offered blocks	1	1	1	1	1	1	1	1	1
Bidded blocks	1	No offers	1	1	1	No offers	1	1	1
Minimum exceeding (profit) oil (%)	41.65%		11.53%	10.34%	22.08%		13.89%	22.87%	21.38%
Offered exceeding (profit) oil (%)	41.65%		11.53%	80.00%	67.12%		76.96%	22.87%	75.86%
Average Local Content Requirement – Exploratory Phase	37%	55.0%	38.0%	35.0%	35.0%	18.0%	18.0%	18.0%	18.0%
Average Local Content Requirement – Development Phase	55%	65.0%	60.0%	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
Consortium	Petrobras (40%), Shell (20%), Total (20%), CNPC (10%) and CNOOC (10%)		Operator Shell (80%) and Total (20%)	Operator Petrobras (45%), Shell (30%) and Repsol Sinopec (25%)	Operator Statoil (40%), Petrogal (20%) and ExxonMobil (40%)		Operator Petrobras (40%), CNODC (20%) and BP (20%)	Operator Shell (55%), CNOOC (20%) and Qatar Petroleum (25%)	Operator Petrobras (50%) and BP (50%)

Table 3: Summary PSA Bidding Rounds (Continuation)

Year	**2018**				**2018**			
Round	**4st PSA**				**5st PSA**			
Sedimentary Basin	Santos		Campos		Santos			Campos
Offered blocks	Três Marias	Uirapuru	Itaimbezinho	Dois Irmãos	Saturno	Titã	Pau-Brasil	Sudoeste de Tartaruga Verde
# Offered blocks	1	1	1	1	1	1	1	1
Bidded blocks	1	1	No offers	1	1	1	1	1
Minimum exceeding (profit) oil (%)	8.32%	22.18%		16.43%	-	-	-	-
Offered exceeding (profit) oil (%)	49.95%	75.49%		16.43%	70.20%	23.49%	63.79%	10.01%
Average Local Content Requirement – Exploratory Phase	18.0%	18.0%	18.0%	18.0%	18.0%	18.0%	18.0%	18.0%
Average Local Content Requirement – Development Phase	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
Consortium	Operator Petrobras (30%), Chevron (30%) and Shell (40%)	Operator Petrobras (30%), Petrogal (14%), Statoil (28%) and Exxon (28%)		Operator Petrobras (45%), Statoil (25%) and BP (30%)	Operator Shell (50%), Chevron (50%)	Operator Exxon Mobil (64%), QPI (36%)	Operator BP (50%), Ecopetrol (20%), CNOOC (30%)	Petrobras (100%)

Therefore, ANP and the Federal Government have been conducting regulatory changes to make Brazil´s oil industry more attractive in the country, expand exploration areas and encourage new investors (ANP, 2018e)[15]. As non-technical uncertainties can be greater inhibitors to investment, some works have analyzed the regulatory framework and fiscal regime in Brazil (Braga and David, 2018; Costa et al., 2018; Mariano et al., 2018a). Costa et al. (2018) argue that regulatory issues may retard investments in the upstream sector: unitization, local content, arbitration, and government take. According to Braga and David (2018), technical, legal and commercial factors dampen the unitization process, which is further complicated in the Brazilian pre-salt zone by those three fiscal regimes mentioned earlier — production-sharing agreements, concession agreements, and onerous assignment agreements. Mariano et al. (2018) highlight the regulatory challenge of establishing criteria for the unitization and individualization of production in pre-salt areas already tendered before fields start to produce; otherwise, it could bring damages to the reservoirs resulting in lower recovery factors.

Moreover, the Federal Law No. 13,365/2016 excluded the obligation of Petrobras to act as the exclusive pre-salt operator while setting out Petrobras's pre-emptive right. Upon exercising the pre-emptive right, the minimum share in the consortium would be 30%. In May 2017, Federal Decree No. 9,041 regulated the preference of Petrobras to act as operator in the consortia formed for the exploration and production of blocks to be contracted under

[15]Among these changes, they have eased some rules for the latest bid round of blocks, such as the removal of the minimum local content as a basis of offer in the bidding criteria; distinct royalties for new frontier areas and mature basins of more substantial risks; and incentives to increase the participation of small and medium-sized companies. Besides, ANP has established the open acreage, which consists of the permanent offer of relinquished fields (or in the process of relinquishment) and exploration blocks offered in past bids that were not awarded.

PSA. On March 28, 2017, the Industry and Competitiveness Development Secretariat of the Brazilian Ministry of Industry and Foreign Trade (MDIC) published Resolution No. 1 proposing reforms to the Local Content Requirements for the 3rd Production Sharing Bidding Round. The reforms have lowered the percentage of Brazilian-made goods and services required for oil and gas E&D.

Furthermore, at the end of 2017, the tax legislation applied to the Brazilian oil and gas industry was reviewed. This study summarizes Brazil's latest tax legislation updates for the oil and gas industry, obtained from ITR (2018). The Federal Law No. 13,586/2017 extended the suspension and relief of federal taxes and administrative fees levied on the importation of goods used in oil and gas activities until December 31, 2040 (both formerly scheduled to end on December 31, 2020). It also establishes a new special tax regime waiving federal taxes on importation, on a permanent basis, of goods to be used in E&D activities. The Normative Instruction No. 1,781/2017 introduced this new special tax regime and the so-called "new Repetro (Repetro-Sped)," according to which some equipment would benefit from tax relief in permanent importation only, and others would benefit from tax relief on permanent or temporary importation.

Then, through the ICMS Agreement No.3, the National Council of Fiscal Policy authorized at the beginning of 2018 the Brazilian states to grant reductions in the basis of calculation and exemption from ICMS — State Tax on Circulation of Goods and Services — levied on transactions involving goods used in upstream activities and carried out under Repetro-Sped.

These tax exemption measures are in line with Kleinberg et al. (2018), who states that when oil prices fall, Governments tend to make tax concessions to maintain the viability of their industry, and not to drive producers out of business or to other countries. On the other hand, the increasing social problems in Brazil in the light of an economic recession and commitments to a low carbon agenda arise the question if these fiscal incentives are genuinely needed to make pre-salt projects economically attractive.

Pedra and Szklo (2018) show that, given the current long-lasting fiscal incentives to the industry in Brazil, there are projects that do not need any fiscal incentives to be profitable, resulting in extra rent to the contractor in detriment of the State (or the Brazilian society).

2.1.3. Analysis of driven factors for well development

In the last decade, Brazil has become a significant producer of crude oil. By 2016, Brazilian oil production surpasses the output of Mexico and Venezuela (BP, 2017), which makes Brazil the largest oil producer in Latin America. Combined with increases in the price, crude oil is the second most crucial good (in value) exported by Brazil in 2018[16] (MDIC, 2018). Currently, Brazil is the world's tenth-largest producer of oil and the third-largest in the Americas (BP, 2018). Much of this success is due to the development of deep-water and ultra-deep-water projects, especially those in the pre-salt layer (EIA, 2016). Brazilian petroleum (oil and condensate) production from the pre-salt layer increases by 1.45 mbd between 2010 and 2018 (ANP, 2018a, 2018f). According to IEA (2018a), rising oil production from the US, Brazil, Canada, and Norway can supply the market through 2020.

[16] First data available for 2019 indicate crude oil became the first good (in value) exported by Brazil (MDIC, 2019).

After 2020, more investments will be necessary to maintain growth. However, additional investments can be disrupted by many factors such as financial issues, regulatory enforcement, cost of regulatory compliance, quality of infrastructure, quality of the geological database, legal system, uncertainty over environmental regulations and trade barriers (Stedman and Green, 2017).

In Brazil, a variety of elements can slow the development of an abundant resource base. These include the volatility of oil prices (Raza et al., 2016), which affect the market value of Brazilian companies (Raza et al., 2016). Hurn and Wright (1994) argue that after discovering a field, economic factors influence the decision to develop it. Oil production in the post-salt zones off the Brazilian coast deviates from the Hubbert curve based on oil prices with a lag of 4 to 5 years, which suggests that oil production from this layer adjusts slowly to oil prices (Hallack et al., 2017). Starting in September 2014, crude oil price volatility spills over onto sovereign credit risk (Pavlova et al., 2018). This result is corroborated by Bouri et al. (2018), who document significant effects of oil price volatility on sovereign risk in Brazil, Russia, India, and China (BRIC). Nevertheless, sovereign credit risk also depends on the efficiency of collecting fiscal revenues, which is referred to as government effectiveness (Jeanneret, 2018).

Understanding the factors that influence investment decisions is critical to offshore drilling in Brazil. There, the rig count for offshore wells, offshore wells completions, and completion of exploratory wells off the coast, decline since 2011 (Table 4). Despite these declines, production from the pre-salt zone increases with the number of oil producing wells (ANP, 2018f, 2018g).

Table 4: Rigs and wells drilled offshore in Brazil

	Rigs offshore	Wells concluded offshore	Exploratory wells concluded offshore	Development wells concluded offshore	Pre-salt development wells concluded
2010	434	212	92	120	1
2011	556	242	135	107	5
2012	502	219	106	113	21
2013	491	213	60	153	22
2014	317	156	48	108	35
2015	259	126	30	96	53
2016	140	93	12	81	39
2017	132	63	6	57	33

Source: Based on (Baker Hughes, 2018; ANP, 2018c, 2018f, 2018g)

Although there is no inherent contradiction between resource abundance and economic development, there are economic and political challenges (Oliveira, 2011). Unless institutions and governments are constrained by strong institutions, rents from the oil industry can generate adverse economic and political outcomes (Oliveira, 2017).

After the first significant discovery in a pre-salt layer in August 2006 (the Lula field), Brazil raises its perspective with the boom of commodities, overcoming global crisis' effects in 2009. Brazil outperforms the other BRIC nations in some aspects: "it is a democracy, it has no insurgents, no ethnic and religious conflicts, it exports more than oil and arms and treats foreign investors with respect" (The Economist, 2009). The positive characteristics are codified in a country of risk 3 (from a scale of 0 to 7) between 2007 and 2015 by the OECD.

After a period of strong growth (2002-2013), a drop in commodities prices, consumption, and investment (Société Générale, 2018) cause a recession, which slows economic growth in Brazil between the second quarter of 2014 and the fourth quarter of 2016 (IPEA, 2018), when the Gross Domestic Product (GDB) shrinks about 8%. This economic recession, corruption scandals, and loss of investment grade (The Economist, 2016), increases the score for national risk from 3 to 4 to 5 between 2016 and 2017 (OECD, 2018).

Despite the potential for the price, price volatility, productivity, and country risk to affect development in the deep-water pre-salt zone in Brazil, their effects have not been fully quantified. Much of this effort follows the methodology described by Ansari and Kaufmann (2019). The novelty of this thesis lies in the inclusion of the country credit risk as a measure of governance to estimate the cointegrated vector autoregressive (CVAR) models and the use of two benchmark crude oils to measure prices.

2.2. Technical background

2.2.1. Reserves and resources

The proper classification and categorization between reserves and resources are fundamental because this classification derives indicators that subsidize the company's business plan, information for investors and country’s policies.

The Stock-Tank Oil Initially in Place (STOIIP) [17] refers to the total volume of oil stored in a reservoir prior to production measured at surface pressure and temperature (as opposed to reservoir conditions). The recovery factor (RF) is the recoverable fraction of STOIIP.

The multiplication between STOIIP and RF results in the URR of oil, which represents the amount of oil from a region or field that is estimated to be recovered over time. This term includes any volumes that are estimated to be undiscovered, are not recoverable with current

[17] The HCIIP is an acronym for Hydrocarbons Initially in Place, and analogously GIIP is an acronym for Gas Initially in Place. To calculate the STOIIP, it is considered the bulk volume of the reservoir rock, the net/gross ratio of formation thickness, the porosity, the water saturation and the oil formation volume factor.

technology, and are not currently economic but which are expected to become so before production ceases (McGlade, 2013). That is, the URR is equivalent to the sum of cumulative production, the remaining reserves and resources already discovered and the estimated recoverable resources from undiscovered deposits (generally termed as 'yet-to-find').

The remaining recoverable volumes are the recoverable reserves and resources which have not yet been produced, i.e., the difference between the URR for a given region and that region's cumulative production. The oil that is not expected to become recoverable is not included in the URR but within the STOIIP. Figure 6 illustrates the different components of the URR.

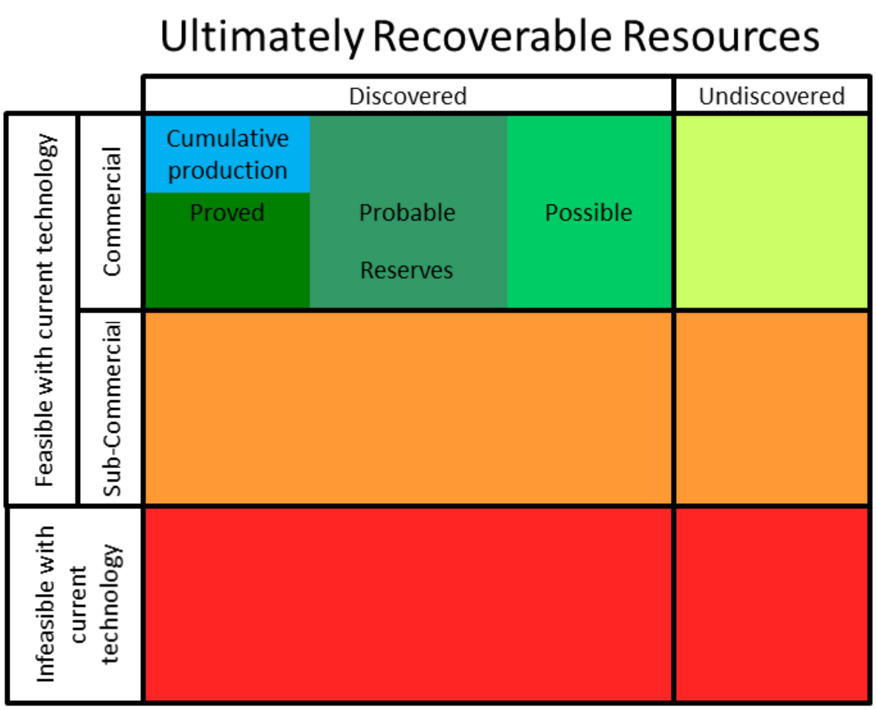

Figure 6: The relationship between reserves and resources
Source: Adapted from McGlade (2013)

The final subset of resources is 'reserves.' This term refers to the volume of discovered hydrocarbons (oil or gas) technically possible and economically feasible to recover and estimated to have a specific probability of being produced at a given time in the productive life of a reservoir. The term ‘resources’ refers to the volume of hydrocarbons, whether discovered or not, which are dependent on economic viability or technology development.

The Petroleum Resources Management System (PRMS) classifies and categorizes all hydrocarbon reserves and resources according to the chance of commerciality and the range of uncertainty in the quantities that are forecasted to be produced and sold in the future from a development project (SPE et al., 2018). The distinction between the three classes (reserves, contingent resources, and prospective resources) is based on the definitions of uncertainty and commerciality for projects (Figure 7).

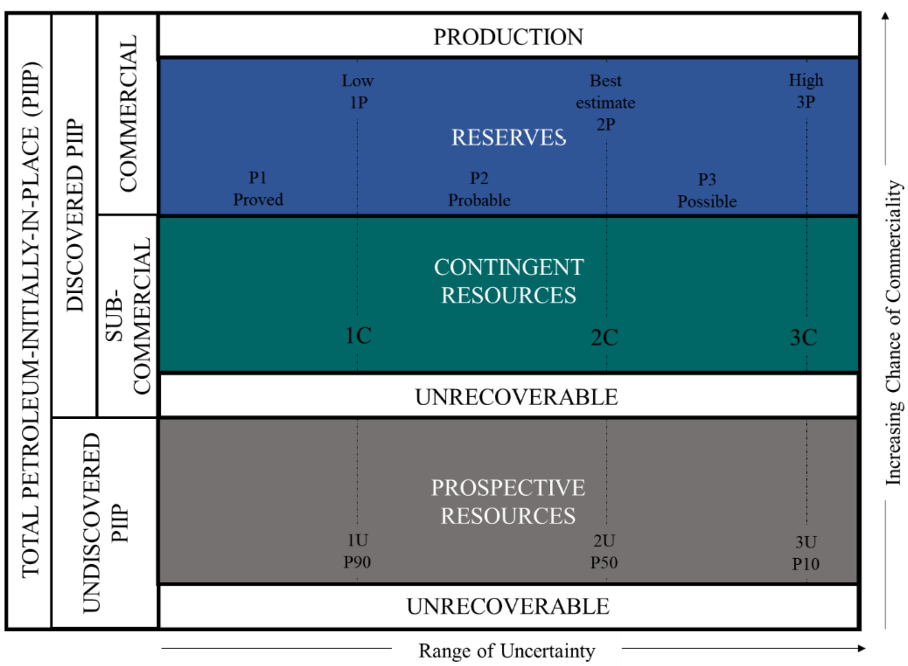

Figure 7: Resources classification framework from the PRMS
Source: Based on SPE et al. (2018)

Reserves may be assigned to projects that must satisfy four requirements: discovered, recoverable, commercial, and remaining[18] based on the development project(s) applied.

Contingent Resources may be assigned for projects that have an associated chance of development. Contingent Resources are those quantities of petroleum estimated to be potentially recoverable from known accumulations, by the application of development project(s) not currently considered to be commercial owing to some contingency.

Prospective Resources may be assigned to those quantities of petroleum estimated to be potentially recoverable from undiscovered accumulations by application of future development projects. Prospective Resources have both an associated chance of geologic discovery and a chance of development.

Unrecoverable Resources are those volumes of either discovered or undiscovered petroleum-initially-in-place (PIIP) evaluated to be unrecoverable by the currently defined project(s). "A portion of these quantities may become recoverable in the future as commercial circumstances change, technology is developed, or additional data are acquired. The remaining portion may never be recovered because of physical/chemical constraints represented by subsurface interaction of fluids and reservoir rocks" (SPE et al., 2018). According to SPE et al. (2011, p. 12), volumes are unrecoverable in two situations: (1) when the technology has been demonstrated to be commercially viable in other reservoirs that are not analogous, and there is no pilot project currently planned to demonstrate commerciality

[18] Cumulative production forecast from the effective date forward to cessation of production.

for this reservoir, (2) when the technology has not been demonstrated to be commercially viable and is not currently under active development, and/or there is not yet any direct evidence to indicate that it may reasonably be expected to be available for commercial application within five years.

As estimates of URR depends on assumptions about commercial viability and technical feasibility, it changes over time due to oil prices variation, geological knowledge improvement, and better knowledge of a technology that leads to RF increasing.

In oil companies, oil production forecast is often made individually for each reservoir based on geological models submitted to a flow simulator, which represents in detail the entire dynamic of the fluids (oil, gas, and water) in the reservoir.

Geologists, geophysics and engineering professionals are required to develop such models, as well as information from wells drilled in the area, seismic, laboratory tests, formation testing, as well as all information on surface facilities (e.g., platform capacity, manifolds, and pipelines). The use of these models considers the strategy of fields' exploitation: some producing and injecting wells, injected fluids (such as water, gas, and CO_2), well location, well geometry, use of advanced recovery methods, among other aspects.

These models need a refined view of the area, as they are used as a day-to-day decision-making tool for the field, and strategic decisions to optimize production.

As more knowledge about a region is gained, information becomes more accurate for the production forecast. The geological uncertainties are worked for each field probabilistically, generating production curves P10, P50 and P90, or deterministically, generating optimistic, base and pessimistic production curves. When the range of uncertainty is represented by a probability distribution, a low, best and high estimate shall be provided such that:

i) There should be at least a 90% probability (P90) that the quantities recovered will equal or exceed the low estimate;

ii) There should be at least a 50% probability (P50) that the quantities recovered will equal or exceed the best estimate;

iii) There should be at least a 10% probability (P10) that the quantities recovered will equal or exceed the high estimate.

In this context, besides the geological uncertainties, the reserve estimation process requires several judgments: how much is technically possible to extract and how much is economically feasible to extract.

Thompson et al. (2009) describe the following steps and uncertainties inherent in the reserve estimation process: geological assessment, engineering assessment, economic assessment, institutional influences, and political and market influences.

Within the geological assessment, geologists involved in the exploration and discovery stage make the initial estimate of oil contained in a reservoir. The most geologically promising regions for the existence of oil are identified. Then, an exploratory well is drilled in that region, allowing the verification of the existence of oil and the estimation of rocks properties. However, with the drilling of an exploratory well, it is necessary to judge the extent and size of the reservoir or field discovered.

This point can be very controversial since an area with several disconnected reservoirs can be interpreted as having a single larger field or several smaller fields. Moreover, this judgment may change over time, with previously distinct fields being merged into a larger field, and larger fields being divided into smaller ones.

Sorrell et al. (2009) define a field as an area consisting of a single reservoir or multiple reservoirs all related to a single geological structure, whereas a reservoir is defined as a subsurface accumulation of oil and gas which is physically separated from other reservoirs and which has a single natural pressure system. However, in the case of Brazil, the definition of a field is not geological, requiring only the sharing of production systems (Article 6 of the Law 9,478/1997).

However, the interpretation of a field's size affects the payment of government take, especially the special participation tax[19], which affects the cash flow of a field and consequently the estimation of the volume of reserves. Divergences between Petrobras and ANP are observed in Lula and Cernambi fields in Santos Basin (Ordoñez, 2014) but also in Parque das Baleias' fields (the Whale Park), in which ANP considers the concessions of Baleia Anã, Baleia Azul, Baleia Franca, Cachalote, Caxaréu, Jubarte and Pirambu as a single field (Petrobras, 2015a, 2015b). Such latter divergence resulted in an agreement between

[19] The special participation tax (the so called *Participação Especial* – PE) is considered a kind of Windfall Profit Tax, which is associated with Ricardian rents (Goldemberg et al., 2014). Such payments are calculated based on a reference oil price informed by ANP, which depends mainly on the quality of the oil from each producing field (Goldemberg et al., 2014). "Ricardian rent is a type of economic rent basically created by variation in resource quality" (Szklo et al., 2007). "Economic rent is a payment to a factor of production or input in excess of that which is needed to keep it employed in its current use" (Szklo et al., 2007). Essentially, windfall tax is a tax levied by governments when economic conditions allow industries to experience excess profits.

Petrobras and ANP on the unification of Parque das Baleias fields (Petrobras, 2019). These episodes reveal the geological assessment can lead to regulatory and legal issues.

Within the engineering assessment, petroleum engineers seek to estimate the RF after the estimation of STOIIP by geologists. Reserves estimation is based on existing technologies, but with the development of new technologies or with a better knowledge of reservoir response to the application of technology, reservoir's productivity and recovery can rise.

Hite et al. (2003) clear up some definitions concerning recovery techniques: primary recovery, secondary recovery, tertiary recovery, Enhanced Oil Recovery (EOR) and Improved Oil Recovery (IOR).

Fluids contained in a rock-reservoir must have natural energy so that they can be produced by the primary recovery mechanism, the main ones being: solution-gas drive, gas-cap drive, and water drive. In the production process, there is a dissipation of the primary energy by the decompression of the reservoir fluids and the resistances encountered as they flow towards the production wells.

Secondary recovery is the oil obtained by supplementing this primary energy. This additional energy derives from the use of injectants that re-pressurize the reservoir and displace oil to producers, usually by waterflooding, although gas reinjection for pressure maintenance is also possible.

Chemical, miscible and thermal flooding processes were developed for oil left behind or not recovered by the secondary recovery (usually waterflooding), referred to as tertiary recovery. Hite et al. (2003) consider the tertiary recovery as referring to the third round of recovery processes to be developed. EOR refers to the same tertiary methods, but it can be used as the first or second recovery process. For this reason, the concept of EOR has replaced tertiary recovery. IOR refers to any practice to increase oil recovery. It can include EOR processes, as well as practices for increasing sweep such as infill drilling, horizontal wells, and polymers.

The characterization, simulation, monitoring, management and control of reservoirs are considered support activities for any recovery method, although they can increase the RF and involve new technologies (Ferreira, 2016).

The critical point is that all these techniques mentioned are directed to reservoirs with specific characteristics, that is, not all techniques are appropriate or even implementable in all reservoirs. Moreover, the RF can vary widely, and even if it is estimated a minimum RF from technology, there is still uncertainty concerning the STOIIP (Thompson et al., 2009).

Within institutional influences, the different criteria used for the estimation of reserves and the different interpretations of the same criterion may lead to variations in the estimates of reserves of a country and a company.

In Brazil, Petrobras' reserves are disclosed accordingly to the guidelines of Resolution no. 47 disclosed by ANP (2014), which establishes that the criteria for classification as reserves and resources should follow the PRMS (SPE et al., 2011, 2018). Petrobras also estimates its proved reserves according to the criteria of the Securities and Exchange Commission – SEC (2008).

In 2017 Petrobras' petroleum proved reserves were 10,533 billion barrels according to the Society of Petroleum Engineers (SPE) and ANP criteria, whereas according to the SEC criteria proved reserves were 8,435 billion barrels.

Within economic assessment influences, the analysis of economic viability required to classify a volume as a reserve depends on the current and anticipated future price of oil, but also the estimated operational costs and required investments.

The oil price considered in calculating the economic feasibility of reserves is the main difference between ANP/SPE and SEC criteria volumes of proved reserves in 2017, according to Petrobras (2018d).

Another economic aspect that may interfere in the estimation of reserves is the level of aggregation in which the economic analysis is done. For example, analysis of cash flow at the field level may result in different reserve volumes than when this analysis is done by the level of the platform producing this field.

Within political and market influences, Thompson et al. (2009) comment that privately-owned companies can be subject to market incentives since reserve estimates and the rate of reserve additions can affect their share price. National Oil Companies (NOCs) and International National Oil Companies (IOCs) [20] have no similar responsibility to shareholders

[20] NOCs concentrate on domestic production and IOCs have both domestic and a significant international operation. These categories cover companies that are fully or majority-owned by national governments. Among the privately-owned companies This thesisdistinguish seven large international oil companies (referred to as the “Majors”) from the rest (referred to as “Independents”) (IEA 2013).

but may have political motives for choosing a particular definition or interpretation of reserves.

McGlade (2013) defines ‘political reserves’ as the volumes of oil announced by a country or company that do not coincide to the reserves it has but preferably those which it would like to communicate to the rest of the world.

Besides political interests and market influences, other political aspects can disrupt oil production. Bøe et al. (2018) argue that political factors can be particularly important because of the close link between politics and crude oil. They find that political instability[21] increases the expected time to invest, i.e., the time lag between discovery and government approval to develop a field. Bøe et al. (2018) assess political stability by The International Country Risk Guide (ICRG) Political Risk. The ICRG Political Risk includes twelve weighted variables covering both political and social attributes, such as corruption, the military in politics, religious tensions, socioeconomic conditions, ethnic tensions, and democratic accountability. Al-Kasim et al. (2013) review of the resource curse and oil production literature indicates the theoretical feasibility of a connection between instances of corruption and suboptimal oil production. Beyond political instability, Oliveira et al. (2018) highlight the political and economic crisis in Brazil makes it more challenging to predict sources of risk in the period 2014 to 2016.

[21] Political stability can be defined as a predictable political environment. Feng (1997) investigates the interactions between democracy, political stability and economic growth. He differentiates between three types of political instability: ‘irregular’ government change (regime-level change); ‘major regular’ (within-regime) government change; and ‘minor regular’ (within-regime) government change.

2.2.2. Hubbert

Hubbert's theory of oil depletion (Hubbert, 1956) is the pioneer curve-fitting model. Hubbert's well-known projection of the US future oil production, proposed in the late 1950s, used a bell-shaped curve and showed to be accurate once US oil production in the lower 48 states peaked in 1970 (Laherrère, 1997) and, therefore, motivated a variety of Hubbert-like curve fitting models.

Hubbert's theory for modeling oil production is based on physical phenomena. He assumes that the first discovery well is drilled, and oil production begins. Then, as additional wells are drilled and the rate of production increases, further exploration is stimulated, and new fields are discovered. However, as more and more fields are discovered, and the number of fields is fixed, the last fields are the most expensive and the smallest ones. Finally, the undiscovered fields become too scarce to justify exploratory drilling (Hubbert, 1982).

Hubbert assumes the cumulative discovery cycle has the same format as the cumulative production cycle, both following a logistic function, with the former preceding the latter by some time interval (Figure 8). Thus, when the discovery rate begins to decline, as the production rate continues to rise, the reserve additions (until then increasing) start to decrease, and when production rates overpass the discovery rate, then reserve additions (until then positive but already decreasing) become negative (Figure 8).

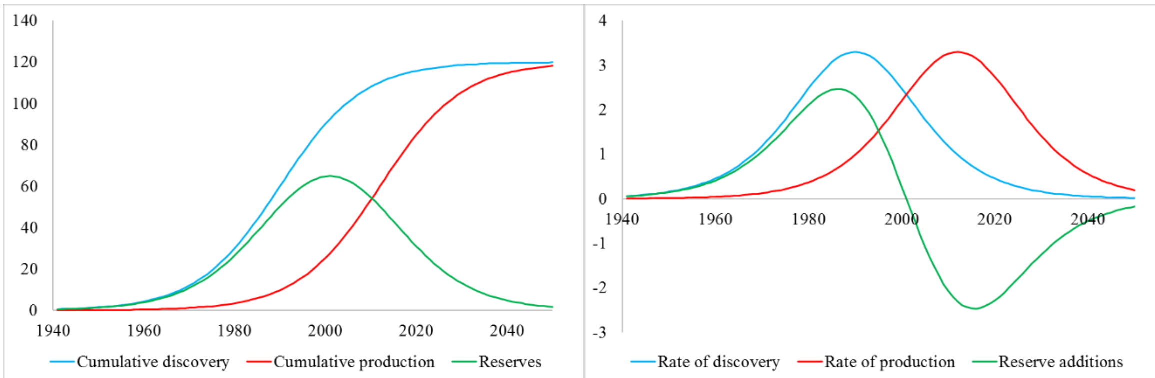

Figure 8: Hubbert's relationship between discoveries, production, and reserves as a function of time

Besides the symmetrical and single cycle production profile considered by the Hubbert logistic model, Hubbert (1982) recognized that the curve could have more than a single maximum or be asymmetrical.

Several authors have used Hubbert models (or their variants) to forecast world oil production (Al-Jarri and Startzman, 1997; Bartlett, 2000; Brecha, 2012; Campbell and Laherrère, 1998; Gallager, 2011; Hubbert, 1962; Maggio and Cacciola, 2009; Nashawi et al., 2010; Rehrl and Friedrich, 2006; Reynolds, 2014; Wang et al., 2011) and some of these studies are based on Hubbert to assess the production in single countries or regions in parallel with worldwide projections (Table 5).

Table 5: Studies that used the Hubbert methodology to project a country's oil production

Country	**Reference**
US	Hubbert (1956)
The former Soviet Union	Laherrère (2002)
France and the Netherlands	Laherrère (1997)
Brazil	D. Ferreira (2005); Szklo, Machado, and Schaeffer (2007); Rueda et al. (2013); Saraiva, Szklo, and Chavez-Rodriguez (2014)
Main fields of the North Sea	Blanchard (2000)
Nigeria	Kingsley-Akpara and IIedare (2014)
Organization of the Petroleum Exporting Countries (OPEC)	Nashawi, Malallah, and Al-Bisharah (2010); Ebrahimi and Ghasabani (2015)
Colombia	Mahecha (2014)
Peru	Chavez-Rodriguez, Szklo, and Lucena (2015)
The United Kingdom and Norway	Fiévet et al. (2015)

As the Hubbert theory of oil depletion states that oil production in large regions follows a bell-shaped curve over time, Brandt (2007) tested the quality of fit of Gaussian, linear and exponential models, being symmetric or asymmetric, to oil production data of 139 producing areas. The results showed that the asymmetrical exponential model is the most useful model and that they show better fits than the symmetric models in most cases, with slower rates of decline than rates of increase.

The multi-Hubbert cycle analysis of oil production in the US developed by Patzek (2008) emphasizes the existence of new populations of reservoirs, in which the main cycle provides the original Hubbert estimate of URR. The smaller cycles describe the new populations of reservoirs, for example in Alaska, the Gulf of Mexico, the Austin Chalk and the California Diatomites, and new recovery processes, such as waterflood, EOR, and horizontal wells. In this regard, multi-cycles can be especially useful to model new reservoirs.

Multi-cycle approaches can explain the production patterns in many countries, which have more than one peak in their production profiles, as shown by Nashawi, Malallah, and Al-Bisharah (2010). Nevertheless, these authors recognize that oil production is affected by ecological, economic, and political factors over the years. Hubbert’s theory embodies the physical aspects of oil formation, but economic and political events may cause annual rates of production to deviate from Hubbert’s curve in a systematic way (Kaufmann, 1991). Once little effort is spent to explore and produce oil resources unless a cost-effective recoverability can be expected, all discovery and production cycles depend on the expected profitable recoverability of the oil resources (Rehrl and Friedrich, 2006).

According to Kaufmann and Cleveland (1991), much of the success of Hubbert's approach could be explained by the fact that the price of oil (benchmark), the average cost of production, and the decisions of the Railroad Commission of Texas (RCT) (2016) have evolved in a way to allow for a symmetrical bell-shaped curve of production over time in the US. A different pattern of any of these variables would have resulted in a curve different from a bell-shaped curve. In brief, the study highlights the importance of economic and institutional aspects of oil production dynamics.

Pesaran and Samiei (1995) argue that when economic factors ─ the same analyzed by Kaufmann and Cleveland (1991) ─ are considered, estimates of URR vary over time and are higher than those obtained when the economic aspects are neglected, i.e., as in the traditional Hubbert model.

Kaufmann (1991) proposes a two-stage approach to analyze the impact of geological, economic, political, and institutional variables on the production of 48 American states by combining the Hubbert curve fitting with econometric methods. The first stage consists of fitting cumulative oil production data and a logistic curve based on the methodology developed by Hubbert. In the second stage, the difference between actual rates of production and rates of production predicted by the bell-shaped production curve is used as a dependent variable in an econometric model in which political and economic factors attempt to explain the deviation of the actual production data in relation to the Hubbert model, i.e., these factors serve as independent variables.

A single Hubbert approach was proposed by Szklo, Machado, and Schaeffer (2007) for Brazil. Furthermore, Saraiva, Szklo, and Chavez-Rodriguez (2014) estimated Brazil’s oil production curves with different URR scenarios by adding productive cycles following a Hubbert variant proposed by Maggio and Cacciola (2009).

To improve the previous analyses and test a variant of the Hubbert model for the case of Brazil ─ that could also be adopted in other countries ─ this study developed:

i) Primarily, single and multi-cycle Hubbert models to project Brazilian post-salt oil offshore production, including asymmetrical production cycles ─ based on the adapted methodology proposed by Brandt (2007) ─ to estimate endogenously the URR in post-salt oil offshore production cycles;

ii) Secondly, a hybrid model ─ based on the methodology developed by Kaufmann (1991) ─ that considers the influence of techno-economic parameters to production cycles seeking to understand how these parameters influence the residuals of the Hubbert model.

iii) Thirdly, a single Hubbert model to estimate the pre-salt oil production peak accordingly to different scenarios of URR.

The classic Hubbert methodology was improved in this study by considering the asymmetry of production, which arises if the ramp-up does not follow the same dynamics as the decline in production. Additionally, a hybrid model considering techno-economic aspects aims to explain the deviation of the crude oil production rate from the prediction of the Hubbert curve employing regression analysis. Preliminary estimates of pre-salt production peak are obtained from URR scenarios derived from different RF.

2.2.3. Creaming curves

Creaming curves can assist to understand the exploratory cycles (e.g., to propose multiple cycles or to identify influences leading to deviations from the logistic form) and indicate the marginally decreasing productivity of an exploratory cycle (e.g., to forecast future discoveries). The creaming curve format is presented by the cumulative discovery cycle in Figure 8.

Such curves have been used concerning plotting the cumulative size of discoveries over time or against the number of new field wildcat wells. Some authors, in contrast, consider 'the true creaming curve' as the cumulative discovery against the number of new field wildcats, to eliminate the ups and downs of exploration when plotting versus time (Laherrère, 2004; Bentley et al., 2007).

Cumulative discoveries result from the sum of cumulative production and reserves. It consists of the amount of discovered oil that is estimated to be recovered over time. Reserves refer to the volume of hydrocarbons technically possible and economically feasible to recover, and that is estimated to be produced at a given time in the productive life of a reservoir with a certain probability.

The extrapolation of cumulative discoveries anticipates the yet-to-find discoveries. However, it does not consider the phenomenon of reserve growth, by which fields ultimately produce more oil than was initially estimated as reserves (Sorrell et al., 2009). McGlade (2013) presents the principal drivers of reserve growth, among them: i) growth due to improvements in, or the application of new production technologies; ii) better understanding of the reservoir geology; iii) upward changes in oil prices or reductions in production costs; iv) reserves definitions changing and/or the inclusion of new/revised data in reporting estimates.

Sorrell et al. (2009) indicate that the extrapolation of discovery trends should provide more reliable estimates of the URR than production trends because the discovery cycle is more advanced, so discoveries' data are available sooner than production data. Since the peak rate of discovery anticipates the peak in production, identification of the former allows for predicting the later (Sorrell et al., 2009). On the other hand, discovery data is less accessible than production data, and it is estimated according to different levels of confidence – the uncertainty in the potential recovery from a project leads to the subdivision of reserves into a low (1P), best (2P) and a high (3P) estimate[22].

The curve-fitting methods have an essential role to play when field-level data is not accessible and also have much in common with more sophisticated techniques (Sorrell et al., 2009). Such methods are better applied to geologically homogenous and well-explored areas (Sorrell et al., 2009). Consequently, this thesis relies on fields within the same basin and the basins where there have been a reasonable number of wildcat wells concluded.

Although some limitations of 'curve-fitting' methods, several curve-fitting studies have applied the Hubbert model to predict future trend in fossil fuel production worldwide (Laherrère, 1997; Blanchard, 2000; Laherrère, 2002; Mohr and Evans, 2009; Maggio and Cacciola, 2009; Höök et al., 2010; Nashawi et al., 2010; Maggio and Cacciola, 2012;

[22] This thesisclarify that 1P reserves refer to proven reserves (or P90 estimate, whereby there is an estimated 90% probability that the actual Reserves will lie somewhere between the P90 and the P0 (maximum) outcomes). Analogously, 2P reserves refer to proven and probable reserves (P50), and 3P to proven, probable and possible reserves (P10).

Kingsley-Akpara and IIedare, 2014; Chavez-Rodriguez et al., 2015; Fiévet et al., 2015; Ebrahimi and Ghasabani, 2015), including Brazil (Szklo et al., 2007; Saraiva et al., 2014; Hallack et al., 2017), but just a few have assessed the exploration history through creaming curves (Bentley et al., 2007; Laherrère, 2008, 2009; Soderbergh et al., 2009; Beglinger et al., 2012; Hackley and Karlsen, 2014; Korsvold, 2015; Chavez-Rodriguez et al., 2016).

Bentley et al. (2007) show Germany's oil creaming curve has been flattening out since about 1960. Soderbergh et al. (2009) notice that the fields discovered in Norway are getting smaller and smaller by applying a creaming curve for Norwegian gas discoveries. Laherrère (2008) applies a creaming curve for four fields shared with the United Kingdom: Frigg, Statfjord, Peik, Alpha. Assuming a new significant cycle is unlikely, he finds the yet-to-find oil and gas discoveries will be of small size. Laherrère (2009) plots creaming curves by continents, but at the time of his study, no pre-salt discoveries in Brazil were used as input within Latin America's curve of historical discoveries. Beglinger et al. (2012) made predictions on the remaining potential for yet undiscovered hydrocarbons accumulations in West African South Atlantic basins. They find that the Douala basin appears to be under-explored with relatively small discoveries. The Rio Muni basin potential remains poorly defined. In the North Gabon syn-rift section and the post-rift section of South Gabon, explorations appear to be immature. Basin future discoveries are expected mainly in ultra-deep waters for the Lower Congo. Exploration in the Congo Fan basin is immature for both oil and gas. Kwanza basin remains substantially undrilled. For the onshore northern Gulf of Mexico basin (in the US), Hackley and Karlsen (2014) find that cumulative discovered oil volumes follow a mature creaming curve with the size of discoveries decreasing with advancing exploration. Korsvold (2015) proposes an analysis of the creaming phenomenon as a way of indicating government influence through past exploration trends in Norway. Chavez-Rodriguez et al. (2016) observe the shape of the creaming curve for Bolivia denotes its immaturity concerning exploration.

It is observed that the bulk of literature regarding the creaming curve method is skewed towards developed countries. To partly fills this research vacuum, this thesis assesses future exploration potential in Brazil building 'true creaming curves.'

2.2.4. Econometric theory

Some studies have applied econometric models of oil supply (Table 6), although its use has not been popular worldwide.

Table 6: Studies applying an econometric methodology

Country	**Reference**
World (major oil producers)	Pickering (2008)
UK	Pesaran (1990); Pickering (2002)
US	Uri (1982)
US- Lower 48 states	Kaufmann (1991); Pesaran and Samiei (1995); Moroney and Berg (1999); Kaufmann and Cleveland (2001)

This section describes briefly the econometric basis which supports the application of two techniques. The first one is a regression model to assess the influence of techno-economic parameters to the post-salt oil production in Brazil, inspired by Kaufmann (1991). The second one is a series of CVAR models to quantify how prices, price volatility, well productivity, and country risk affect the number of development wells that are completed in the pre-salt zones of Brazil, inspired by Ansari and Kaufmann (2019).

Econometric datasets come in a variety of types, one of them is time series data. A time series data set consists of observations on a variable or several variables over time. The influence of past events on future events and the prevalence of lags in the social sciences make time an essential dimension in a time series data set (Wooldridge, 2013). A sequence of random variables[23] (i.e., its values cannot be controlled or known a priori) indexed by time is called a stochastic process. The stochastic process is stationary if its mean and variance are constant over time and the value of the covariance between two periods depends only on the distance or lag between the two periods and not on the actual time at which the covariance is computed.

The econometric theory below is presented according to Gujarati and Porter (2009). Symbolically, letting Y_t represent a stochastic time series, it is stationary if the following conditions are satisfied:

$$\text{Mean:} \quad E(Y_t) = \mu \tag{1}$$
$$\text{Variance:} \quad E(Y_t - \mu)^2 = \sigma^2 \tag{2}$$
$$\text{Covariance:} \quad \gamma_k = E[(Y_t - \mu)(Y_{t+k} - \mu)] \tag{3}$$

Where γ_k, the covariance at lag k, is the covariance between the values of Y_t and Y_{t+k}, that is, between two values of Y, k periods apart.

A non-stationary time series has a time-varying mean or a time-varying variance or both. In this case, its behavior can only be studied for the period under consideration, not being possible to generalize it to other periods. Therefore, using trending variables in regression analysis may result in spurious results, in the sense that superficially the results look good, but on further investigation they are suspicious.

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[23] If there is at least one value of r for which $0<p(y=r)<1$, a discrete variable is said to be random variable. If there is some r for which $p(y=r)=1$, y is deterministic rather than random.

Among some tests of stationarity, the unit root test has become widely popular over the past several years. Letting Y_t represent the stochastic time series of interest, that is:

$$Y_t = \rho Y_{t-1} + u_t \qquad -1 \leq \rho \leq 1 \tag{4}$$
$$Y_t - Y_{t-1} = (\rho - 1)Y_{t-1} + u_t \tag{5}$$
$$\Delta Y_t = \delta Y_{t-1} + u_t \tag{6}$$
$$\Delta Y_t = \beta_1 + \delta Y_{t-1} + u_t \tag{7}$$
$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + u_t \tag{8}$$

Where u_t is a white noise error term.[24] The Dickey-Fuller (DF) test is used to find out if the estimated coefficient of Y_{t-1} in Equation 6 is zero. Dickey and Fuller have shown that under the null hypothesis that δ=0 (i.e. $\rho = 1$), the t value of the estimated coefficient of Y_{t-1} in Equation 6 follows the τ (tau) statistic. The DF test is estimated under three different null hypotheses, presented by Equation 6, 7 and 8. In each case, the null hypothesis is that δ=0, which is another way of saying there is a unit root and the time series is nonstationary. The alternative hypothesis is that δ is less than zero. If the null hypothesis is rejected, it means Y_t is a stationary time series with zero mean (case of Equation 6), Y_t is stationary with a nonzero mean (case of Equation 7), and that Y_t is stationary around a deterministic trend (case of Equation 8). In the DF test, it was assumed that the error term u_t was uncorrelated. In case u_t are correlated, the augmented Dickey-Fuller (ADF) test is used. This test is conducted by "augmenting" the three previous equations by the lagged values of the dependent variable ΔY_t. In this case, the ADF test consists of estimating the following regression:

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + \varepsilon_t \tag{9}$$

[24] An error term with the following properties is called a white noise error term: E(u_t)=0; var(u_t)=σ_u^2; cov(u_t, u_{t+s})=0, s≠0. That is, a sequence u_t is a "white-noise process if each value in the sequence has a mean of zero, a constant variance, and is serially uncorrelated" (Enders, 1995).

Where ε_t is a pure white noise error term and $\Delta Y_{t-1} = (Y_{t-1} - Y_{t-2}), \Delta Y_{t-2} = (Y_{t-2} - Y_{t-3})$, and so on. In the ADF test, whether δ=0 is still the null hypothesis verified.

An important assumption of the classical linear regression models is that the variance of each disturbance term u_t is some constant number equal to σ^2, that is the assumption of homoscedasticity. Symbolically, var(u_t)= σ^2, t=1, 2…, n. Applying the method of ordinary least squares (OLS) in the presence of heteroscedasticity are likely to give inaccurate results because the confidence intervals based on the variance of OLS estimators will be unnecessarily larger leading to t and F tests' incorrect results. In short, if the usual testing procedures carry on to be used despite heteroscedasticity, whatever conclusions may be misleading (Gujarati, 2003).

The Breusch-Pagan (BP) tests for heteroskedasticity in a linear regression model. It tests whether the variance of the errors from a regression is dependent on the values of the independent variables, in this case, heteroskedasticity is present. If the test statistic has a p-value below an appropriate threshold (e.g., $p<0.05$) then the null hypothesis of homoskedasticity is rejected, and heteroskedasticity assumed.

Another assumption of the classical linear regression models is that there is no correlation between the two error terms u_i and u_j (disturbances). This means that given any two X values, X_i and X_j, the correlation between any two u_i and u_j is zero. Symbolically, $cov(u_i,u_j|X_i, X_j)=0$, where i and j are two different observations, and cov stands for covariance. In other words, the classical model assumes that the disturbance term relating to any observation is not influenced by the disturbance term relating to any other observation. "Under both heteroscedasticity and autocorrelation, the usual OLS estimators, although linear, unbiased, and asymptotically normally distributed, are no longer minimum variance among all linear unbiased estimators. Put differently; they are not efficient relative to other linear and unbiased estimators. As a result, the usual t, F and χ^2 may not be valid" (Gujarati and Porter, 2009, p. 413).

A two-variable regression model can illustrate the Breush-Godfrey (BG) test of autocorrelation (also known as the LM test) (Equation 10).

$$Y_t = \beta_1 + \beta_2 X_t + u_t \qquad (10)$$

If the error term u_t is assumed to follow the pth-order autoregressive, AR(p) scheme as follows (Equation 11):

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \cdots + \rho_q u_{t-p} + \varepsilon_t \qquad (11)$$

In which ε_t is a white noise error term. The null hypothesis H_0 to be tested is that H_0: $\rho_1 = \rho_2 = \cdots = \rho_p = 0$. That is, there is no serial correlation of any order.

However, any heteroscedasticity or autocorrelation test has yet been judged to be unequivocally more potent in the statistical sense. Therefore, this study applied the BP test and BG test to detect, respectively, heteroskedasticity and autocorrelation. The Newey-West method can be applied to obtain standard errors of OLS estimators that are corrected for autocorrelation and heteroskedasticity.

The classical linear regression models also assume that there is no perfect multicollinearity, that is, there are no perfect linear relationships among the explanatory variables. The term multicollinearity refers to the existence of a "perfect" (Equation 12) or "not perfectly so"

(Equation 13) linear relationship among some or all explanatory variables of a regression model.

$$\lambda_1 X_1 + \lambda_2 X_2 + \cdots + \lambda_k X_k = 0 \tag{12}$$

$$\lambda_1 X_1 + \lambda_2 X_2 + \cdots + \lambda_k X_k + \nu_i = 0 \tag{13}$$

Where ν_i is a stochastic error term. If multicollinearity is perfect (Equation 12), the regression coefficients of the X variables are indeterminate, and their standard error is infinite. If multicollinearity is less than perfect (Equation 13), the regression coefficients, although determinate, possess large standard errors which means the coefficients cannot be estimated with high precision or accuracy.

Despite the problems that multicollinearity poses to the classical linear regression models, multicollinearity does not lead to imprecise estimates of the cointegration relations (Juselius, 2018). The CVAR theory – cointegrated vector autoregressive – forthcoming is presented according to Enders (1995) and based on Kaufmann and Juselius (2013). For more detailed information about the CVAR model, see Juselius (2007).

Due to the obstacles posed by strong correlations among trending variables to a better understanding of economic systems, economists developed statistical techniques based on the idea of cointegration and error correction. This approach allows evaluating whether correlations among trending variables correspond to statistically meaningful long-run relations as measured by the cointegrating relations and to detect the dynamics by which variables adjust to deviations from these long-run relations back to equilibrium after having been pushed away by exogenous shocks, as measured by the error correction. The power of this methodology is the basis for Clive Granger's 2003 Nobel Prize in economics.

Cointegration is a statistical concept defined as a stationary linear combination among nonstationary variables. A cointegration has the property that it eliminates a common stochastic trend among stochastically trending variables.

Stochastic trends can be eliminated partly by differencing, that removes the long-run information, and partly by cointegration (linear combinations of time series that cancel the stochastic trend), that ensures the long-run information is preserved. The CVAR model built by combining the first difference terms and cointegration describes short-run adjustment and long-run relation in the data. It enables the correct use of standard inference based on (χ^2, F, t), and ensures the validity of R^2.

Briefly, it is possible to summarize two advantages of the CVAR model over the classical linear regression models: (1) there can be multicollinearity among explanatory variables, and (2) it eliminates a common stochastic trend among trending variables preserving the long-run information.

The conventional wisdom was to differentiate all nonstationary variables used in a regression analysis so nonstationary (trending) series could become stationary by differencing. It is now recognized that is possible there to be a linear combination of integrated variables that is stationary; these variables are said to be cointegrated. According to Enders (1995), "any equilibrium relationship among a set of nonstationary variables implies that their stochastic trends must be linked". Such an equilibrium relationship means that the variables cannot move independently from each other (Enders, 1995, p. 355).

To illustrate the theory of CVAR models, Equation 14 specifies the first example for the number of wells drilled in a region.

$$w_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + \beta_3 r_t + e_t \qquad (14)$$

Where w_t is the long-run number of wells drilled, p_t is the oil price level, y_t is the yield, r_t is the interest rate, e_t is the stationary disturbance term and β_i are parameters to be estimated, being the variables price level, yield and interest rate nonstationary integrated of order one[25], I(1). The theory expressed in Equation 14 asserts that there exists a linear combination of these nonstationary variables that is stationary, shown in Equation 15.

$$e_t = w_t - \beta_0 + \beta_1 p_t + \beta_2 y_t + \beta_3 r_t \qquad (15)$$

The linear combination of integrated variables in the right-hand side of Equation 15 must be stationary since $\{e_t\}$ is stationary. Such a linear combination of nonstationary variables illustrates the concept of cointegration. Equation 16 presents when a set of economic variables are in long-run equilibrium, according to Engle and Granger (1987).

$$\beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \cdots + \beta_n x_{nt} = 0 \qquad (16)$$

In which β denotes the vector $(\beta_1, \beta_2, \beta_3, \ldots \beta_n)$ and x_t denotes the vector $(x_{1t}, x_{2t}, x_{3t}, \ldots, x_{nt})'$. The system is in long-run equilibrium when βx_t=0. The deviation from long-run equilibrium is called equilibrium error (e_t), so that:

$$\beta x_t = e_t \qquad (17)$$

[25] If a nonstationary time series has to be differenced *d* times to make it stationary, that time series is said to be integrated of order *d*, I(d).

The components of the vector x_t are said to be cointegrated of order d, b, denoted by CI(d,b) if all components of x_t are integrated of order d and there exists a vector β such that the linear combination $\beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \cdots + \beta_n x_{nt}$ is integrated of order (d-b), where b>0. The vector β is called the cointegrating vector. In terms of Equation 15, the variables price level, yield and interest rate are cointegrated of order (1,1), that is CI(1,1), if e_t is stationary and such variables are nonstationary I(1).

If x_t has n components, there may be as many as n-1 linearly independent cointegrating vectors. The number of cointegrating vectors is known as the cointegrating rank of x_t. A characteristic of cointegrated variables is that their time paths are influenced by the deviation from long-run equilibrium, which influences the short-run dynamics. This dynamic model is the one of error correction. Deviation from equilibrium influences the short-term dynamics of variables in a system. For instance, Equation 18 and Equation 19 illustrates a simple error correction model.

$$\Delta r_{st} = \alpha_S (r_{Lt-1} - \beta r_{St-1}) + \epsilon_{St} \qquad \alpha_S, \beta > 0 \tag{18}$$

$$\Delta r_{Lt} = -\alpha_L (r_{Lt-1} - \beta r_{St-1}) + \epsilon_{Lt} \qquad \alpha_L, \beta > 0 \tag{19}$$

Where r_{Lt} and r_{st} are the long- and short-term interest rates, respectively. The terms $\epsilon_{St}, \epsilon_{Lt}$ are white-noise disturbance terms, and by assumption Δr_{st} and Δr_{Lt} are stationary. The Equation 18 shows that the short-term interest rate changes in response to the stochastic shock ϵ_{St} and to the previous period's deviation from long-run equilibrium. Everything else equal, if $r_{Lt-1} - \beta r_{St-1} > 0$ the short-term interest rate would rise, and the long-term interest rate would fall. The long-run equilibrium is observed when $r_{Lt} = \beta r_{St}$. Notice that α_S and

α_L interprets the *speed of adjustment* parameters; the larger α_S is (in absolute terms), the greater the response of r_{st} to the previous period deviation from long-run equilibrium. In the case of Equation 18, if the *speed of adjustment* coefficient α_S is considered statistically equal to zero, then the $\{\Delta r_{st}\}$ is unaffected by the long-term interest rate. Enders (1995) generalizes the relationship between cointegration, error correction, and cointegration rank, resulting in Equation 20:

$$x_t = A_1 x_{t-1} + \epsilon_t \tag{20}$$

Where x_t is a (n x 1) vector $(x_{1t}, x_{2t}, \dots, x_{nt})'$, ϵ_t is the (n x 1) vector $(\epsilon_{1t}, \epsilon_{2t}, \dots, \epsilon_{nt})'$, A_1 is an (n x n) matrix of parameters. Subtracting x_{t-1} from each side of Equation 21 and letting I be an (n x n) identity matrix, it is obtained:

$$\Delta x_t = -(I - A_1)x_{t-1} + \epsilon_t = \pi x_{t-1} + \epsilon_t \tag{21}$$

Where π is the (n x n) matrix $-(I - A_1)$ and π_{ij} denotes the element in row i and column j of π. If the rank of the (n x n) matrix π is zero, then Equation 21 becomes equivalent to an n-variable VAR in first differences. If the rank is one (r=1), there is a single cointegrating vector given by any row of the matrix π. In this case, each sequence $\{x_{it}\}$ can be written in the error-correction form (Equation 22):

$$\Delta x_{1t} = \pi_{11} x_{1t-1} + \pi_{12} x_{2t-1} + \cdots + \pi_{1n} x_{nt-1}\, \epsilon_{1t} \tag{22}$$

This work applies a CVAR model to evaluate the role that oil prices, oil price's volatility, well productivity and country risk play in the development of pre-salt using the ideas of cointegration and equilibrium error correction. To quantify the oil price's volatility, this thesis estimates Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) models of crude oil prices.

To quantify the oil price's volatility, this thesis estimates GARCH models of crude oil prices. That is, just as the error term u_t follows the pth-order autoregressive AR(p) scheme presented by Equation 11, there can be autocorrelation in the variance of the error term at time t with the squared values of the error term lagged one or more periods. The idea is similar to the autocorrelation of the error term, but in the Autoregressive Conditional Heteroskedastic (ARCH) model "it is the (conditional) variance of u_t that depends on the (squared) previous error terms, thus giving the impression of autocorrelation" (Gujarati and Porter, 2009, p. 794).

The variance of the disturbance term is assumed to be constant in conventional econometric models. However, the assumption of a constant variance (homoskedasticity) is inappropriate for several econometric time series that exhibit periods of large volatility and tranquility. In these cases, one approach that helps to forecast the variance is to introduce an independent variable (Equation 23).

$$y_{t+1} = \epsilon_{t+1} x_t \tag{23}$$

Where y_{t+1} is the variable of interest, ϵ_{t+1} is a white-noise disturbance term with variance σ^2, x_t is an independent variable that can be observed at period t. When the realizations of the $\{x_t\}$ sequence are not all equal, the variance[26] of y_{t+1} conditional on the observable value of x_t is Var($y_{t+1}|x_t$)=$x_t^2\sigma^2$.

[26] Let X be a random variable, $Var(X) = \sigma_X^2 = E[X - E(X)]^2$. The expected value of y_{t+1}, $y_{t+2}\ldots$, conditioned in the observed values of y_1 through y_t is a conditioned mean or expected value of y_{t+i}. This is denoted by $E_t(y_{t+i}|y_t, y_{t-1}, \ldots, y_1)$ or $E_t y_{t+i}$.

Considering that $\{\hat{\epsilon_t}\}$ denotes the estimated residuals from the model $y_t = a_0 + a_1 y_{t-1} + \epsilon_t$. The conditional variance of y_{t+1} is represented by Equation 24:

$$Var(y_{t+1}|y_t) = E_t[(y_{t+1} - a_0 - a_1 y_t)^2] = E_t \epsilon_{t+1}^2 \tag{24}$$

Thus far, the variance of ϵ_{t+1} is a constant equal to σ^2. Equation 25 models the condition variance as an AR(q) process using the square of the estimated residuals.

$$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\epsilon}_{t-1}^2 + \alpha_2 \hat{\epsilon}_{t-2}^2 + \cdots + \alpha_q \hat{\epsilon}_{t-q}^2 + v_t \tag{25}$$

In which v_t is a white-noise process. If all values of α_i are equal to zero, the estimated variance is constant and equal to α_0. Otherwise, Equation 26 represents how to forecast the conditional variance at time t according to an autoregressive process given by Equation 25.

$$Var(y_t|y_{t-1}) = E_t \hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\epsilon}_{t-1}^2 + \alpha_2 \hat{\epsilon}_{t-2}^2 + \cdots + \alpha_q \hat{\epsilon}_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \hat{\epsilon}_{t-i}^2 \tag{26}$$

Equation 25 is called an ARCH model, but this linear specification (Equation 25) is not considered the most convenient. It is "more tractable to specify v_t as a multiplicative disturbance" (Enders, 1995, p. 142). Engle's (1982) proposes Equation 27 as the simplest example from the class of multiplicative conditionally heteroskedastic models (an ARCH(1) process).

$$\epsilon_t = v_t \sqrt{\alpha_0 + \alpha_1 \epsilon_{t-1}^2} \tag{27}$$

Where v_t is a white-noise process such that $\sigma_v^2 = 1$, v_t and ϵ_{t-1} are independent of each other, and α_0 and α_1 are constants such that α_0>0 and $0 < \alpha_1 < 1$. The ARCH process given by Equation 27 has been extended, resulting in ARCH (q) processes (Equation 28):

$$\epsilon_t = v_t \sqrt{\alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2} \tag{28}$$

Bollerslev (1986) extended Engle's (1982) work by developing the generalized ARCH (p,q) model, called GARCH(p,q). Equation 29 represents the error process in a GARCH model.

$$\epsilon_t = v_t \sqrt{h_t} = v_t \sqrt{\alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}} \tag{29}$$

Where $\sigma_v^2 = 1$, $\{v_t\}$ is a white-noise process independent of past realizations of ϵ_{t-i}, Eϵ_t=E$v_t\sqrt{h_t}$=0 and the conditional variance of ϵ_t is given by h_t. The main idea is that the conditional variance of ϵ_t depends on the squared error term in the previous periods (as in the ARCH(q) model) but also on its conditional variance in the previous periods (Equation 30).

$$E_{t-1}\epsilon_t^2 = h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{30}$$

3. Methodology

This thesis addresses three different models seeking to answer the questions stated in the first chapter of this work (Figure 9).

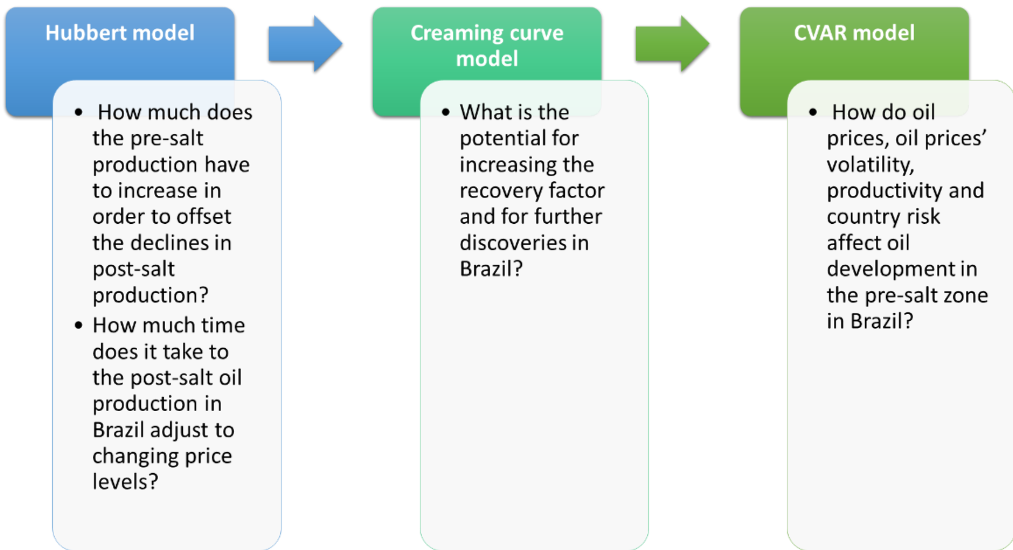

Figure 9: Process flow chart

3.1. Database for the Hubbert model

To forecast Brazil's oil production with a single-cycle and multi-cycle Hubbert model, to back-test Hubbert's model, and to explain residuals of Hubbert model, annually observations between 1954 and 2015 for the offshore oil production at field level by water-depth were obtained from ANP (2016). The Brent oil price (deflated to dollar 2015) was obtained from BP (2016).

However, oil production of some Brazilian oil fields, such as Marlim, Barracuda, Caratinga and Marlim Leste, comes from both post-salt and pre-salt layers. Thus, the production of the post-salt deep-water oil fields was obtained by reducing the offshore production in deep waters by pre-salt production — obtained from the Monthly Oil and Natural Gas Production

Bulletin disclosed by ANP (2018a) ─ once all pre-salt producing oil fields are in deep water (water-depth deeper than 250m).

Pre-salt fields' STOIIP are obtained from the Development Plan summaries disclosed by ANP (2018h) to estimate the oil production peak from the pre-salt zone.

Eleven pre-salt fields with STOIIP available (Table 7) are used to estimate the STOIIP from the pre-salt zone. This study considers the sum of STOIIP from the following pre-salt fields: Lula, Sapinhoá, Jubarte, Mero, Lapa, Búzios, Sépia and Itapu. Fields which produce in pre-salt but mostly from post-salt are not included as it not possible to differ the STOIIP from pre-salt and post-salt layer: Marlim Leste, Barracuda, and Caratinga.

Table 7: STOIIP from pre-salt fields

Pre-salt producer fields*	STOIIP (Mbbl of oil)[2]	Percentage of production from the pre-salt layer
Lula	17,791	
Sapinhoá	3,311	
Jubarte[1]	9634.58	63%
Lapa	1,676.86	
Baleia Franca[1]	─	89%
Baleia Azul	─	
Búzios	29,889	
Marlim Leste[1]	5,804.6	27%
Sururu	─	
Barracuda[1]	3,053	6%
Caratinga[1]	2,255	22%
Voador	─	
Marlim[1]	─	1%
Pirambu	─	
Pre-salt non-producers' fields	**STOIIP (Mbbl of oil)[2]**	
Sépia	4,959.8	
Itapu	1,316.14	
	URR (Mbbl of oil)	
Mero**	3,300	

**Information from the Monthly Oil and Natural Gas Production Bulletin (ANP, 2018a) - July 2018*
***Total recoverable volume of Mero obtained from Petrobras (2017a)*
[1]Fields that produce from both pre-salt and post-salt layer
[2]Information of STOIIP available in the Development Plan disclosed by ANP (2018h)

3.2. Hubbert model

Except for the pre-salt analysis (in section 3.2.4), the methodology within section 3.2 is the same applied by Hallack et al. (2017). Following the discussion in section 2.1.1, the Brazilian offshore post-salt oil production was therefore divided into two classes: shallow water (up to 250m water depth) and deep water (more than 250m water depth). The asymmetric Hubbert model based on a Gaussian curve, developed by Brandt (2007), was adapted to the original Hubbert model. It allows the curve-fitting of the post-salt oil production at shallow water as this production profile presents a long enough smooth decline. Then, this study uses a regression model to explain the differences between the Hubbert model and observed production data, inspired by Kaufmann (1991). After that, a logistic curve is fitted to pre-salt historical oil production using different scenarios of URR to estimate pre-salt peak.

The following sections give a detailed insight into the applied methodology.

3.2.1. Forecasting Brazil's oil production with a single-cycle and multi-cycle Hubbert model

This study estimates the production profile and the URR for offshore post-salt oil production in Brazil. The annual production Q'_t is represented by the first differential of cumulative production Q_t as shown by Equation 31 (Sorrell and Speirs, 2009).

$$Q'_t = \frac{dQt}{dt} = \frac{aQ_\infty e^{-a(t-t_m)}}{\left(1+e^{-a(t-t_m)}\right)^2} \tag{31}$$

Three parameters are used to explain the cumulative production Q_t: the URR is represented by Q_∞, *a* is the "steepness" of the curve, *t* is the variable time (year), and t_m is the midpoint of the growth trajectory.

A single cycle model may however not be appropriate for the case of Brazil. This study developed a curve that shows the cumulative oil produced (Np) for each field in the year of its discovery accumulated until 2015, aiming to capture the evolution of the size of discoveries over time. As shown in Figure 10, a new cycle of post-salt discoveries started in the mid-1980s. This cycle results from deepwater discoveries.

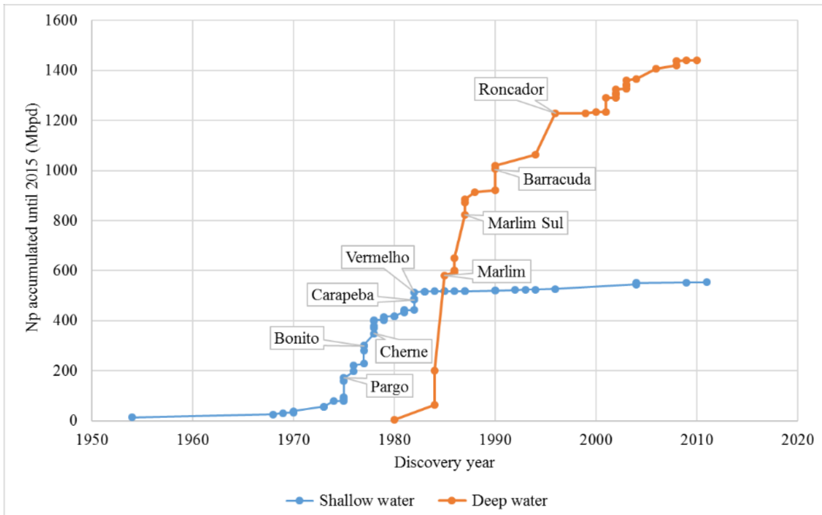

Figure 10: Np accumulated until 2015 (Mbpd) from offshore oil fields separated in shallow and deep water
Source: Based on ANP (2016)

It can be observed in Figure 9 that a new cycle of deep-water discoveries brought a significant increase in Np in the mid-1980s due to the technological development of exploration and production in deep waters.

As the Hubbert's theory considers that the production cycle follows the same format of the discovery cycle, it can be inferred that the technological development of exploration and production in deep waters, which occurred in the 1980s, may have contributed to deviate observed production from the estimates of the single cycle Hubbert curve. Also, it can be inferred that the deep-water technology caused the beginning of a new discovery and production cycle.

The multi-Hubbert model applied to the Brazilian oil production, therefore, results from the sum of curves from two cycles, in which 'i' represents a cycle and 'N' represents the total number of cycles (i.e., two) modeled ─ see Equation 32 and Equation 33:

$$Q'_t = \sum_{i=1}^{N} \frac{a_i Q_{\infty i} e^{-a_i(t-t_{mi})}}{(1 + e^{-a_i(t-t_{mi})})^2} \tag{32}$$

$$URR = \sum_{i=1}^{N} Q_{\infty i} \tag{33}$$

The total production Q'_t is explained by the following variables: $Q_{\infty i}$, which is the URR of a cycle, a_i, which is the "steepness" of a cycle curve, and t_{mi}, which is the time when production reaches a cycle peak.

Brandt (2007) evaluated some models (Gaussian, linear and exponential, symmetrical and asymmetrical) fitting them to production data. Among the models tested by Brandt (2007), there is the asymmetrical Gaussian curve of production in which a different standard deviation on the increasing and decreasing sides of the production curve is allowed.

To adapt the Brandt (2007) formulation to the logistic curve hitherto used (Equation 31), a parameter 'a' in function of time and a 'k' factor was adopted, as shown in Equation 34. The parameter 'a' derives from the slope of the curve to the left (a_{inc}) and right (a_{dec}) side of peak production and the 'k' factor represents the transition between the left and right side of peak production.

$$a(t) = a_{dec} - \frac{a_{dec}-a_{inc}}{1+e^{k(t-t_m)}} \text{, so that: } \quad \begin{array}{l} t \ll t_m \rightarrow a(t) = a_{inc}; \\ t \gg t_m \rightarrow a(t) = a_{dec}; \\ t = t_m \rightarrow a(t) = \frac{1}{2}(a_{inc} + a_{dec}) \end{array} \tag{34}$$

This yields a new formulation of the asymmetric Hubbert model (Equation 35).

$$Q'_t = \frac{dQt}{dt} = \frac{a(t)Q_\infty e^{-a(t)(t-t_m)}}{(1 + e^{-a(t)(t-t_m)})^2} \tag{35}$$

As reported by Sorrell and Speirs (2009), geology and techno-economic factors affect the URR, which can only be estimated to a reasonable degree of confidence when exploration is well advanced. For less-explored regions, estimates must rely upon geological analysis.

Thus, the dynamics of economic and political change becomes nontrivial for the estimation of the URR. Many curve-fitting models are based on URR scenarios to mitigate this weakness. However, the difficulty in anticipating future discoveries and production cycles cannot be sufficiently mitigated by URR scenarios. Therefore, this study endogenously estimates the URR, which is possible in a stable way as both cycles, i.e., shallow and deep-water offshore production, have passed their production peak already.

Also, the importance of pre-salt development in a post-salt declining context is evaluated by determining the average annual growth rate of pre-salt oil production, which is necessary to maintain the offshore oil production volume at the level of 2015. This thesis first estimates how much pre-salt production must increase to compensate for the decline in post-salt production in absolute levels and subsequently derive the annual growth rate for the period 2016-2025 necessary to achieve this production volume.

A nonlinear least-squares approach was used minimizing the root-mean-square of the errors (RMSE) of the historical oil production data and the Hubbert curves. For that purpose, this thesis used the function nls (nonlinear least squares) in the software R. The quality of the fit is assessed by determining the coefficient of variation (CV) factor, defined as the ratio of the RMSE to the estimated peak in oil production of the cycle i (${Q'}_i^{\text{peak}}$):

$$CV = \frac{RMSE}{{Q'}_i^{peak}} \tag{36}$$

3.2.2. *Back-testing Hubbert's model*

The descriptive effect of the Hubbert curve can be observed by comparing the single and multi-cycle curves with the actual observed post-salt offshore production. However, the stability of the Hubbert forecast over time remains to be tested. This test is performed by the application of the back-testing technique, computed each year, for each period T. Back-test steps are:

1. The production data, P_i to P_N, is truncated at a specific date in the past T;
2. The extrapolation of the oil production rate is made based on the truncated historical data P_i to P_T;
3. The future production predicted by extrapolation is compared to the actual production from the date T in the past up to the present date N;
4. The average relative error is calculated for date T;
5. The URR is estimated;
6. The steps above are repeated until the T=N.

The percentage error (Equation 37) is obtained by the sum of the relative differences, in absolute terms, between the Hubbert oil production forecast (using historical data from 1954 until T) and the observed oil production data (from 1954 to 2015).

$$Error(T) = \sum_{t=1954}^{2015} \frac{\left| P_{T,t}^{estimated} - P_t \right|}{P_t}; \qquad \text{in which T} \in [2000;2015] \qquad (37)$$

The back-test permits to compare the stability of endogenously estimated URR from all oil production cycles: post-salt offshore (single cycle), shallow water post-salt offshore (symmetrical and asymmetrical first cycle), and deep-water post-salt offshore (symmetrical and asymmetrical second cycle).

3.2.3. Explain the residuals of the Hubbert model with Kaufmann's hybrid approach

The impact of Hubbert's work inspired the incorporation of economic, institutional and technological variables to the original model, resulting in hybrid models to forecast oil production (Kaufmann, 1991; Kaufmann and Cleveland, 2001; Pesaran and Samiei, 1995).

Therefore, aiming at finding the main reasons of deviation between the Hubbert single-cycle and multi-cycle models related to the historical oil production in post-salt offshore in Brazil, this study applied a methodology inspired by Kaufmann (1991) for the period 1954-2015.

The relative difference between the observed annual oil production $\boldsymbol{P_t}$ and the annual oil production estimated by Hubbert $\boldsymbol{Q'_t}$ generates the residue $\boldsymbol{R_t}$ (Equation 38).

$$R_t = \frac{P_t - Q'_t}{Q'_t} \qquad (38)$$

The residue is used as a dependent variable to identify the factors that might have caused the difference between modeled and observed production (Equation 39).

This study focuses on the most important economic factor for oil production to explain these differences, i.e., the oil price, on structural factors that estimate the role of symmetry in the Hubbert curve, and on a term that controls past changes in production levels. Other economic factors, such as the GDP or the exchange rate of the Brazilian Real against the Dollar would have been highly interesting to analyze. However, consistent time-series for these indicators are only available for a very short time, which would have made statistical analysis spurious. The original regression equation is shown in the following:

$$R_t = \alpha + \sum_{i=0}^{L} \beta_i BP_{(t-i)} + \beta_{L+1} PC'_t + \beta_{L+2} \Delta P_t + \epsilon_t \qquad (39)$$

Where α is the intercept coefficient. $\boldsymbol{BP_{(t-i)}}$ is the Brent oil price at year t-i, deflated to 2015-dollars. Different lags in the dollar price are allowed to test the time of adjustment necessary to adapt production to price changes. $\mathbf{PC'_t}$ is a dummy variable to test the symmetry of the production curve (the variable is zero before the peak in the production curve, and after the peak, it is equal to the difference between the production in the year next to the peak and the peak). $\mathbf{\Delta P}$ denotes the absolute production variation between two consecutive years (in a million barrels), and ϵ is the error term.

Therefore, $\mathbf{PC'_t}$ evaluates if the production curve modeled should be altered in a way to capture the asymmetry of the production curve. In other words, if the slope of the curve to the left and right side of peak production is different, $\mathbf{PC'_t}$ will take it into account. If the applied Hubbert model itself is asymmetric, it is expected that the corresponding coefficient is insignificant, as the asymmetry is already captured by the original model.

The $\mathbf{\Delta P_t}$ variable aims to capture the actual inertia effect on production. In other words, it would be the natural tendency of oil production to continue the previous years' profile without a sharp break behavior, as this usually requires massive investment volumes.

This study uses the Augmented-Dickey Fuller (ADF) test to check stationarity of the involved time-series. If necessary, it takes differences of the time-series to obtain stationary ones. Additionally, it checks for heteroscedasticity and autocorrelation in the residuals of the regression with the Breusch-Pagan (BP) and Breusch-Godfrey (BG) tests (Wooldridge, 2013). If heteroscedasticity and auto-correlation cannot be rejected, this study includes a lagged R_t to reduce auto-correlation and applies the heteroscedasticity and auto-correlation robust estimator by Newey and West (1987) subsequently.

3.2.4. Preliminary effort to estimate the oil production peak from the pre-salt province

This study estimates the pre-salt peak of production using different scenarios of URR as a constraint for fitting the pre-salt historical oil production to a logistic curve.

According to ANP (2017), there is a 20% RF for deep water in Brazil and a 30%-60% RF range for the primary/secondary recovery mechanism. For that reason, four different URR scenarios are elaborated according to different scenarios of RF: i) 15% RF in the pessimistic scenario; ii) 20% RF in the base scenario; iii) 30% RF in a primary recovery scenario; iv) 60% RF in a secondary recovery scenario (optimistic). The $URR_{pre-salt}$ is estimated for each RF scenario as shown in Equation 40. After that, each URR scenario is used to fit pre-salt historical oil production to the logistic curve. The URR_{Mero} is added separately into Equation 40 because ANP does not disclose the Development Plan summary for Mero field. Thus, the information of STOIIP for Mero is not publicly disclosed. On top of that, Mero is the only field developed under the PSA regime within the dataset. For this reason, this thesis includes separately the information of total recoverable volume of Mero field obtained from Petrobras (2017a), as shown in Table 7.

$$URR_{pre-salt} = \left(\sum STOIIP_{field}\right) x\ RF + URR_{Mero} \tag{40}$$

3.3.Database for the creaming curve model

The number of wildcat wells concluded by basin are disclosed by the Brazilian petroleum regulatory agency, ANP (2018c). The fields' STOIIP are obtained from the Development Plan summaries provided by ANP (2018h). The field's date of discovery is obtained from the Development Plan summaries (ANP, 2018h) as well as by ANP (2016). The accumulated production of oil from onshore and offshore fields is obtained from ANP (2016). Such information is also disclosed by ANP (2019). Between August 2016 and December 2017, the accumulated production is obtained by basin from ANP (2018a). The basin in which the field is located is obtained from the Development Plan summaries (ANP, 2018h) as well as by ANP (2016). The 1P and 3P reserves by basin are provided by ANP (2018i). The term 'oil' referred by this study does not embed natural gas resources, i.e., oil equivalent is not considered by this study.

3.4.Creaming curve model

This methodology is obtained from Hallack and Szklo (2019).

In several ways, this study extends previous exploratory efforts analysis in Brazil (Szklo et al., 2007; Almeida and Arruda, 2017). This thesis measures the exploratory effort by the number of wildcat wells constructed. First, this thesis identifies the basins that concentrate most of the exploratory effort and discovered STOIIP. Secondly, this thesis infers the potential to increase discoveries from known fields by improving recovery factors in the four basins that concentrate 95% of the discovered STOIIP. Thirdly, this thesis weights the STOIIP data with the estimated recovery factor to estimate the size of discoveries and build the creaming curves. To estimate the size of discoveries, this thesis calculates the recovery factor using the historically accumulated production and reserves 1P and 3P. The choice of reserves 1P and 3P relies exclusively on the availability of data, as the information of 2P reserves is not disclosed by the Brazilian petroleum regulatory agency, ANP. Finally, the creaming curve is fitted by three different functions, which are extrapolated to allow a projection of what is likely to be discovered in the future vs. increasing wildcat wells drilled, as well as to identify frontier areas in Brazil. Besides, this thesis compares the creaming curves' shape and the three function's extrapolation for the four basins.

The estimated recovery factor for each basin in the analysis is derived as shown in Equation 41.

$$RF_j^{basin} = \frac{j\_Res^{basin} + \sum_{t,field} Prod_{t,field}^{basin}}{\sum_{field} OOIP_{field}^{basin}} \tag{41}$$

in which *j* can be 1P reserves or 3P reserves, j_Res^{basin} represents the volume of *j* reserves by basin, RF_j^{basin} is the recovery factor for a basin considering *j* reserves, $Prod_{t,field}^{basin}$

represents the field's historical petroleum production at time t assembled by basin and $STOIIP_{field}^{basin}$ represents the fields' STOIIP grouped by basin.

To estimate the size of discovery for each field ($URR_{j,field}^{basin}$) this work multiplies its STOIIP ($STOIIP_{field}^{basin}$) by the previously estimated recovery factor of the respective basin RF_j^{basin}, as shown in Equation 42:

$$URR_{j,field}^{basin} = STOIIP_{field}^{basin} * RF_j^{basin} \tag{42}$$

After the size of discovery is identified for each field, this thesis builds the creaming curves. For each basin, this thesis plots the cumulative number of wildcat wells drilled in the abscissa axis and the cumulative fields' size of discoveries in the ordinate axis.

Then, this thesis fits the creaming curve to three functions: logistic, Gaussian and Gompertz (see Appendix B). The choice of these functions relies on the fact that the curve starts at zero and exhibits asymptotic behavior. Moreover, these functions were applied in a previous curve-fitting analysis (Brandt, 2007; Sorrell et al., 2009; Brandt, 2010). After that, this thesis extrapolates the function's curve to estimate the potential for yet-to-find discoveries in each basin. For that purpose, this thesis uses the function *predict* in the RStudio software (Version 1.1.463).

As proposed by Nashawi, Malallah, and Al-Bisharah (2010) for assessing the fit of different Hubbert models, the goodness of the fit for the different basins is appraised using the coefficient of variation (CV_j^{basin}) factor. Such a coefficient is defined by that work as the ratio of the square root of the estimated variance of the random error (σ_j^{basin}) to the parameter representing the asymptote ($asym_j^{basin}$), as presented in Equation 43. Because higher values of discovery data result in higher variances regardless of the quality of fit, the use of a ratio becomes fundamental to overcome this issue.

$$\mathrm{CV}_j^{basin} = \frac{\sigma_j^{basin}}{asym_j^{basin}} \tag{43}$$

Finally, this work obtains the uncertainty in the yet-to-find estimative from the standard error of the asymptote parameter within the non-linear least squared estimates derived from the function *nls* at the stats package in the RStudio software.

3.5.Database for the CVAR model

This thesis compiles monthly observations for the price of crude oil, the number of petroleum producing wells and completed development wells, and production in the pre-salt zone. Significant rates of production in the pre-salt region start in 2010. Therefore, models are estimated from a sample that includes observations from January 2010 to March 2018 (Figure 11).

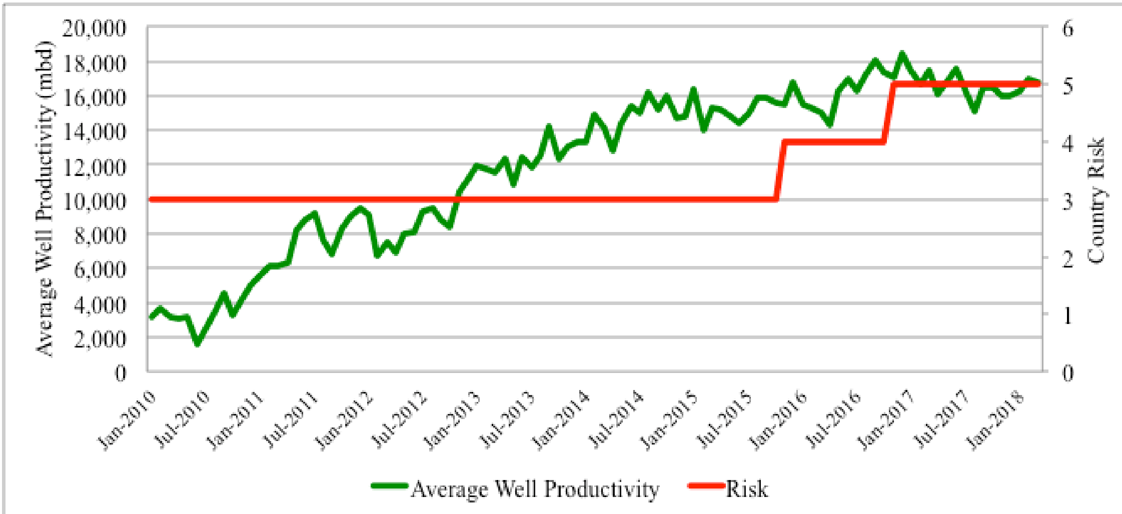

Figure 11: Brazil's country risk measured by OECD and Pre-salt's Average Well Productivity (mbd) over time

Monthly observations for the petroleum production in the pre-salt zone are obtained from the Monthly Oil and Natural Gas Production Bulletin (ANP, 2018a). Monthly observations for the number of petroleum producing wells are derived from the Brazilian petroleum regulatory agency, ANP (2018f, 2018g). Information for the number of offshore development wells is provided by ANP (2018c). Petrobras[27] operates all development wells drilled in the pre-salt layer, according to the information provided by ANP (2018c).

This thesis compiles a time series for well completions in the pre-salt layer using observations from Santos and Campos Basin. To separate offshore development wells between the pre-salt and post-salt layers, this work considers the geological group/formation and the pre-salt fields. According to ANP (2018g), wells in the geological group/formation Guaratiba and

[27] The Brazilian state-controlled oil company – initially a state-owned monopoly – historically certifies about 95% of its reserves by the U.S. Securities and Exchange Commission (SEC) criteria. Currently, the certifying company is DeGolyer and MacNaughton (D&M) (Petrobras, 2018a). Petrobras is controlled by the Federal Government, which is the majority shareholder, and shall be governed by the terms and conditions of the Corporation Law. Its Board of Directors has autonomy to define the pricing policy, and the Federal Government has decision-making power in the Company's Fiscal Council Board.

Lagoa Feia, respectively, in Santos Basin and Campos Basin are from the pre-salt layer. This thesis also considers the pre-salt fields that possess development wells: Lula, Sapinhoá, Búzios, Lapa, Sururu, Mero, Sépia, Sul de Lula, Itapu, Atapu, Sul de Berbigão, and Berbigão (Petrobras, 2014; Braga and David, 2018; ANP, 2018c).

Prices for crude oil (dollars per barrel) are measured by spot and future prices (contracts with maturity dates of one-month, six-month, one year and two years) for both WTI and Brent crudes. Monthly averages for the spot price of crude oil are measured by the WTI Free on Board (FOB) Price at Cushing Oklahoma, and the spot price of Brent is measured by the Europe FOB Price. Prices for WTI and Brent futures contracts measure monthly averages for the end-of-day price. Prices for WTI come from future contracts that are traded in the New York Mercantile Exchange, whereas prices for Brent come from futures contracts that are traded in the ICE Futures Europe Commodities. All prices inputted into the model are deflated (base year 1982) by monthly values of the U.S. city average for all items (CUUR0000SA0) that is obtained from the Bureau of Labor.

This thesis seeks to identify the BEP by estimating models that specify BEP's that vary between $5 and $50 (real 1982 dollars) per barrel at $5 increments[28]. These values are used to calculate proxies for profitability, net revenue, and perceived volatility although these calculations treat BEP as a constant, BEP changes over time as a function of technology and reservoir quality. Some of these changes are proxied by the productivity of oil-producing wells. As such, the BEP represents a break-even price at the average sample value for

[28] It represents a BEP varying between about $12.5 and $125 per barrel (in 2018 prices) at $12.5 increments.

productivity (and other independent variables). This thesis' top-down estimate for the BEP represents a price that generates proxies for profitability, net revenue, and perceived volatility that best describe the number of development wells drilled in pre-salt formations per month. In other words, it represents how the industry responds to oil prices (and price volatility, technology and reservoir quality, and country risk).

To proxy for the cost of borrowing foreign capital, this thesis uses the measure for the country risk that is compiled by the Organization for Economic Cooperation & Development (OECD) for Brazil (Figure 11). This index measures the country credit risk and the likelihood that a country will service its external debt. In other words, the OECD country risk weighs the chance that a government would "prevent an entity from converting local currency into foreign currency and/or transferring funds to creditors located outside the country" (OECD, 2019). Based on this definition, this thesis expects that this measure of country risk has a negative relation with well completions in the pre-salt zones in Brazil because an increase in risk increases the cost of borrowing foreign capital.

3.6. CVAR Model

3.6.1. Overview

This thesis estimates a series of models to quantify how prices, price volatility, productivity, and country risk affect the number of development wells that are completed in the pre-salt zones of Brazil.

This methodology is obtained from Hallack et al. (2019), which extends the work of Ansari and Kaufmann (2019) by including the country risk analysis into the models. Nonetheless, we analyze the number of development wells in the pre-salt layer as the endogenous variable in the CVAR model, whereas Ansari and Kaufmann (2019) analyze the number of rigs active to drill oil and gas wells in tight formations.

To explore the effect of price volatility on the number of development wells completed in the pre-salt zones of Brazil, This thesis uses Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models (Bollerslev, 1986; Schaeffer et al., 2012) of crude oil prices to estimate conditional volatilities from monthly observations of real rates of return. The use of GARCH-class models to characterize crude oil price volatility is widely observed in the literature (Sadorsky, 2006; Narayan and Narayan, 2007; Agnolucci, 2009; Kang et al., 2009; Mohammadi and Su, 2010; Wei et al., 2010; Hou and Suardi, 2012; Klein and Walther, 2016; Bos et al., 2018). Because GARCH models have a good record in providing accurate estimates for the volatility of returns from financial time series data (Agnolucci, 2009), this thesis proxy the volatility of oil prices by using the plain GARCH model. Tests of the residuals from the GARCH (1,1) model suggest that it can be used to proxy the volatility of oil prices (Appendix D.1).

Prices are measured using spot and future contracts with different maturities for two benchmark crude oils: WTI and Brent. To quantify the effect of price, this thesis calculates profitability per barrel and net revenue. Both variables require a BEP, which is unknown. To identify a BEP, this thesis uses a range of values. For each BEP and measure of price, this work calculates proxies for profitability, net revenue, and perceived volatility. This thesis uses these variables along with productivity and country risk to estimate a series of CVAR models (Johansen, 1996; Gebre-Mariam, 2011; Naser, 2015) for the number of development wells completed in the pre-salt zone of Brazil. The nature of these variables (endogenous or weakly exogenous) and their long- and short-run relations are determined using statistical criteria. For each CVAR, this work calculates an in-sample simulation for the monthly change in development wells. The accuracy of these simulations is used to identify the measure of price and BEP which best explains the number of development wells completed in the pre-salt zones of Brazil.

3.6.2. Modelization

To explore the effect of price volatility on the number of wells drilled, this thesis follows standard econometric practice (e.g., Enders (1995)) and use a standard general autoregressive conditional heteroskedasticity GARCH (1,1) model. The general specification of a GARCH (p,q) model is given by Equation 44.

$$Y_t = \varepsilon_t = \sigma_t z_t \tag{44}$$

in which Y_t represents the return $\left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)$ to real prices for crude oil associated with the price on the spot or future market, the disturbance term ε_t is normally distributed with zero mean, and the variance of ε_t follows a GARCH (p,q) process, z_t is i.i.d. – independent and identically distributed – with zero mean and mean unit variance.

Equation 45 specifies the conditional variance of a GARCH (p,q) process, σ_t^2:

$$\sigma_t^2 = \mu + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \omega_j \varepsilon_{t-j}^2 \tag{45}$$

In which the constants $\mu > 0$, $\alpha_i \geq 0$, $\omega_j \geq 0$. $\sqrt{\sigma_t^2}$ represents the conditional volatilities of the underlying price series (*Vol*), which vary by the time till maturity. For all measures of oil prices, volatility increases in 2008/2009 and again after the price drops from about \$100 per barrel at the end of 2014 (Figure 12).

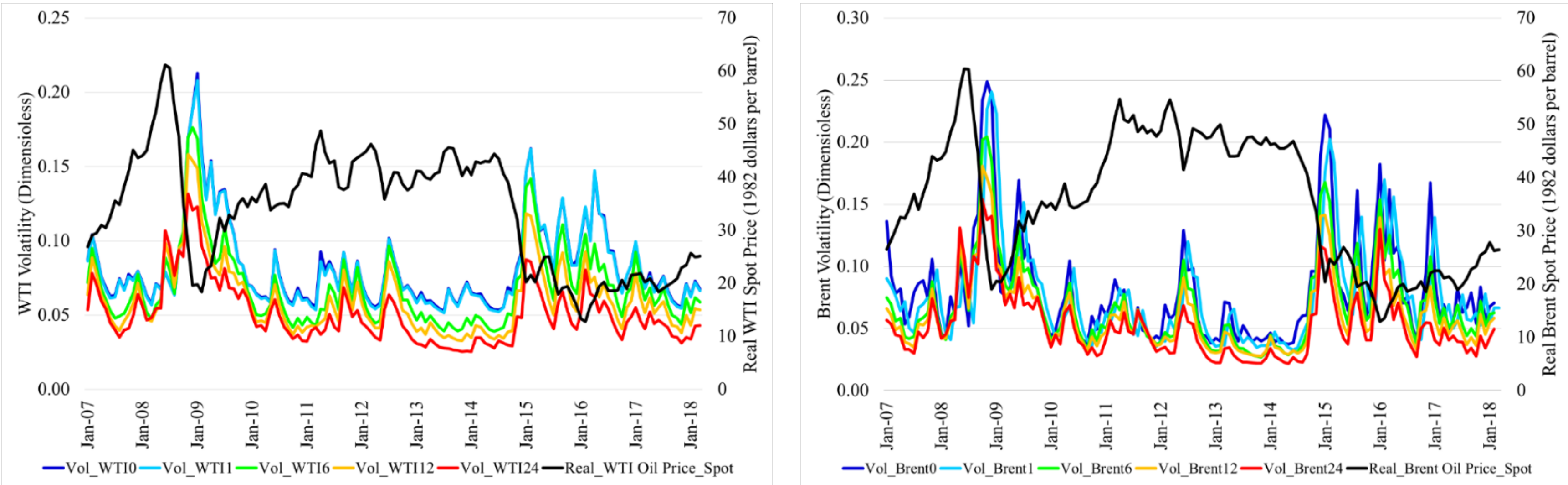

Figure 12: Oil price volatility for WTI and Brent.
The volatility estimated by the GARCH model (Equation 45) from the spot price of Brent crude oil (dark blue line), the price on the future contract with a maturity of 1 month (light blue line), 6 months, (green line), 12 months (orange line), and 24 months (orange line) and the real Brent spot price of crude oil (black line).

Volatility declines as the maturity of the futures contract increases. The volatility of Brent and WTI generally are similar, although the peak values for Brent are slightly greater. After a price drop at the end of 2014, the increase in volatility for Brent shows greater persistence.

This thesis postulates that the effect of price volatility on well completions is modified by how firms perceive volatility. Perceptions of volatility may increase when prices are near the BEP, as opposed to periods when the price for crude oil is much higher than or less than the BEP. For example, a price increase from $55 per barrel to $60 per barrel at a BEP of $50 per barrel means the profit increases from $5 per barrel to $10 per barrel, which represents a 100% increase. That same $5 increase represents a 10% and 12.5% increase in profit for a BEP of $5 and $95 per barrel respectively.

To create a proxy for how this perception alters the effect of volatility, this thesis weights the volatility quantified by the GARCH model with a perception index that is based on the difference between the current price and the BEP. This perception index is calculated using a normal distribution in which the mean is the BEP, and the variance of this normal distribution is the variance of the price for oil over the previous 24 months (Figure 13).

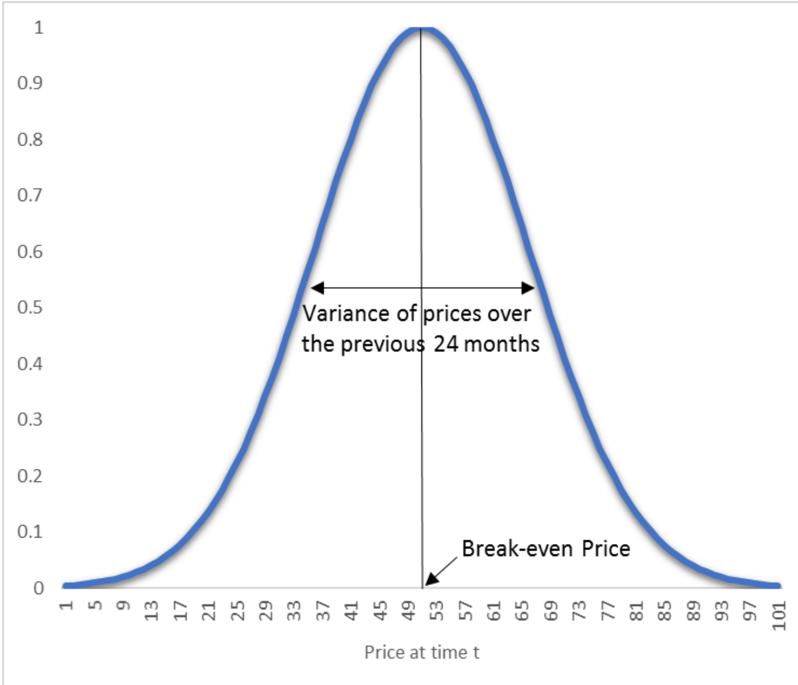

Figure 13: Normalized perception index.
The shape of this index changes across CVAR models based on the break-even price and changes over time-based on the variance of prices over the previous 24 months. The value of the normalized perception index at a price at time t is multiplied by the GARCH estimate for volatility at time t to calculate perceived volatility (PerVol) at time t.

The price at time t (P_t) is used as the abscissa value to identify the ordinate value of the normalized perception index that is multiplied by the volatility estimated by the GARCH model. This product *PV* represents 'perceived volatility.' Perceived volatility equals volatility from the GARCH model *Vol* when the price at time t equals the BEP. However, when the price at time t is far from the BEP (P_t<<BEP or P_t>>BEP), the normalized perception index is well below one, which reduces *PV* relative to *Vol*.

In addition to *PV*, this thesis uses BEP to calculate two other variables that represent the economic return associated with drilling an oil well. Equation 46 approximates the profitability (dollars per barrel), Pr_t:

$$Pr_t = (P_t - BEP) \tag{46}$$

In which profitability at time t (Pr_t) is the difference between the real price of oil at time t (P_t) and the real BEP. Equation 47 calculates the net revenue, Rv_t:

$$Rv_t = (P_t - BEP) * AWP_t = Pr_t * AWP_t \tag{47}$$

The average productivity of wells drilled into pre-salt zones (AWP), which is measured by barrels per day per well, is presented by Equation 48:

$$AWP_t = \frac{Prod_t}{W_t} \tag{48}$$

In which *Prod* is the production from the pre-salt zone (barrels per day), and *W* is the number of petroleum producing wells in the pre-salt zone. This proxy represents the effect of technology and reservoir quality.

To eliminate the effects of inverting matrices with elements that differ greatly in size due to different units of measure, each of the time series described above is standardized as follows (Equation 49):

$$n_t = \frac{(y_t - \bar{y})}{\sqrt{Var(y)}} \tag{49}$$

In which y_t is the value (in original units), $\bar{y}$ is the average value over the sample period, and $Var(y)$ is the variance over the sample period.

To explore the relation among pre-salt development wells (*W*), price volatility (*Vol*), perceived volatility (*PV*), proxies for economic returns (*Pr*, *Rv*), proxies for technology and reservoir quality (*AWP*), and country credit risk (*Risk*) for each of ten BEP and ten measures of price (five measures for Brent and five measures for WTI), this work estimates 100 CVAR models (50 for Brent prices and 50 for WTI prices). The general form of a CVAR model is given by Equation 50:

$$\Delta x_t = A_0 \Delta w_t + A_1 \Delta w_{t-1} + \Gamma_{11} \Delta x_{t-1} + \Pi(x_{t-1}, w_{t-1}) + \mu_0 + \Theta M + \varepsilon_t \qquad (50)$$

In which x_t is a vector of p endogenous variables whose behavior is being modeled, w_t is a vector of e exogenous variables, μ_0 is a vector of constant terms, M is a vector that includes eleven monthly dummy variables (Jan-Nov), A_0, A_1, Γ_{11}, Θ, and Π are matrices of regression coefficients, Δ is the first difference operator ($\Delta x_t = x_t - x_{t-1}$), and ε is Niid (0, Ω).

When the time series x_t are nonstationary, the long-run matrix Π can be formulated as presented in Equation 51:

$$\Pi = \alpha\beta' \qquad (51)$$

Where α is a p x r matrix of adjustment coefficients and β is an r x (p + e) matrix of cointegration coefficients that define stationary deviations from long-run equilibrium relationships, and r is the number of long-run cointegrating relations. For more information about the CVAR model, see Juselius (2007).

To minimize the degree to which biases affect the statistical estimates of the CVAR model, this work uses a standard set of 'rules' to identify each model. First, This thesis determines the number of cointegrating relations using the likelihood-based trace test (Johansen, 1996). Based on the number of cointegrating relationships, this thesis tests whether each variable is weakly exogenous. Weakly exogenous variables are assigned to w (Equation 50); if the null hypothesis is rejected, then they are are weakly exogenous, and assigned to x.

For combinations of BEP and price measures that generate CVAR with one or more cointegrating relations and reject the null hypothesis that W is weakly exogenous (i.e., W is in the x vector), this thesis imposes overidentifying restrictions. This thesis imposes the highest number of restrictions that as a group do not reject the null hypothesis that (1) the restrictions do not change the number of stationary relations (evaluated against a χ^2 distribution) and (2) rejects the null hypothesis that the coefficient in β' equals zero (evaluated against a t distribution).

After the model is identified, this thesis recovers the sample residuals (ε_t in Equation 50). The residuals are used to calculate an in-sample forecast for the monthly change in well completions, which is the endogenous variable in the CVAR model, as presented by Equation 52:

$$\Delta\widehat{W_t} = \Delta W_t - \varepsilon_t \tag{52}$$

In which ΔW_t is the first difference of the (normalized) W time series and $\Delta\widehat{W_t}$ is the forecasted first difference of wells (normalized).

The in-sample forecasts are used to identify the price measure and the BEP that generates the most accurate simulation. To determine the CVAR model that simulates ΔW_t most accurately, this thesis uses a general-to-specific automatic model selection procedure (Hendry and Doornik, 2014). This procedure retains/eliminates in-sample simulations based on statistics that measure the retention of irrelevant variables and the retention of relevant variables. Besides, the procedure tracks mean squared errors before and after model selection. Because the number of models is large (fifty models use Brent prices, and fifty models use

WTI prices) relative to the number of observations (99 between January 2010 to March 2018), identifying the most accurate model occurs in three steps. In the first step, the automatic model selection procedure compares all fifty in-sample simulations to determine the most accurate model. That uses either Brent or WTI. In the second step, this most accurate model is compared to the other 49 models in a head-to-head comparison. If another model is more accurate than the previous one, this thesis repeats the head-to-head comparisons until This thesis identifies the most accurate model that measures price using WTI and Brent. In the third step, this thesis recognizes the single most accurate model by comparing the most accurate model that measures oil prices using WTI to the most accurate model that measure oil prices using Brent.

Figure 14 includes all steps explained in this section up to now in order to obtain the data necessary to identify the CVAR models.

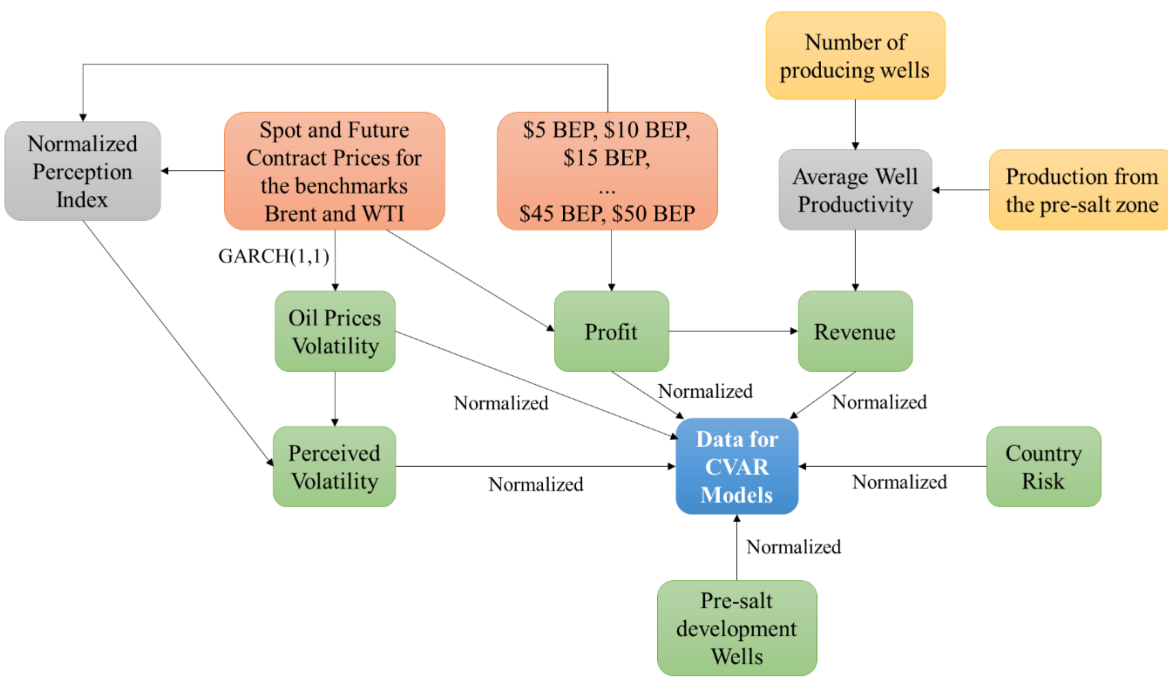

Figure 14: Flowchart that summarizes the steps to obtains the data to identify the CVAR models

Figure 15 shows the main steps to identify the CVAR models and obtain the most accurate one.

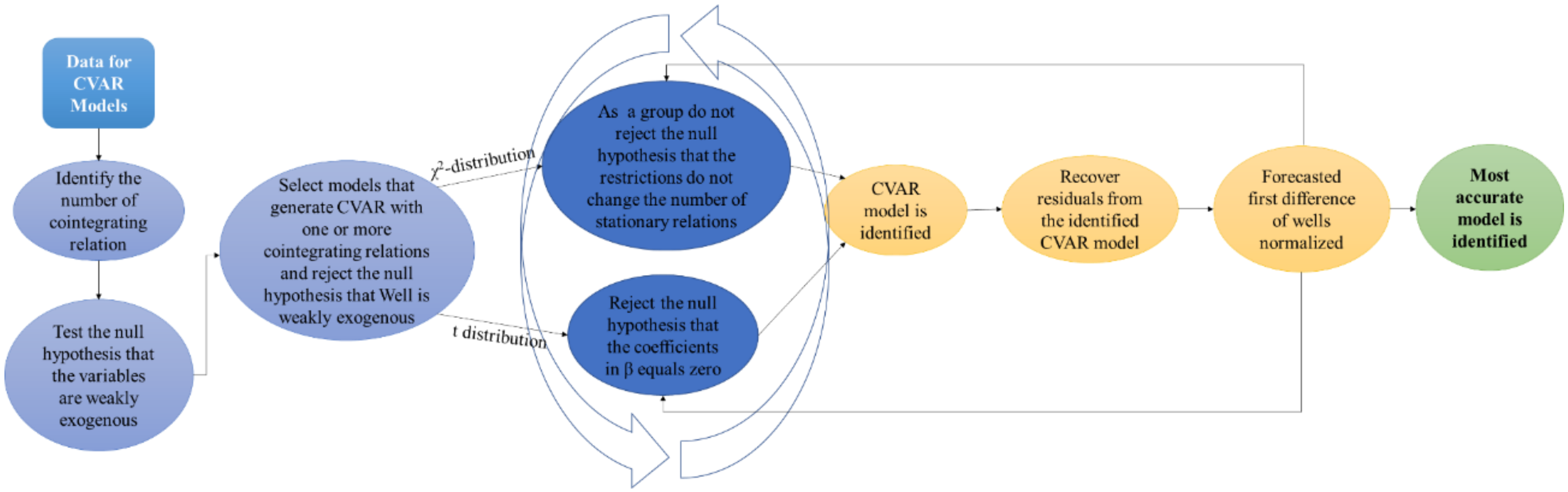

Figure 15: Flowchart that summarizes the steps to identify the CVAR models and obtain the most accurate one

4. Results

4.1. Hubbert

4.1.1. Results from post-salt Hubbert models and back-testing

Figure 16 shows the comparison of the three models with the observed production data. Figure A1 in Appendix A additionally shows the comparison of the three models with the accumulated observed production data. It can be observed that the three models have a good fit for the oil production data. In addition, it can be recognized the two-cycle models better estimate the production peak (in 2010), but that all the three models fail to capture it fully. The asymmetric two-cycle model additionally was better able to capture the deviation from a normal logistic curve in the 1980s due to a slower decrease in production from the shallow water offshore oil fields. The asymmetrical model has, in the long-term, a slower decline than the symmetrical model and the single cycle model.

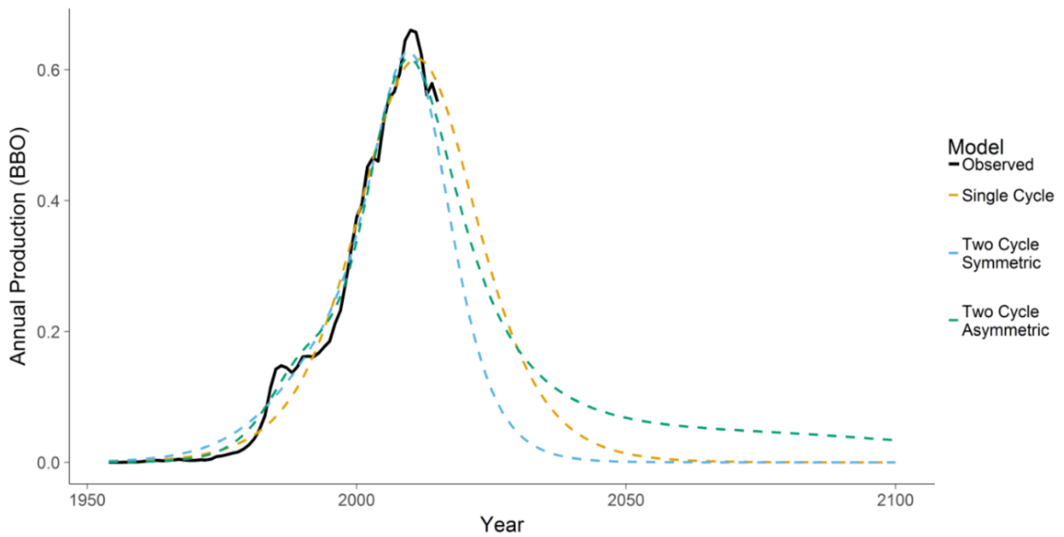

Figure 16: Post-salt offshore oil production compared with variants of the Hubbert curve.

The URR estimates of the Hubbert curve, the peak year, and the residual standard error of the three models are shown in Table 8. The asymmetrical two cycle model estimates the highest URR of all models, the single cycle model comes in second, and the two-cycle symmetrical model third. According to the three models, between 56% and 78% of the URR of Brazil´s post-salt has already been produced up to 2015. The estimated decline rate of Brazil’s post-salt hover between 6.7% and 13.7% per year in the period 2016-2025 (see Table 8). The lower decline rates (from the single and two cycle asymmetric models) are below the average decline rate of 13% based on a global database of 603 offshore oil fields (Sallh et al., 2015).

Moreover, the necessary annual growth rate in pre-salt to maintain current offshore production levels hovers between 7.1% and 8.9%, depending on the model. This statement answers the first question established in the introduction of this thesis. This annual growth rate is well below the observed average increase in the production rate of pre-salt oil, which was 54% in the period 2010 to 2015. Another option (alternative or complementary) would be to start a new cycle in the post-salt basins based on EOR, as happened in the US (Alvarado and Manrique, 2010; Manrique et al., 2010). The post-salt peak year is estimated to be around 2010/2011 by all models. The asymmetric two cycle model has a slightly lower RMSE than the one cycle model, while the two-cycle symmetric model has the lowest RMSE, and the CV of 3%-4% indicates a coherent fit of all three models.

Table 8: Summarized results from Hubbert cycles

	Single Cycle Model	**Two-Cycle Model Symmetric**	**Two-Cycle Model Asymmetric**
URR *(BBO)*	18.4	15.2	21.0
Np/URR	64%	78%	56%
Remaining recoverable offshore post-salt oil reserves (BBO)	6.6	3.4	9.1
t_m *(year)*	2011	2010	2010
RMSE	25E6	20E6	24E6
CV (%)	4%	3%	4%
Average necessary annual growth rate of pre-salt to maintain 2015 offshore production levels in the period 2016-2025	7.4%	8.9%	7.1%
Annual decline rate estimated for post-salt offshore oil production in the period 2016-2025	6.7%	13.7%	7.2%

The individual results for the two-cycle models are shown in Table 9. This table shows that the asymmetrical model estimates a considerable higher URR for shallow and deep water than the symmetrical one. In any case, the remaining recoverable offshore post-salt oil reserves in shallow water reservoirs are lower in absolute and relative terms than in deep water. The peak for shallow water in the asymmetrical model is earlier, by three years, while the RMSE is significantly lower. The estimate of the peak in deep water production (2010) is the same as the peak in total post-salt offshore production shown in Table 8. The RMSE in shallow water is lower than in deep water, which is consistent with the lower levels of oil production from shallow water. However, shallow water cycles have higher CV levels than deep-water cycles, which indicates the goodness of fit for deep water is better than for shallow water. Figure A2 in Appendix A shows the three models fail to capture the production peak fully.

Table 9: Detailed results for two-cycle models

	Shallow Water		**Deep Water**	
Model type	*Symmetrical*	*Asymmetrical*	*Symmetrical*	*Asymmetrical*
URR (BBO)	3.78	4.30	11.47	16.68
Np / URR	92%	81%	73%	50%
Remaining recoverable offshore post-salt oil reserves (BBO)	0.3	0.8	3.1	8.3
t_m *(year)*	1994	1991	2010	2010
RMSE	21E6	14E6	25E6	22E6
CV (%)	17%	10%	4%	4%

The application of the back-testing methodology allows deriving the relative error (Figure 17) and the URR (Figure 18) from the Hubbert estimates. The relative error declines almost invariably the longer the time-series. URR declines for the single cycle model from 2004 on but increases (significantly) before and proves to be unstable. Figures A3-A5 in Appendix A show in detail the forecasts and the overshoot of the single and two cycle models in some years. Figures A6-A7 in Appendix A show in detail the fit of the two cycles for the symmetrical and the asymmetrical model.

The two-cycle models are much more stable and URR increases (except for the first two years), when the used time series is extended. This clearly shows the advantage of multi-cycle models over single-cycle ones and demonstrates the limitations of estimating a Hubbert curve with a short time series of historical production data. This applies, for example, to the Brazilian pre-salt oil production, which has recently started production.

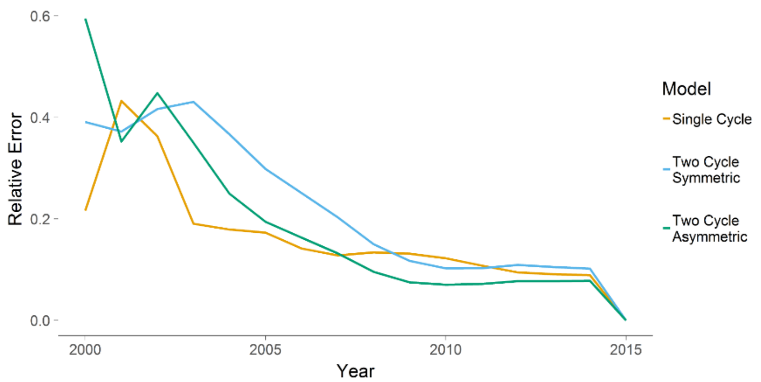

Figure 17: Relative error defined by the production from time T to 2015.

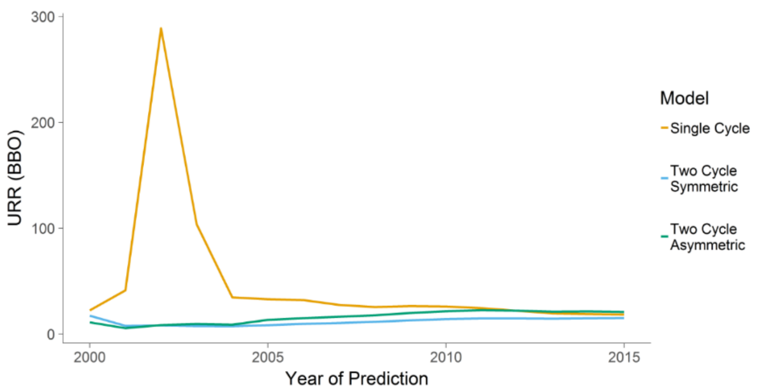

Figure 18: Estimated URR defined by the production from time T to 2015.

4.1.2. *Results from Kaufmann's approach*

As outlined in the methodological section, this study first applies the ADF test to the non-differenced time-series to test for stationarity. None of the tested time-series is stationary. Hence, this study uses the first difference to estimate the model. For the first differences, the ADF test rejects the null hypothesis of nonstationary on a 10% significance level for all variables. After estimating the regression model in differences, this study tested for auto-correlation and heteroscedasticity in the residuals of the models. As no auto-correlation in the residuals was rejected by the BG-test, this study included the differenced R_t with lag 1 in the model. In the resulting models, the BG-test does not reject the null hypothesis that there is no serial correlation and neither the BP-test rejects the null hypothesis of homoskedasticity. Table 10 shows the respective results. This study, therefore, estimated coefficients with the standard OLS (ordinary least squares) estimation procedure. In total, this study estimated three models: one for each of the three Hubbert variants. As the F-statistic (p-value) of the overall significance test is less than 5% for the three models, the null hypothesis that the fit of the intercept-only model is equal to this model is rejected.

In general, the estimates of the three models are similar: the first lag of R_t is highly significant, as is ΔP_t, which controls for the change in production from year to year. The oil price is significant on lags 5, and partly on lag 4 – except for the asymmetrical model. Still, these results indicate that with a delay of 4-5 years, oil production starts to deviate from the Hubbert estimate in the direction of the oil price change. This statement answers the second question established in the introduction of this thesis.

All coefficients are positive, except for the one-year lag of the oil price, which is however not significant. The factor PC'_t controls for the asymmetry of the curve. The asymmetry is more evident in the first cycle; however, it is not significant for any of the models.

Table 10: Results of regression. Observation: all variables in first differences

	Single Cycle Model		**Two-Cycle Model Symmetric**		**Two-Cycle Model Asymmetric**	
	Coefficient Estimate⁺		*Coefficient Estimate*⁺		*Coefficient Estimate*⁺	
Intercept	9.20E-04		2.92E-03		6.61E-03	
R_{t-1}	5.53E-01	***	6.23E-01	***	4.65E-01	***
$BP_{(t)}$	5.46E-04		2.22E-04		3.38E-04	
$BP_{(t-1)}$	-1.12E-04		-2.70E-04		-5.36E-04	
$BP_{(t-2)}$	4.14E-04		1.95E-04		1.52E-04	
$BP_{(t-3)}$	7.84E-04		3.58E-04		2.93E-04	
$BP_{(t-4)}$	1.70E-03	.	1.02E-03	.	8.98E-04	
$BP_{(t-5)}$	3.49E-03	*	2.18E-03	*	1.95E-03	*
PC'_t	6.72E-03		-3.28E-03		6.45E-03	
ΔP_t	2.65E-03	***	2.38E-03	***	2.23E-03	***
BG-test#	0.28		0.40		0.10	
BP-test#	0.78		0.61		0.99	
Adjusted R^2	0.53		0.63		0.16	
F-statistic (p-value)	<0.0001		<0.0001		0.0337	

+ Auto-correlation and heteroscedasticity robust estimator by Newey and West (1987)
#BP-test: p-value of Breusch-Pagan test, BG: p-value of Breusch-Godfrey test.
****,*,. indicate significance on the 0.001, 0.05, and 0.1 levels*

4.1.3. Results from preliminary pre-salt Hubbert models

Lula, Sapinhoá, Jubarte, Mero, Lapa, Búzios, Sépia and Itapu hold a STOIIP of 68.6 billion barrels of oil. After applying the different scenarios of RF and adding the URR from Mero field, four scenarios of URR for pre-salt are obtained: 14, 17, 24 and 44 billion barrels of oil (Figure 19).

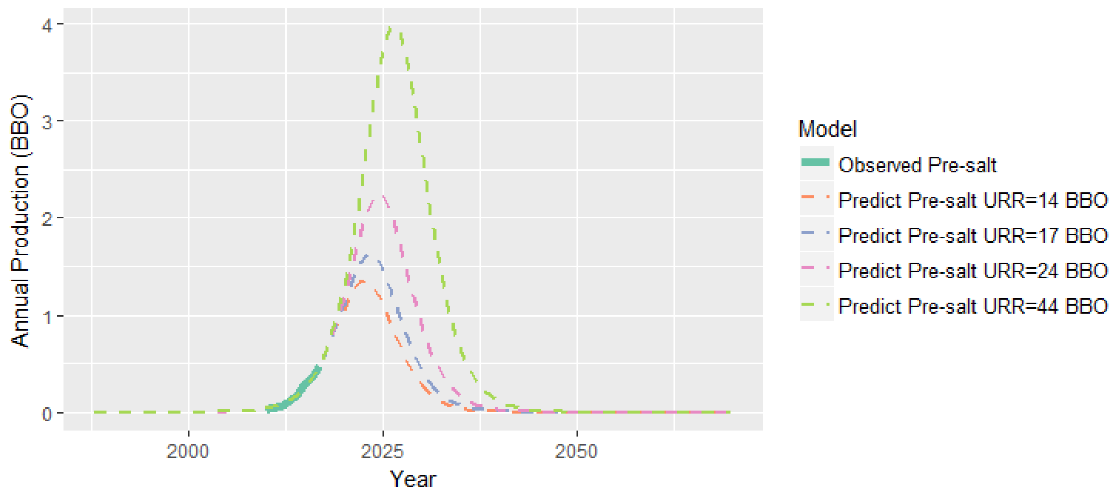

Figure 19: Pre-salt oil production curves

In the four scenarios analyzed, the peak of pre-salt production would occur up to the next decade, when it would start to decline. In the pessimistic and base scenarios (respectively, 15% RF and 20% RF), the peak of production would be about 4 mbd of oil in 2022-2023 and in the most optimistic scenario (60% RF) the peak of oil production would be around 11 mbd of oil in 2026. The year of peak hover between 2022 and 2026. It is worth mentioning that the production from pre-salt is approximately 1.4 mbd of oil in December 2018. The high degree of variability is expected since the uncertainty is expressed through four highly different scenarios of RF (from 15% to 60%) to estimate the $URR_{pre-salt}$. Even though a 60% RF is not considered reasonable for production in ultra-deep-water, it is possible to infer from this plain effort that there is potential for pre-salt production at least double over the next decade.

4.1.4. Discussion

Standard classification of oil resources regarding the probability of recovery considers barriers to the commercialization of projects. This approach, therefore, generates ranges of uncertainty for oil production forecasts (SPE et al., 2018). However, this study did not evaluate the uncertainty at a project level once it has applied a top-down methodology, where these bottom-up level data are not used. Therefore, no classification of resources concerning the probabilities of recovery is made by this study.

The top-down approach proposed by Hubbert is well established to obtain an aggregate production scenario, but it does not permit to analyze individual oil fields. In this case, a bottom-up approach deriving from the combination of all individual field behaviors would be necessary to forecast non-trivial oil production profiles. This would reflect more closely the reality of each field as well as the whole system. Also, a fragility of this study lies in not considering future cycles deriving from post-salt EOR/IOR projects. This study does not compare the cost-benefits of pre-salt or EOR/IOR development: both alternatives involve high cost, ultra-deep-water development, new technologies, and risks.

The econometric analysis of the residuals of oil production from the Hubbert model was limited to a few variables: the oil price and structural variables. Nevertheless, significant coefficient estimates were obtained for the lagged oil price. This indicates that future deviations of oil production levels from the Hubbert curve may be forecasted with today's oil prices. Further work in that direction seems to be promising.

The stability analysis obtained from the back-test shows the URR of Brazil´s post-salt stabilizes around 21 billion barrels in the asymmetrical two-cycle model and 15 billion barrels for the symmetrical two cycle model, while the single cycle model is in-between. Table 11 presents the estimated Brazilian offshore post-salt URR obtained from previous studies based on similar Hubbert models. Oddly, the more recent study (Saraiva et al., 2014) – presented in Table 11 – estimates a much higher URR than this analysis and an earlier study by Ferreira (2005).

Saraiva et al. (2014) consider probabilistic scenarios for the Brazilian crude oil URR according to the probabilities of adding reserves in deep waters (including EOR). The different URRs estimated by Saraiva et al. (2014) result from three scenarios of remaining recoverable resources based on USGS (2000) data and the sum of cumulative production. Differently, this study estimates endogenously and deterministically the URR deriving only from historical oil production data.

More recently, USGS (2012) re-developed the geological assessment of undiscovered conventional technically recoverable oil and gas resources in Brazil's sedimentary basins, according to their associated probabilities. However, the geology-based assessment methodology did not separate resources into onshore and offshore neither post-salt and pre-salt, which makes comparison difficult. The post-salt remaining resources off the coast were estimated from Sergipe-Alagoas Basin, Espírito Santo Basin, Campos Basin, and Santos Basin, although it includes pre-salt and coastal resources. The URR is the sum of post-salt cumulative production and the post-salt remaining recoverable resources. From the remaining oil assessment mean, the pre-salt reservoirs have an estimated 55.6 billion barrels of oil (USGS, 2012), resulting in approximately 32.7 billion barrels of URR from post-salt (offshore and onshore).

As EOR processes barely occurred in the post-salt layer, the URR endogenously estimated by this work does not consider the EOR potential (differently from Saraiva et al. (2014), who estimate the URR including the EOR potential based on based on USGS (2000). For this reason, this analysis is mostly coherent with Ferreira (2005), who assumed the offshore peak production occurs in 2010.

Table 11: Previous URR estimates of post-salt offshore production based on Hubbert methodology for Brazil

Source	Scenario	URR (billion barrels)
USGS (2012)	Mean^{+}	32.7
Saraiva et al. (2014)	P95	27.7
	P50	46.3
	P5	105.0
Ferreira (2005)		17.5

$^{+}$ *Includes offshore and onshore post-salt resources*

Some simulation models optimize oil production dynamics in Brazil (Castro and Filho, 1998; Oliveira, 2006; Junior, 2010; Santana, 2012; Paiva, 2012; Nascimento, 2013). However, these previous simulation models were more concerned about the dynamics of the oil production systems than estimating the expected oil to be recoverable for offshore post-salt production. Comparing this work's results to those is therefore not possible.

Other studies do not report the URR, but published decline rates instead. This study therefore also determined decline rates from the fitted curves and compared them to published ones. However, there is no published information on the total post-salt decline rate - instead, this work used the decline rate from the Campos Basin (Table 12). For that basin, Ferreira (2016) estimates that the rate of decline of fifteen oil fields (consisting of the twenty biggest producing oil fields in Campos Basin), weighted by the production in 2015, was 12.6%. Petrobras (2016) indicates in its Strategic Plan 2017-2021 that the oil production in the Campos Basin has a stable decline rate of around 9% (below the industry average of 12% for deep-water wells). Canheu and Sobreira (2014) show the decline rate for the Campos Basin varies with time, being dependent on well maturity. The average rate of decline for the offshore oil production hovers around 8% per year. This analysis derived from the Hubbert estimates results in a decline rate of 6.7%-13.7%, depending on the chosen model (see Table 8). This is well in line with the three results presented below.

Table 12: Previous rate of decline estimates for the Campos Basin

Source	**The annual rate of decline (%)**
Ferreira (2016)	12.6
Petrobras (2016)	9.0
Canheu and Sobreira (2014)*	8.0

**Calculated considering an oil production of 1800 kbd in 2010 and oil production of 400 kbd in 2020.*

This study estimates the peak of pre-salt production using the classic Hubbert methodology due to the increasing and representative amounts of reserves and oil production from the pre-salt layer. Even though, fitting Hubbert models at such an early stage of production can generate unstable results.

This study estimates the peak of pre-salt production considering the data available for the STOIIP. Some fields were not included in this estimative due to the lack of public information of STOIIP. Even though the Hubbert curve takes into account the possibility of future discoveries, the possible lack of information for past discoveries may drive to inaccurate fits and thus to misleading forecasts. For this reason, results can be considered conservatives. Moreover, this preliminary analysis does not include any political, regulatory, economic and technological aspects which can affect the pre-salt cycle of production.

4.1.5. Conclusion

This study estimated Brazilian post-salt offshore oil production curves using single- and multi-cycle Hubbert models. It concluded that technological advances in E&D in deep water contributed to deviating the real production from a single cycle-fitted Hubbert curve. This thesis' URR estimates hover in between 15 and 21 billion barrels and show that pre-salt production must increase by around 7%-9% per year to offset the declines in post-salt production (without considering EOR/IOR in this case).

The analysis clearly shows that the applied multi-cycle models are more stable than the single cycle model. The two-cycle model symmetric produces lower estimates of URR than the single-cycle model, indicating that the URR can be slightly slower than estimated in previous works. However, this may be partially explained by the fact that this study does not consider the possibility of reserve additions through EOR/IOR. With a lag of 4-5 years, the deviation of oil production from the Hubbert curve follows oil prices changes. That means that a significant amount of time elapses before oil production in Brazil adjusts to changing price levels.

Considering the STOIIP from the pre-salt fields Lula, Sapinhoá, Jubarte, Mero, Lapa, Búzios, Sépia and Itapu, the analysis based on the classical model of Hubbert show the peak of production from this layer may occur between 2022 and 2026.

4.2. Creaming curves

This thesis hypothesizes the reason why a significant amount of time elapses before oil production in Brazil adjusts to changing oil prices in the Hubbert model is related to the long-term effect of the oil prices for investments in new projects. This thesis hypothesizes that four-to-five years elapse after a discovery happens to start-up its production. In order to better understand the exploratory cycles and assess potential oil exploration in Brazil, this thesis develops creaming curves.

4.2.1. Results

The number of wildcat wells concluded by basin are shown in Figure 20, where it is observed the exploratory effort of drilling wildcat wells is concentrated in a dozen basins.

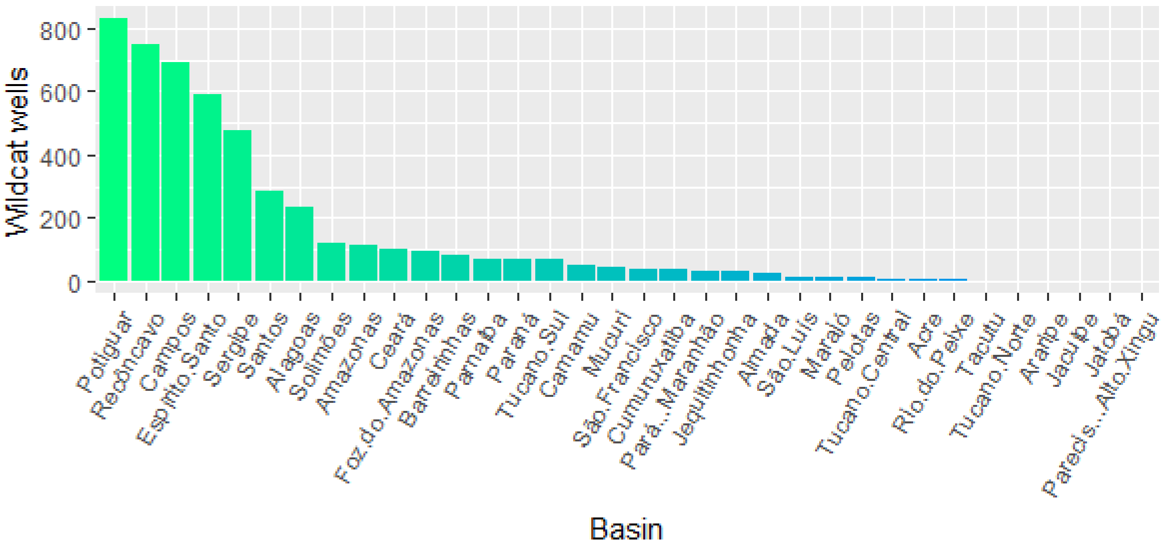

Figure 20: Number of wildcat wells drilled per basin in Brazil

From the Development Plan summaries, this thesis obtains that there are few basins with STOIIP discoveries. Among them, four basins concentrate 95% of the STOIIP discovered: Campos, Santos, Recôncavo, and Potiguar (Figure 21). This study focuses on the analysis of these basins.

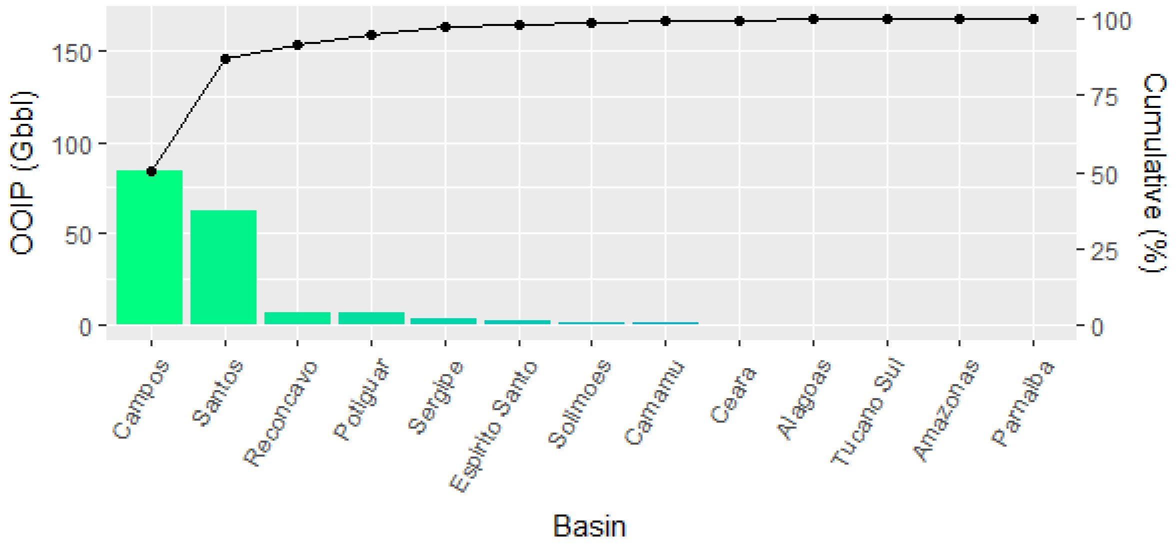

Figure 21: Volume of STOIIP per basin

The estimated recovery factor, respectively, for Campos, Potiguar, Recôncavo and Santos basins results is i) RF_{1P}^{basin} of 21%, 18%, 26%, 13%; ii) RF_{3P}^{basin} of 24%, 20%, 27%, 26%.

The accumulated size of discoveries over exploratory effort is fitted by a Gaussian, Gompertz and logistic functions. As the size of fields' discoveries derives from a basin's recovery factor, the curve-fitting obtained from 1P reserves and 3P reserves results in the same curve shape, only one being a shift from the other. This fact can be observed through the same coefficient of variation between 1P and 3P reserves within the same basin. Figure 22 presents the curve-fitting derived from 3P reserves.

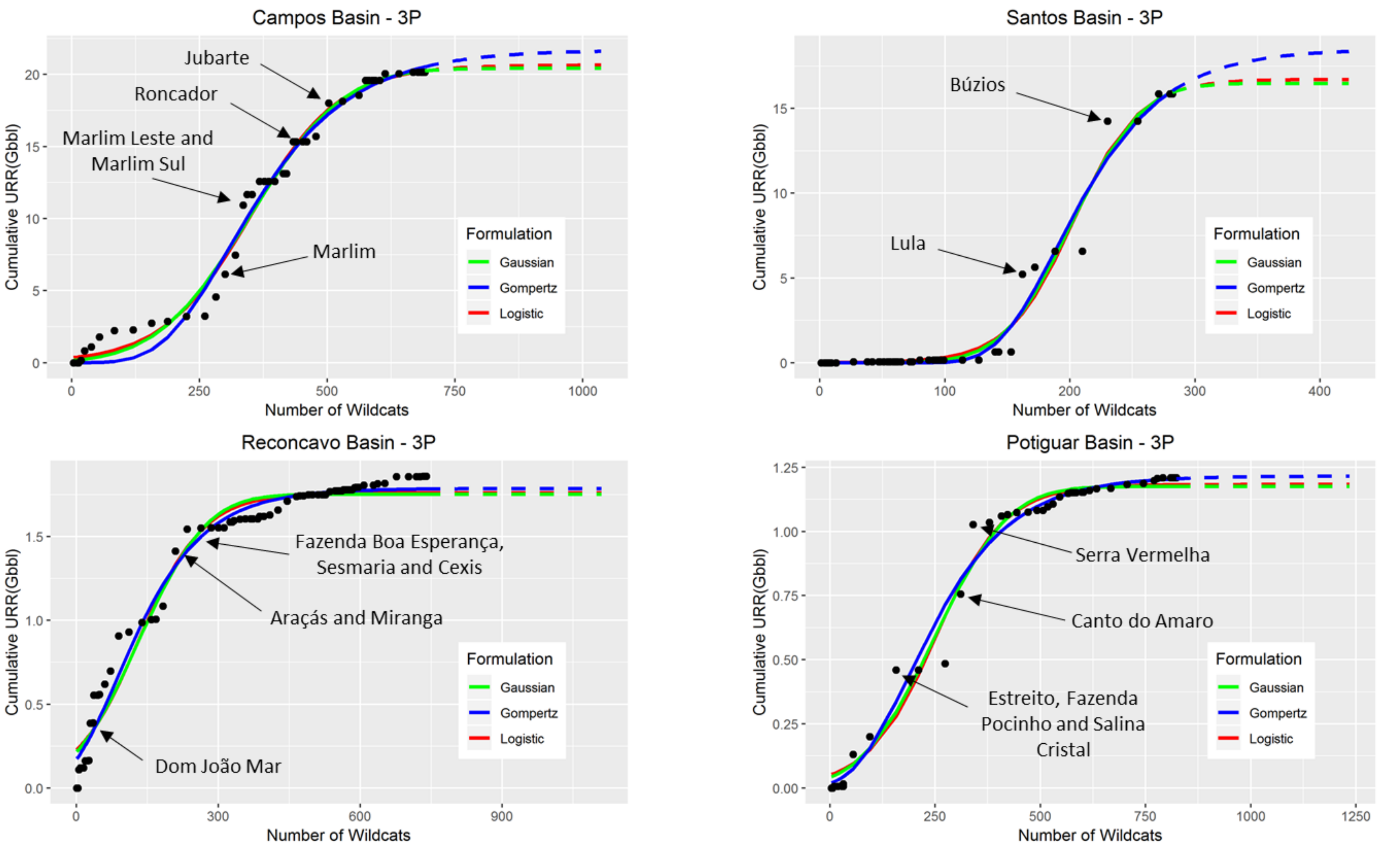

Figure 22: 'True' creaming curves per basin

Tables C1-C6 (in Appendix C) present the statistics of the three functions fitting derived from 1P and 3P reserves. The accuracy of the fitting results in the CV_j^{basin} factor of 4.1%-4.7%, 4.4%-4.6%, 4.8%-6.0%, 4.6%-4.9%, respectively, for Campos, Santos, Recôncavo, and Potiguar basins (Table 1), which shows a reasonable match to the data. For the fit of the rate of production to a Hubbert curve, Maggio and Cacciola (2009) and Saraiva et al. (2014) obtain relative errors around 2.5%, Nashawi, Malallah, and Al-Bisharah (2010) obtain relative errors ranging from 0.59% to 6.57%, whereas Maggio and Cacciola (2012) achieve relative errors from 2.3% to 11.8%. Hallack et al. (2017) obtain relative errors ranging from 3% to 4% for single-cycle, multi-cycle, and deep water; nevertheless, it ranges between 10% to 17% for shallow water.

The potential for yet-to-find discoveries is obtained by the difference between the total cumulative volume of discoveries and the cumulative volume already discovered. This difference is negative for Recôncavo basin regarding the three functions fitted. This difference is also negative for Potiguar basin regarding the logistic and Gaussian functions, whereas the Gompertz function results in a slight potential of yet-to-find discoveries. Such negative difference also suggests a fragility of the method, once the curve extrapolated results in a misleading forecast of discoveries.

For Campos and Santos basins, this thesis estimates a yet-to-find oil potential of, respectively, 233-1336 million barrels (MMbbl) and 314-1350 MMbbl considering 1P reserves (Table 13). If 3P reserves are considered, this thesis estimates a yet-to-find potential of 267-1534 MMbbl and 615-2644 MMbbl, respectively, for Campos and Santos basins (Table 13).

For Santos and Campos basins the uncertainty in the yet-to-find discoveries is more significant; for Rôncavo and Potiguar basins, a small uncertainty in the yet-to-find estimative is obtained (Table 13).

Table 13: Summary of results of model fitting

j	Basin	Total URR (MMbbl)		Uncertainty (MMbbl)	Historical number of wildcats	CV^{basin}
		Current URR	Yet-to-find discoveries			
Logistic function						
1P	Campos	17,563	429	±295	691	4.1%
	Santos	8,100	435	±292	282	4.6%
	Recôncavo	1,784	-96	±16	740	5.9%
	Potiguar	1,124	-25	±13	823	4.9%
3P	Campos	20,157	493	±339	691	4.1%
	Santos	15,859	852	±573	282	4.6%
	Recôncavo	1,860	-100	±17	740	5.9%
	Potiguar	1,210	-27	±13	823	4.9%
Gaussian function						
1P	Campos	17,563	233	±295	691	4.3%
	Santos	8,100	314	±286	282	4.5%
	Recôncavo	1,784	-103	±16	740	6.0%
	Potiguar	1,124	-32	±12	823	4.8%
3P	Campos	20,157	267	±338	691	4.3%
	Santos	15,859	615	±561	282	4.5%
	Reconcavo	1,860	-108	±16	740	6.0%
	Potiguar	1,210	-35	±13	823	4.8%
Gompertz function						
1P	Campos	17,563	1,336	±502	691	4.7%
	Santos	8,100	1,350	±588	282	4.4%
	Reconcavo	1,784	-70	±15	740	4.8%
	Potiguar	1,124	5	±15	823	4.6%
3P	Campos	20,157	1,534	±576	691	4.7%
	Santos	15,859	2,644	±1,151	282	4.4%
	Reconcavo	1,860	-73	±16	740	4.8%
	Potiguar	1,210	5	±16	823	4.6%

For Campos basin, this work predicts a yet-to-find potential of 233 ± 295 MMbbl to 1336 ± 501 MMbbl, respectively, for Gaussian function and Gompertz function considering 1P reserves, whereas a yet-to-find potential of 267 ± 338 MMbbl to 1534 ± 576 MMbbl is predicted considering 3P reserves.

For Santos basin, this work predicts a yet-to-find potential of 314 ± 286 MMbbl to 1350 ± 588 MMbbl, respectively for Gaussian function and Gompertz function considering 1P reserves, whereas This thesis predicts a yet-to-find potential of 615 ± 561 MMbbl to 2644 ± 1151 MMbbl considering 3P reserves.

4.2.2. Discussion

The estimated RF_{1P}^{basin} and RF_{3P}^{basin} for each basin results, respectively, in an average recovery factor of 20% and 24%. Such result is coherent with ANP (2017), which estimates a recovery factor of 19% (for 1P reserves) and 25% (for 3P reserves) in reservoirs with any historical production in Brazil. The average recovery factor suggests potentials to increase reserves from known fields at Recôncavo, Santos, Campos, and Potiguar basins if it is considered that the average recovery factor in the North Sea (specifically in Norway and the United Kingdom) is 46% due to the intense use of EOR/IOR projects (ANP, 2017). A critical simplification was made by assuming that the recovery factor is the same for all fields in the same sedimentary basin.

The highest number of wildcat wells drilled among the analyzed basins are in Recôncavo and Potiguar basins (which reserves are mostly from onshore), where findings do not indicate a potential for yet-to-find discoveries (except a slight potential identified for the Potiguar basin by the Gompertz function). Their creaming curves have been flattening out over the last drilled wells, which is coherent with the onshore oil production decline in Brazil. Such curve behavior suggests incremental additions are small and exploration success rates decline, leading to declining prospectivity and higher risks associated with exploration (Kaiser and Narra, 2018).

In Santos and Campos basins the findings suggest an increase in exploratory investments could promote the incorporation of yet-to-find discoveries. Despite the last considerable volumes of discoveries in the Santos basin, findings show it can still be promising to invest in exploration therein because the discovery curve is rising steeply. Such curve behavior suggests exploration is efficient and prospectivity is high because vast reserves are being found quickly (Kaiser and Narra, 2018). This result is consistent with the fact Brazil's exploration intensity indicators suggest its sedimentary basins are still frontier exploration areas (Szklo et al., 2007).

The URR estimated for Campos' basin is 17.5-22.3 billion barrels of oil, respectively for 1P reserves fitted to the Gaussian function and 3P reserves fitted to the Gompertz function. These URR estimates (17.5-22.3 billion barrels) are calculated as the sum of current URR plus estimated yet-to-find discoveries including uncertainties.

Considering that most pre-salt resources are located in Santos basin and the majority of post-salt offshore resources are located in Campos basin, this thesis can infer that this estimation is coherent with the 15-21 billion barrels of oil estimated for the post-salt offshore layer by Hallack et al. (2017).

The yet-to-find potential estimated by this study for Campos and Santos Basin is much lower than the mean of the geological assessment of undiscovered technically recoverable oil estimated by USGS (2012): 14,736 MMbbl of oil and 59,689 MMbbl of oil, respectively, for Campos and Santos basins. This thesis infers that this difference can be due to three reasons. The first one is the lack of data regarding the size of some discoveries that are consequently affecting the fit of the logistical curve. The second one is the use of a historical recovery factor, which does not consider possible future increments in the recoverability by the application of EOR/IOR techniques. The third one is the application of the creaming curve methodology, which results in more conservative estimates than predicted by the USGS' geology-based methodology.

For Santos basin (where huge discoveries are situated, and which holds around 95% of the petroleum pre-salt production), the yet-to-find potential estimated may be more prominent as important discoveries were not included in the creaming curve – e.g., Júpiter and Carcará prospects did not declare commerciality yet, and Mero field (the former Libra prospect) does not have its Development Plan disclosed. For Campos Basin, important pre-salt discoveries were not included in the creaming curve as well, because they did not declare commerciality yet – e.g., Gávea, Pão de Açúcar, and Seat (within the block BM-C-33) (Wood Mackenzie, 2018).

The resulting yet-to-find discoveries and URR estimates are almost comparable for two functions (logistic and Gaussian), but very different (higher) in the case of Gompertz function. The Gompertz approach provides more optimistic results (for the four Brazilian basins investigated) because this functional form is asymmetric; the right-hand period (between the inflection and the final asymptote) is approached much more gradually by the curve than the left-hand period (between the first asymptote and the inflection). According to Sorrell and Speirs (2009), the point of inflection is around 35-40% of the asymptotic for the Gompertz curve. This behavior is in contrast to the logistic and Gaussian functions, in which the asymptotes are approached by the curve symmetrically.

The differences in the estimates by using different functions are not negligible. Despite the application of the findings of this paper to prediction efforts are less certain, the results of this work give an overall picture of the exploration history and potential in Brazil. Moreover, it highlights the uncertainty in URR estimates, which is typical in the oil and gas industry, especially for oil frontier areas. Indeed, estimating this uncertainty provides additional information for policymakers.

Overall, the strength of this work lies in its relevance. Brazil is the main frontier of deep-water oil resources (EIA, 2016) and there is a need for scientific studies embedding Brazil's pre-salt oil resources. The creaming curves technique is well studied and depicted in the literature, however, this is one of the few studies with a transparent methodology available for Brazil.

This study has several limitations. The four basins focused on this study (Campos, Santos, Recôncavo, and Potiguar) represent 97% of 3P reserves in Brazil (ANP, 2018i). However, it uses a sample of 357 fields in which the information of STOIIP and date of discovery is publicly available. Some discoveries were not included due to the lack of information within the Development Plan summaries, because it is not disclosed or the field’s commerciality declaration was not made yet.

Hubbert assumes the cumulative discovery cycle has the same format as the cumulative production cycle, both following a logistic function (Hallack et al., 2017). Firstly, Kaufmann (1991) assumes economic and political events may cause annual rates of production to deviate from Hubbert's curve systematically. Secondly, Korsvold (2015) suggests the creaming curves of a petroleum province take a logistic form when they are not affected by technical, economic, geological, and political variables. Thirdly, the curve fitting technique does not allow for anticipating future cycles of discoveries, and it can be susceptible to the selection of functional form (Sorrell and Speirs, 2009; Sorrell et al., 2009). Fourthly, this thesis neglects the effect of reserve growth, which leads to changes in the shape of the creaming curve (Sorrell et al., 2009; McGlade, 2013).

Though this thesis recognizes many aspects can have an influence on the creaming curve's shape, this study aimed at examining the general behavior of discoveries through the exploratory effort in Brazil.

4.2.3. Conclusion

This thesis shows that a few basins concentrate the exploratory effort and there is an unknown potential to be explored in Brazil. The developed methodology permits to model the yet-to-find discoveries by basin. Due to the lack of information for reserves' size by field, this methodology proposes and applies a creaming curve variant that can be suited by many other countries within this issue. Such variant applies an average recovery factor at a basin-level to estimate the discoveries' size by field.

This study highlights that different functional forms fit the data well and similarly, but they lead to different estimates of remaining resources. This work observes that Santos basin estimates are the most uncertain regarding the choice of the function (considering 3P reserves). This greater uncertainty is coherent with the fact that Santos basin is the most recent frontier exploration area in Brazil. Besides this, it is observed that the uncertainty in reserves estimates has an essential role in the uncertainty of yet-to-find discoveries. For Campos basin, where there are not many differences between 1P and 3P reserves, the choice of the function plays a significant role in the estimate of remaining resources.

Even though the Santos basin has a broader range of yet-to-find discoveries, most remaining resources are identified there for all scenarios of reserves and functions used as input to fit the creaming curve. The proposed estimate enables one to design a policy that meets the challenges to accelerate the development of this basin.

The steep rise in the discovery curve for Santos basin is coherent with the new exploratory cycle of pre-salt discoveries. This identification enforces the need to set up an appropriate regulatory framework for such an essential cycle of discoveries.

The results can benefit financial institutions interested to invest in exploratory activities in Brazil as well as experts concerned about applying the creaming curve variant for other countries where field-level data is not accessible. In the light of increasing social problems and a recent period of economic recession in Brazil, the estimates of remaining resources provide further information for policymakers and help them to envision the revenues that can derive from fostering this activity in the country.

Finally, policymakers can adjust the fiscal regimes that encompass the petroleum exploratory activities in order to represent better the discovery potential of different areas and what should be done in there. As mentioned before, in Brazil three fiscal regimes co-exist.

However, for oil frontier areas the most relevant are the concession (tax/royalty) and the production sharing regimes. The concession regime is also relevant for mature basins requiring EOR/IOR. By better knowing remaining oil resources and understanding the creaming phenomena, policymakers can attract more investments and even refine the terms of contracts to different types of oil operators. For instance, this study has shown that without EOR/IOR mature areas in Brazil have no addition in remaining resources. Nevertheless, these areas can still be attractive to specialized minor operators, whose investment capacity and focus are not associated with very productive areas that require a lot of fixed investment, such as the Brazilian pre-salt.

4.3. CVAR Models

This thesis proposes an econometric model for a better understanding of how some variables affect the number of pre-salt development wells. The primary reason for choosing an econometric model lies on the fact that curve fitting is not suitable for pre-salt layer since pre-salt oil has only recently been started to produce and curve fitting models at such an early stage generate unstable results (Sorrell and Speirs, 2009). The secondary reason lies in the fact that there are advantages of the CVAR model over the classical linear regression models.

Moreover, findings from the creaming curve show potential for further discoveries in the two basins where there are pre-salt fields.

4.3.1. Results - CVAR Models for development wells

Of the one hundred CVAR models for development wells, all models reject the null hypothesis of zero cointegrating relations and the null hypothesis that pre-salt development wells (W) are weakly exogenous. These results indicate that the 100 CVAR models contain a statistically meaningful long-run relationship for the endogenous variable, W.

4.3.2. Results - Most accurate models

Comparisons of the in-sample forecasts identify a single 'most accurate' model for $\Delta Well$. For CVAR models that use WTI to measure prices, the 'most accurate' has an r^2 of 0.79 and uses prices from a future contract with six-month maturity and a BEP of $62 per barrel in 2018 prices ($25 in 1982 dollars) (Table 14). For models that use Brent to measure oil prices, the most accurate has an r^2 of 0.74 and uses prices from a future contract with one-year maturity and a nominal BEP of $25 per barrel in 2018 prices ($10 in 1982 dollars) (Table 15). Of these two models, the more accurate uses WTI to measure oil prices. The most accurate model that measure oil prices using Brent is illustrated in the Appendix D.2.

Table 14: Regression results for the CVAR model chosen as the most accurate

Price	WTI
BEP	$25
Price measure	Six months
Over-identifying restrictions	$\chi^2(3) = 1.856$
α	-0.857
	CR #1
Π	
Well	1**
AWP	-0.705**
Profit	--
Revenue	--
PerVol	-0.32**
Vol	--
Risk	0.443**
μ_0(Constant)	-0.497*
Alpha	
CR #1	-0.857**
Γ_{11}	
$\Delta Well_{t-1}$	-0.238**
A_0	
ΔWell	--
ΔAWP	-0.472
ΔProfit	-0.261
ΔRevenue	0.930*
ΔPerVol	0.507*
ΔVol	-0.443**
ΔRisk	0.675*
A_1	
$\Delta Well_{t-1}$	--
ΔAWP_{t-1}	-0.546^{+}
$\Delta Profit_{t-1}$	-0.643
$\Delta Revenue_{t-1}$	-0.549
$\Delta PerVol_{t-1}$	-0.745**
ΔVol_{t-1}	-0.206
$\Delta Risk_{t-1}$	0.598*
Diagnostics statistics	
ARCH	$\chi^2(2) = 4.289$ [0.117]
Normality	$\chi^2(2) = 3.191$ [0.203]
R^2	0.788

*Test statistics reject the null hypothesis at the **1%, *5%, +10% level.*

Table 15: Regression results for the CVAR Brent model chosen as the most accurate

Price	Brent		
BEP	$10		
Price measure	12 months		
Over-identifying restrictions	$\chi^2(8) = 7.954$		
	CR #1	**CR #2**	
Π			
Well	1**	--	
AWP	-0.7**	--	
Profit	--	--	
Revenue	--	-1.247*	
PerVol	--	1**	
Vol	--	-3.596**	
Risk	0.198*	--	
Alpha	**ΔWell**	**ΔPerVol**	**ΔVol**
CR #1	-0.799**	0.241**	0.093
CR #2	-0.064**	0.036*	0.101**
Γ_{11}			
$\Delta Well_{t-1}$	-0.293**	-0.011	0.109
ΔAWP_{t-1}	--	--	--
$\Delta Profit_{t-1}$	--	--	--
$\Delta Revenue_{t-1}$	--	--	--
$\Delta PerVol_{t-1}$	0.216	-0.056	-0.199
ΔVol_{t-1}	-0.59**	0.038	0.241*
$\Delta Risk_{t-1}$	--	--	--
A_0			
ΔWell	--	--	--
ΔAWP	-0.779	0.234	0.086
ΔProfit	0.218	-0.691	-0.917
ΔRevenue	0.567	-0.198	-0.201
ΔPerVol	--	--	--
ΔVol	--	--	--
ΔRisk	0.868*	-0.007	-0.006
A_1			
$\Delta Well_{t-1}$	--	--	--
ΔAWP_{t-1}	0.310	0.670	1.586**
$\Delta Profit_{t-1}$	0.104	0.102	-0.157
$\Delta Revenue_{t-1}$	-0.927^{+}	-0.518	-1.251**
$\Delta PerVol_{t-1}$	--	--	--
ΔVol_{t-1}	--	--	--
$\Delta Risk_{t-1}$	1.115**	-0.351	-0.325
Diagnostics statistics			
ARCH	$\chi^2(2) = 1.243$ [0.537]	$\chi^2(2) = 26.741$ [0.000]	$\chi^2(2) = 4.487$ [0.106]
Normality	$\chi^2(2) = 6.451$ [0.04]	$\chi^2(2) = 103.318$ [0.000]	$\chi^2(2) = 5.597$ [0.061]
R^2	0.739	0.409	0.581

*Test statistics reject the null hypothesis at the **1%, *5%, +10% level*

4.3.2.1.The most accurate WTI model

In this most precise model, the cointegrating relation for *W* has a negative long-run relationship with country risk and a positive long-run relation with the perceived volatility and productivity; overidentifying restrictions eliminate the other variables.

The negative relation between well completions and country risk is consistent with expectations. However, this effect is offset by a positive short-run relationship given by the element of A_o that is associated with Risk. This short-run effect slows the rate of adjustment but does not alter the equilibrium level of well completions that are associated with country risk. In other words, the negative long-run relation between well completions and risk prevails.

The positive relation between well completions and perceived volatility is the opposite of that expected. This positive relation is offset by a substantial and precisely measured negative relationship between the lagged first difference of *PV* in the A_1 matrix (Table 14). This short-run effect slows the rate of adjustment but does not alter the equilibrium level of well completions. In other words, the positive long-run relation between well completions and perceived volatility prevails.

Deviations from equilibrium are eliminated relatively quickly; the point estimate for error correction term (α) is -0.857. This point estimate is not statistically different from -1.0 (t = -1.4944, p > 0.14) which implies an instantaneous rate of adjustment. This rapid adjustment seems inconsistent with the long-term planning horizon for drilling in deep-water in general and the pre-salt zone in particular. The notion of 'drilling queue may explain this seeming contradiction' and the small number of pre-salt development wells completed per month (average 2.22 and a standard deviation of 1.74). At any point in time, several projects are in the queue, and the number of projects in the queue that are initiated is based on current conditions.

4.3.2.2. The most accurate Brent model

The most accurate model that measures oil prices using Brent uses the price from a future contract with a one-month maturity and has a nominal BEP of $25 per barrel (Table 15). The long- and short-run relations are similar to the most accurate model that uses WTI to measure price. The cointegrating relationship for *W* has a negative long-run relationship with risk and a positive long-run relation with productivity; overidentifying restrictions ($\chi^2(8) = 7.954$, $p > 0.438$) eliminate the other variables. The negative long-run relation with risk is offset by a positive relationship with the lagged first difference in risk. Similarly, disequilibrium in the long-run relation among *W*, *Risk*, and *AWP* is eliminated quickly: the point estimate for error correction term (α) is -0.799, which is not statistically different from -1.0 ($t = -1.745$, $p > 0.085$).

A second cointegrating relation includes perceived volatility, volatility, and revenue and represents the long-run relationship for perceived volatility, as indicated by the element of the α matrix associated with perceived volatility. Disequilibrium in this second cointegrating relation also loads into the equation *W* as shown by the component of the matrix α associated with *W* -0.064. This component indicates that *W* has a negative short-run relation with *PV* and a positive short-run relationship with *Rv* and *Vol*. The negative association with *PV* and the positive relation with *Rv* is consistent with expectations. The positive short-run relation with *Vol* in the second cointegrating relation is offset by the negative relationship with the lagged first difference in volatility in the matrix Γ_{11} (Table 15).

4.3.3. Discussion

4.3.3.1. Measuring oil prices

At first glance, it is surprising that the most accurate CVAR model uses WTI to measure oil prices. Brazilian exports of crude oil to the US decline during the sample period while exports to the Asia-Pacific region (mainly to China) increase. The Asia-Pacific region is the largest (54%) export market for Brazilian crude, and China purchases 42% of exports (442 thousand barrels per day) in 2017 (ANP, 2018j). Due to the increasing prevalence of Brent in the consumption patterns of Asian refiners (Platts, 2011), Ohara (2014) uses the price of Brent to calculate the break-even price for exports of crude oil from the pre-salt zone to China. Europe imports another 10% of Brazilian crude oil exports in 2017. Petrobras uses the price of Brent in presentations to investors and within the Business and Management Plan. Together, this suggests that Petrobras uses the price of Brent to plan development in the pre-salt zone.

Instead, the accuracy of the CVAR model that uses WTI may be associated with Brazil's role in the international oil market: more than one-third (38%) of Brazilian crude oil production is exported in 2017 (ANP, 2018j). In 2017, the US is the second-largest importer of Brazilian crude oil (17%). Furthermore, the US is the largest (51%) source of Brazil's petroleum product imports, about 314 thousand barrels per day in 2017 (ANP, 2018j). Finally, the Brazilian oilfield service sector purchases many services in US dollars. These trade patterns suggest that the dollar value of crude oil is critical and is the reason that this work does not convert US dollars to local currency units in this econometric analysis.

Finally, the CVAR model for WTI may be more accurate because the number of endogenous variables is equal to the number of cointegrating relations (i.e., the model is full rank). Under these conditions, the stochastic trends in the weakly exogenous variables fully account for the stochastic trends in the endogenous variable; the number of wells drilled into the pre-salt zone. Conversely, the most accurate CVAR model for Brent is not full rank; the number of cointegrating relations is less than the number of endogenous variables. As such, the stochastic trends in the weakly exogenous variables do not account for all of the stochastic trends in the endogenous variables. The unmodeled trends may be associated with shortcomings in the OECD measure for country credit risk, which does not capture all factors that may dampen investments in the upstream oil and gas sector. Unfortunately, it is difficult to quantify some qualitative factors that may affect investments in the upstream oil and gas sector, such as concerns over regulatory enforcement and uncertainty over environmental regulations.

4.3.3.2.Break-even prices

The 'most accurate' CVAR model has a nominal BEP of \$62 per barrel (\$25 in 1982 dollars). This BEP represents a top-down empirical estimate for the price that firms use to schedule drilling in the pre-salt zone. This \$62 BEP is greater than recent bottom-up estimates (see Table 16). This thesis' higher estimate may be caused by ongoing changes in productivity, price volatility, and country risk. As mentioned previously, the BEP that generates the most accurate model represents an average for the sample period. Over the latter portions of the sample productivity increases, which reduces the BEP relative to the sample average. Furthermore, 'bottom-up' engineering estimates ignore the negative effects of country risk and volatility, which may cause them to understate the BEP. Conversely, the BEP in the 'most accurate' CVAR model that uses Brent to measure price has a BEP of $25 per barrel in 2018 prices.

Table 16: Estimated BEP for pre-salt

Estimate ($/barrel)	**According to**	**Reference**
29-49	Strata Advisors (2016)	Strata Advisors (2016)
30 (for Portfolio 2017)	Petrobras (2017b)	Petrobras (2017b)
~ 30-40	IHS Markit and Petrobras	Parente (2018)
35	Ricardo Bedregal	Sreeharsha (2017)
40 (below)	Shell and Equinor (formerly Statoil)	Solbraekke and Nysveen (2016); Paraskova (2017); Shell (2016)
40-55	Brazilian Ministry of Mines and Energy, Petrobras and Pré-Sal Petróleo	Noon (2016); Leahy and Adams (2016); Stevenson (2018)
45 in Libra field	Wood Mackenzie	Paganie (2018)
47 (mostly around)	Deloitte (2018)	Deloitte (2018)
47-59 in Libra field	OpenOil (2014)	OpenOil (2014)
45*	Pedra and Szklo (2018)	Pedra and Szklo (2018)
65-84	McKinsey (2014)	McKinsey (2014)

**Based on a thorough cash flow analysis, using an Internal Rate of Return (IRR) of 11.3% p.y., and applied to the largest oil fields in pre-salt. It also accounts for the price discounts of the typical pre-salt oil to Brent. Accordingly, smaller and less productive fields in pre-salt would deal with higher BEPs, around 60 US$/bbl (Brent Basis).*

4.3.3.3.Determinants of Pre-salt Well Completions

In this section, this thesis evaluates how changes in prices, volatility, technology, and country risk affect the number of wells completed in the pre-salt zone. Their individual effects are quantified by simulating the CVAR model in which the variable of interest follows its historical evolution while holding the value of the other variables remain constant at their sample mean (Figure 23). For example, this thesis evaluates the effect of country risk by simulating the CVAR model using the observed value for *Risk* while holding *Pr*, *Rv*, *AWP*, *Vol*, and *PV* at their sample mean.

Simulations by the two most accurate models indicate that changes in technology and reservoir quality, as proxied by *AWP*, have the most significant effect on pre-salt development wells. As expected, increases in *AWP* generally are responsible for the increasing number of wells completed in the pre-salt zone during the sample period.

The effect of country risk varies between the two models. Country risk has a more significant impact on the model that uses WTI to measure oil prices than in the most accurate model that uses Brent to measure oil prices. The effect of country risk is most notable during the end of the sample period when higher risk depresses the number of wells drilled. This negative effect is consistent with recent analyses, which indicate that poor governance can offset the effect of an abundant resource base. Kaufmann and Banerjee (2014) find that governance, as measured by country risk, influence whether crude oil is part of a unified world oil market. Specifically, the price for crude oil's from nations with a high country risk is less likely to be part of a unified market. Kaufmann (2016) finds that crude oils from nations with a higher country risk suffer a price penalty.

Except for 2015, when volatility increases the number of wells modeled, volatility and perceptions of the volatility generally have a small impact. This modest effect may be caused by the long lifetime of wells drilled into the pre-salt zone. A long-lifetime implies that volatility at the time of E&D will have relatively little effect on the net present value of the revenue stream over a long period. However, this hypothesis is disrupted by Kleinberg et al. (2018), who argue that uncertainty about future oil prices poses a higher risk to deepwater offshore projects because of their long construction schedules and extended production lifetimes compared to tight oil wells.

Prices can affect the development of pre-salt wells through *Pr* and *Rv*. Figure 4 indicates that significant changes in *Pr* and *Rv* have little effect on the number of wells completed. This behavior is consistent with BEP's that are less than or equal to the current price. The small impact of prices may be generated by Petrobras, which focuses on investments in the pre-salt layer based on its high and increasing well-productivity (Sandrea and Goddard, 2016). These increases are generated by technical improvements (Petrobras, 2015c) and the high quality of reservoirs.

Across the two most accurate models, changes in technology and reservoir quality, as proxied by AWP, have the most significant effect on *Wells* simulated by the model (Figure 23). Additionally, Figures A8-A12 in Appendix A shows the effect of each variable on *Wells* simulated by the model.

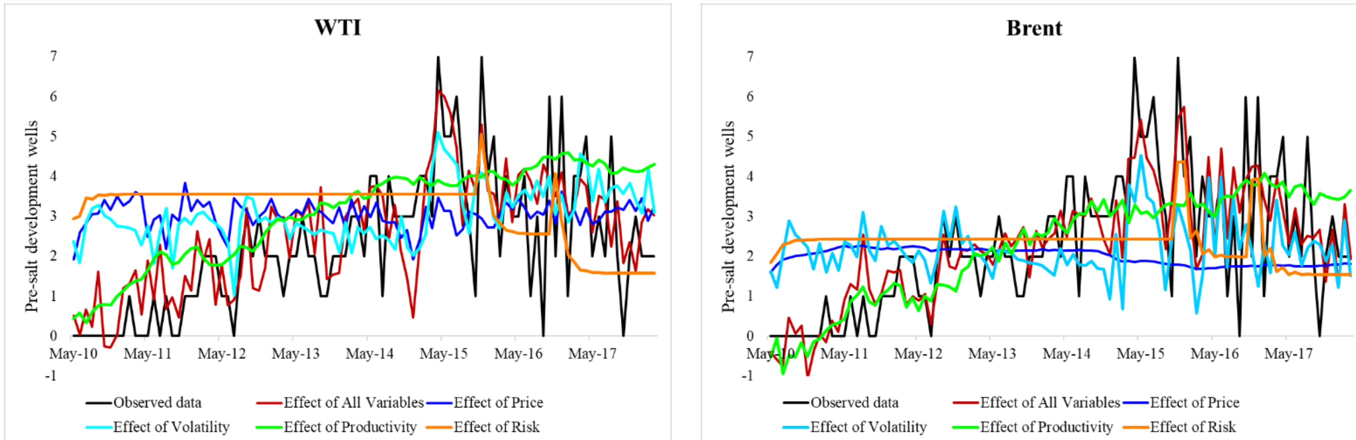

Figure 23: Determinants of pre-salt development wells for WTI and Brent.
The effect of volatility (light blue line), prices (blue line), average well productivity (green line), risk (orange line), and all variables (dark pink line) that is simulated by the CVAR model more accurate between Brent models and WTI models for pre-salt development wells by holding all other variables at their sample mean. Observed values for pre-salt development wells are givens by the black line.

4.3.4. Conclusion

Although prices for WTI and Brent dropped in 2014, the output from the pre-salt zone continues to increase. This increase may be associated with lower costs due to continued learning and standardization (Mariano et al., 2018b). The positive effect of productivity highlights the importance of research and technological development policies for petroleum development in offshore oil frontier areas. This result is consistent with Petrobras' strategy of emphasizing technological development and competence building (Waterworth and Bradshaw, 2018).

Nevertheless, technology and reservoir quality are not the sole determinants of investment decisions in the upstream sector. This thesis' results also indicate that when Brazil's fiscal situation deteriorates (i.e., credit risk rises), economic conditions worsen, the level of indebtedness increases, and oil production slows. Despite technological gains, as measured by average productivity, the most accurate model (and the most accurate model that measures oil prices using Brent) indicates that increases in country risk for Brazil slow the rate at which development wells are drilled into the pre-salt zone. This result suggests that governance in Brazil (measured by the OECD country credit risk) affects pre-salt development. Because pre-salt development is affected by the country credit risk, Petrobras' risk level probably correlates with sovereign risk, as stated by the credit rating agency Standard & Poor's (Petrobras, 2018e).

The stable outlook and slight upgrade in the Petrobras' credit rating by Moody's and Standard & Poor's in the last years (between 2016 and 2018) reflects the ability of the company to improve liquidity and reduce debt.

However, Fitch downgrades Petrobras' credit rating at the beginning of 2018, which suggests something else can prevent Petrobras' credit rating improvement. In this context, a further orientation of Petrobras' policies towards maximizing profit is needed to weaken the market's perception of the correlation between the company's risk level and the sovereign risk. It also indicates that for NOCs or even state-controlled oil companies (partially open capital), the country risk can affect a company's investment-grade rating and its pace of development.

Given China reliance on petroleum imports (as the country is the current world's largest crude oil importer), the higher output from the pre-salt layer creates a favorable landscape for cooperation between China and Brazil (as well as between Brazil and other major crude oil-importing countries) in the oil sector.

The partnership between these countries should not be limited to increasing participation on crude oil imports; China imported 13% of Brazil's crude oil exports in 2008, and this share rose to 42% in 2017. Chinese oil companies participate in the first, third and fifth consortium of companies formed by the previous pre-salt bid rounds. The recent new wave of liberalization in the oil and gas industry sector in Brazil can pave the way for more significant participation of the Chinese oil companies in the next pre-salt bidding rounds.

5. Conclusion and Future Researches

This thesis described the development of curve-fitting and econometric methods to model oil production in Brazil. This work focused on the economic assessment and political influences inherent to the reserve estimation process because the geological and engineering assessments require much more specific data for its evaluation. On top of that, the models developed by this study did not address reserves' criteria defined by different institutions.

This thesis modeled the Brazilian oil production considering asymmetric and multi-cycles adapted from the classical Hubbert model. The developed asymmetric Hubbert model derived from Brandt (2007) based on an asymmetrical Gaussian curve. Inspired by Kaufmann (1991), this thesis used a regression model to explain the differences between the Hubbert model and the observed production data by assessing the influence of techno-economic parameters to the post-salt offshore oil production in Brazil.

Firstly, this thesis indicated that an annual average growth rate by around 7%-9% per year in the pre-salt oil fields can compensate for the decline in post-salt offshore oil production for the period 2016-2025. Additionally, the deviation of the crude oil production rate from the Hubbert curve followed changes in oil price with a four-to-five-year lag. Hence, this thesis hypothesizes that exists a lag of four-to-five-years between discovery and production for post-salt offshore projects in Brazil.

Taking that into account, this thesis used a creaming curve plotted as the cumulative discovery against the number of new field wildcats to assess the potential for further discoveries. The novelty of this methodology lies in the assumption that the recovery factor is the same for all fields within a sedimentary basin to estimate the fields' size of discoveries.

Secondly, the current low recovery factor of 24% in Brazil (compared to the average recovery factor of 46% in Norway and the United Kingdom due to intense use of EOR/IOR projects) will indicate a potential to increase reserves from known fields provided that EOR/IOR projects satisfy reserves' requirements.

The findings did not suggest a potential for further discoveries in the Recôncavo neither the Potiguar basin, whereas an increase in exploratory investments could promote the incorporation of new reserves in Santos and Campos basins. The shape of the creaming curve for the Santos basin indicated its immaturity regarding exploration. Even though the coefficient of variation factor is similar for Santos and Campos basins, the reliability over Santos estimates is lower due to the early stage of exploration in this basin (reflected by the lower number of wildcat wells drilled). Therefore, the volumes associated with further discoveries in the Santos basin (estimated by the three functions in analysis and for 1P and 3P reserves applying creaming curves) are more uncertain than in the Campos basin.

Proceeding on this track, this thesis estimated the dynamic relationship between crude oil prices, price volatility, productivity, and country risk and their effect on the number of development wells that are completed in the pre-salt zones of Brazil. This relation is examined using time series econometric techniques of cointegration and error correction modeling. Such method is more appropriate to analyze the pre-salt development than the curve-fitting method since important volumes of pre-salt discoveries are located in Santos basin and the curve-fitting method is better applied to well-explored areas (Sorrell et al., 2009). Moreover, the technique of cointegration and error correction modeling poses some advantages over the classical linear regression models.

Thirdly, this thesis analyzed the production of oil in the pre-salt layer – which represents 55% of Brazil's oil production in 2018 – twelve years after the discovery of the Lula field. This analysis is conducted in the light of an economic recession, an increase in the country credit risk, and the oil price collapse in 2014. This thesis identified: (1) the break-even price (BEP), (2) the measure of prices (spot or future contract) that can be more accurately used to plan development, (3) the effect of oil price volatility, productivity, oil price and country risk on wells drilled in the pre-salt province.

The most accurate model measured price using the six-months futures contract for WTI and has a BEP of $62 per barrel in 2018 prices ($25 in 1982 dollars). This model indicated that perceived volatility, productivity, and country risk affect the number of development wells drilled into the pre-salt zone, whereas price plays a relatively minor role. The most accurate model that used Brent to measure oil prices had a BEP of $25 per barrel in 2018 prices ($10 in 1982 dollars) and used the twelve-month future contracts and also indicated that productivity and country risk affected the number of development wells drilled in the pre-salt zone.

Despite oil prices collapse in 2014, the oil price did not show a significant role in the observed changes for the number of pre-salt development wells. This result could be due to Petrobras' strategy to concentrate investments in the development of pre-salt, revealed by its recent Business and Management Plans. It could also show the beginning of pre-salt development was carried by high productivity projects with lower BEP, in which the level of prices was not a severe enough constraint to halt pre-salt projects under development. Moreover, albeit oil prices dipped in 2014, the technological learning curve and the process standardization led to reductions in costs in the pre-salt zone (Mariano et al., 2018a). It is also possible to speculate some reasons for such increase in the average productivity of wells drilled into pre-salt zones: wells drilled in better positions due to greater geology knowledge of the pre-salt region; results of water and gas injections; improved acidification (stimulation) of pre-salt wells.

The most accurate model (which uses WTI) had a better coefficient of determination (r^2) of 0.79 whereas the most accurate model that used Brent to measure oil prices had an r^2 of 0.74. Although the most accurate model used WTI to measure oil prices, the difference of 5% in the coefficient of determination suggests the importance of the Brent price for the Brazilian oil industry as well. It is also possible to hypothesize that the deviation of Brazilian oil exports from the US to China in the last decade was not wholly captured by the model applied in this thesis.

Due to the differences in *Vol*, *PerVol*, *Profit*, and *Revenue* between different measures of prices for Brent and WTI models, the lower BEP of $25 per barrel in 2018 prices ($10 in 1982 dollars) from the most accurate Brent model can be partially justified by the prevalence of higher Brent price levels since 2010, especially in the period 2011-2014. The abundant increase in light tight oil production from the US contributes to WTI prices discounts relative to Brent due to the costs of moving WTI from Cushing, Oklahoma to overseas markets where it might compete with Brent (EIA, 2015). Such differential to Brent crude became broader and more volatile after 2010 (EIA, 2014). This trend may continue, as EIA (2018b) expects Brent oil prices will average about $6 per barrel higher than WTI prices in 2018 and 2019.

Future research to estimate CVAR models that analyze the behavior of exploratory effort in the pre-salt layer as well as the E&D in the post-salt layer would improve the knowledge of the dynamic of E&D in the Brazilian oil industry and could evidence the interconnections among the different producing zones. This methodology could still be applied to other countries. Another future topic would be a scenario analysis for pre-salt development by evaluating the individual effect of the country risk, as its increase depresses the number of development wells in the pre-salt zone. Also, a future task would be to estimate CVAR models by evaluating the impact of inflation as a proxy for the Brazilian economy effect. These analyses should be part of upcoming model development efforts.

The availability of existing data was and continues to be a challenge for the accurate representation of Hubbert models, creaming curves and the CVAR models for Brazil. These models overcome the unavailability of data in Brazil, which can be considered as an advantage of this work. However, some previously unavailable data became disclosed by ANP throughout this thesis, which shows some improvement in the availability of data.

Another challenge was to obtain available historical data able to quantitatively represent some of the many factors that undermine investments in the upstream oil and gas sector, such as concerns over political stability, uncertainty over regulatory obligations, and environmental regulations.

Because Petrobras operates all the pre-salt development wells within the period in the analysis, it will be compelling to review the development of pre-salt in the future (e.g., next five years) considering the arrival of new major players in Brazil's oil market, as well as new operators in the pre-salt zone.

The new age of energy abundance shifts the power from sellers to buyers (O'Sullivan, 2017). On the one hand, the US is now the most significant global crude oil producer, and fossil fuels will continue to be the primary source of energy for the foreseeable future (O'Sullivan, 2017). On the other hand, China, the current world's largest crude oil importer and the main destination of Brazilian crude oil exports, has strong policy push towards electric vehicles (EV) deployment and strong momentum in EV sales is expected up to 2030 by IEA (2018b, 2018c).

Within the context of increasing crude oil exports from Brazil to China (from 13% in 2008 to 57% in 2018), future studies could analyze new opportunities of cooperation between these countries, given the higher output from pre-salt layer, the growing dependence of China on the Middle East and the technological developments that will enable the world to move away from fossil fuels towards cleaner energy sources.

Further to this, the events over the last decade evidence the dynamicity of the oil industry. Regarding the national panorama, the increasing amounts of oil exports in Brazil reveal the Brazilian economy is becoming more vulnerable to the oil price volatility and oil price level. Future studies could analyze the use of oil exports revenues to invest in the Brazilian trade balance diversification away from commodities to the extent possible by increasing the share of technological goods. The 'resource curse' is not so much a product of an abundance of resources but rather the dependence on them (Waterworth and Bradshaw, 2018).

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SPE, Society of Petroleum Engineers, WPC, World Petroleum Council, AAPG, American Association of Petroleum Geologists, SPEE, Society of Petroleum

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Appendix A

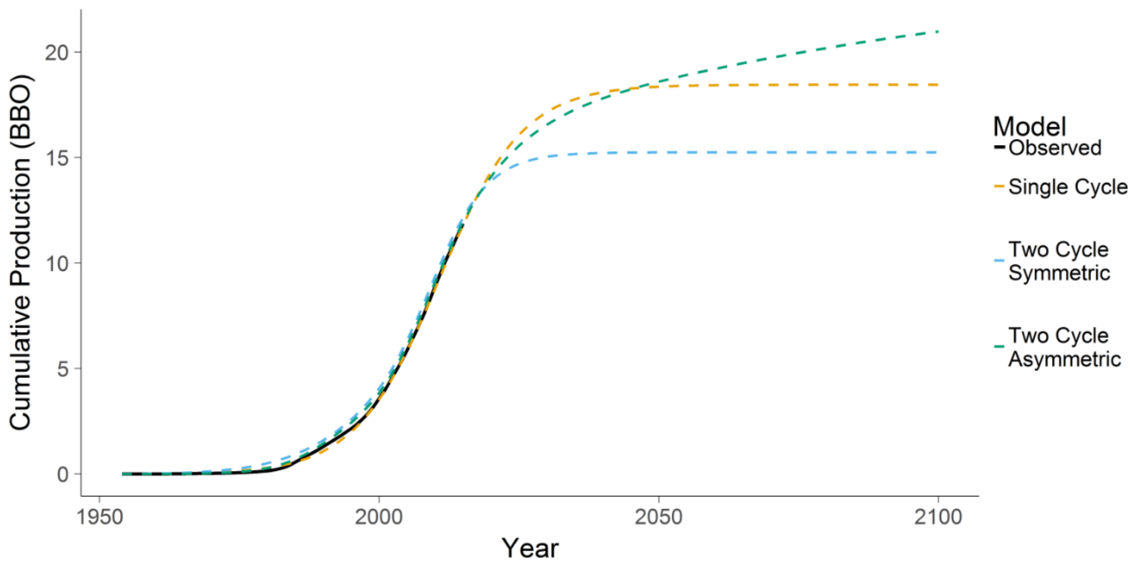

Figure A1: Post-salt offshore cumulative oil production fitted to variants of the Hubbert curve.

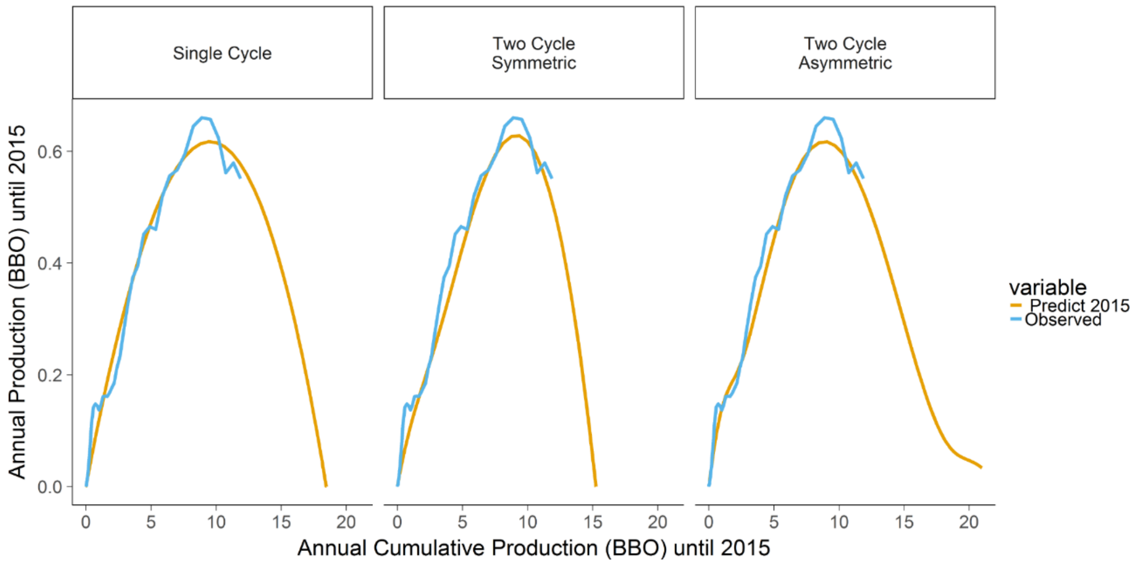

Figure A2: Annual production versus cumulative projected production

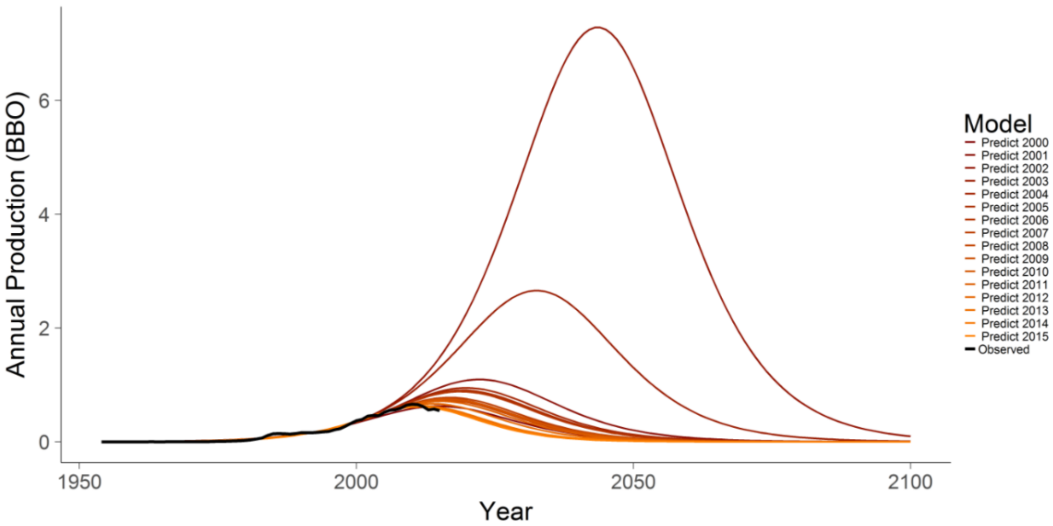

Figure A3: Backcasting of single cycle model.
Observation: The black line shows the observed data, while the colored lines show the fit of the single cycle model, using only data up to the indicated year (i.e., Predict 2000 uses production data from 1954-2000 to fit the model).

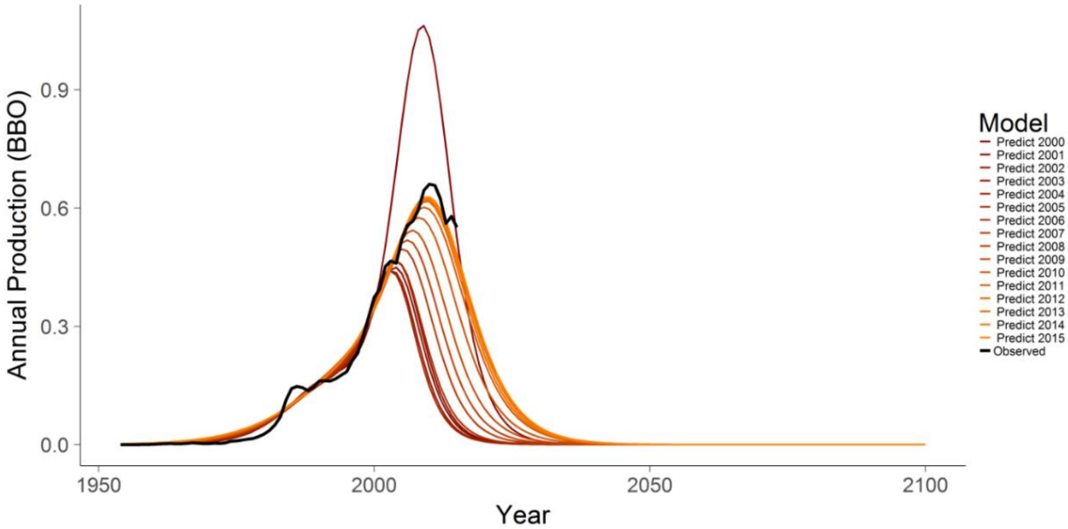

Figure A4: Backcasting of two cycles symmetrical model.
Observation: The black line shows the observed data, while the colored lines show the fit of the two-cycle symmetrical model, using only data up to the indicated year (i.e., Predict 2000 uses production data from 1954-2000 to fit the model).

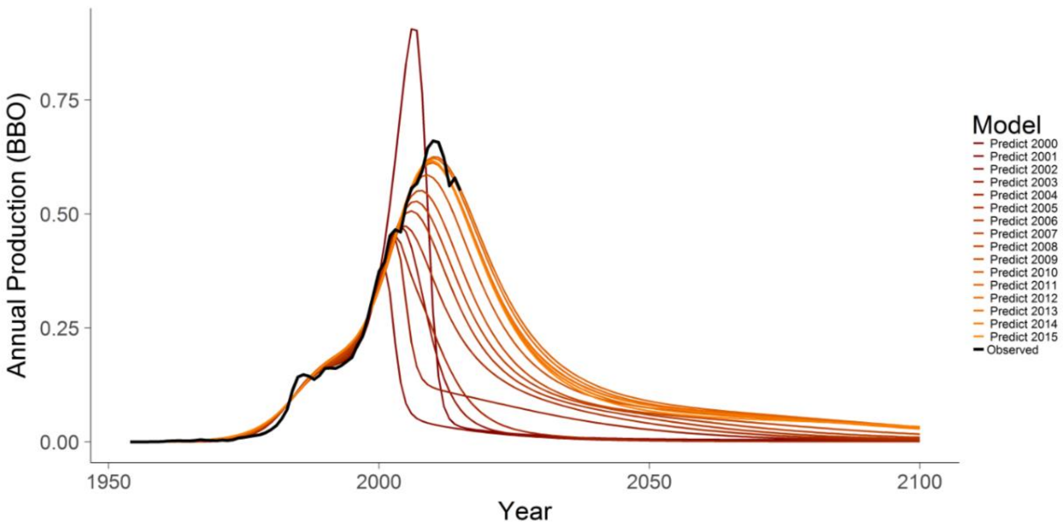

Figure A5: Backcasting of the two-cycle asymmetrical model.
Observation: The black line shows the observed data, while the colored lines show the fit of the two-cycle asymmetrical model, using only data up to the indicated year (i.e., Predict 2000 uses production data from 1954-2000 to fit the model).

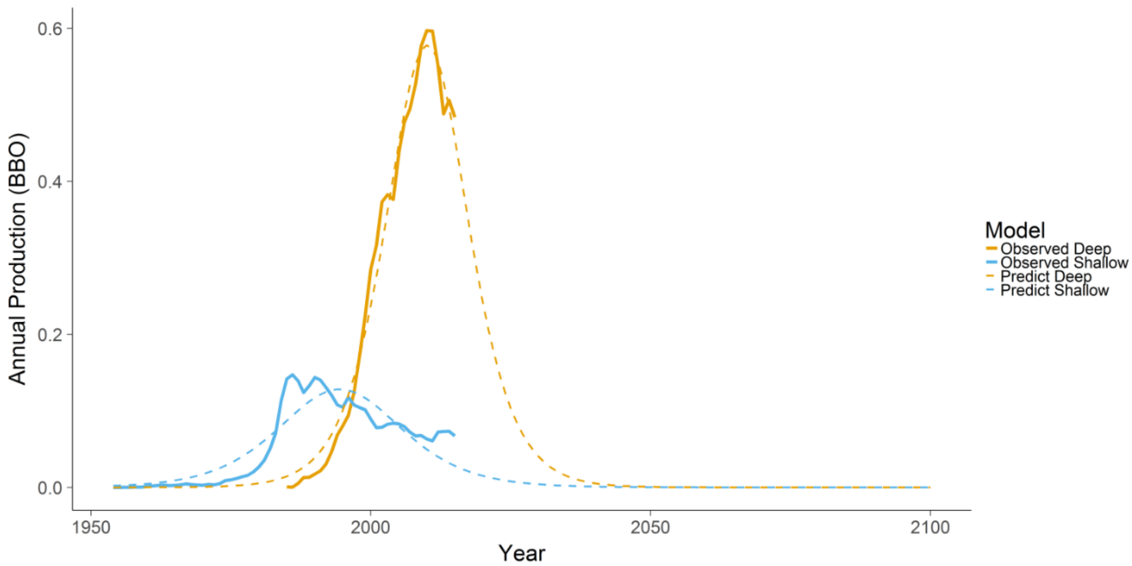

Figure A6: Symmetrical Hubbert forecast with two cycles

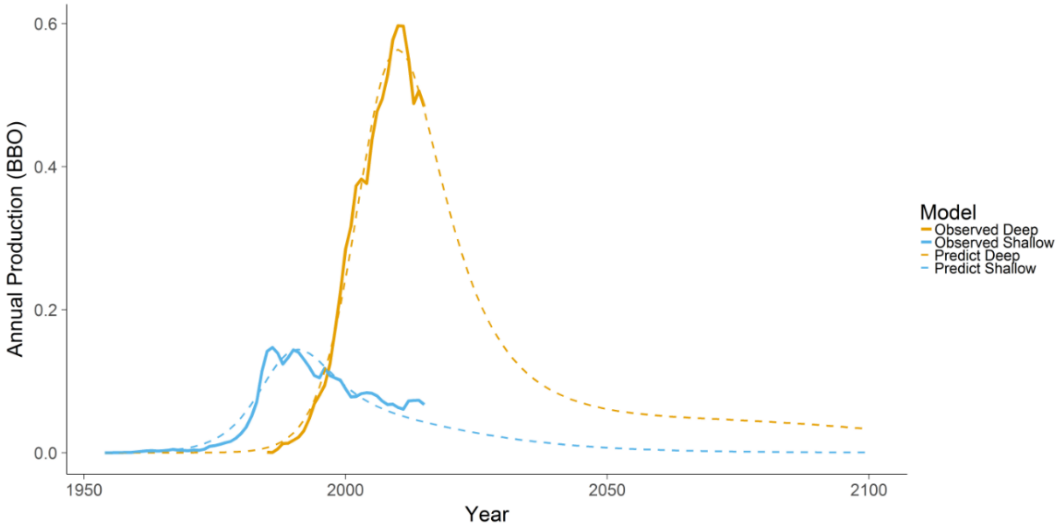

Figure A7: Asymmetrical Hubbert forecast with two cycles.

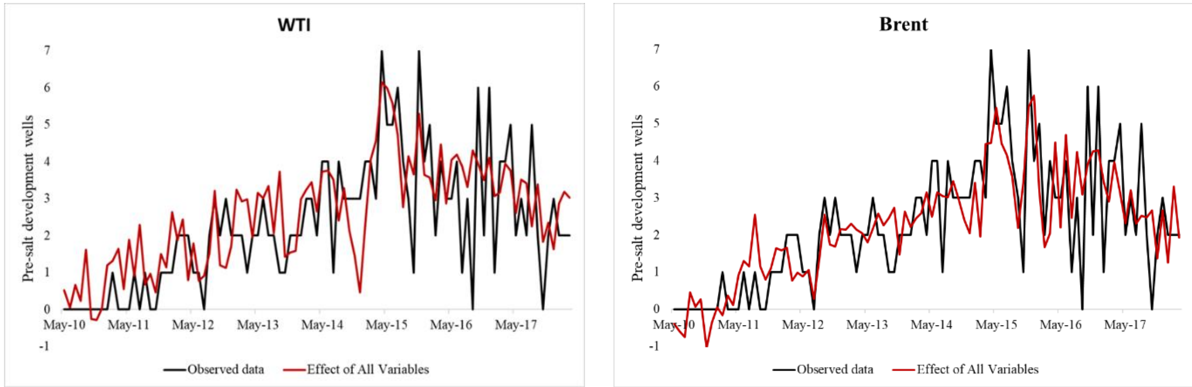

Figure A8: Determinants of pre-salt development wells for WTI and Brent. The effect of all variables (dark pink line).

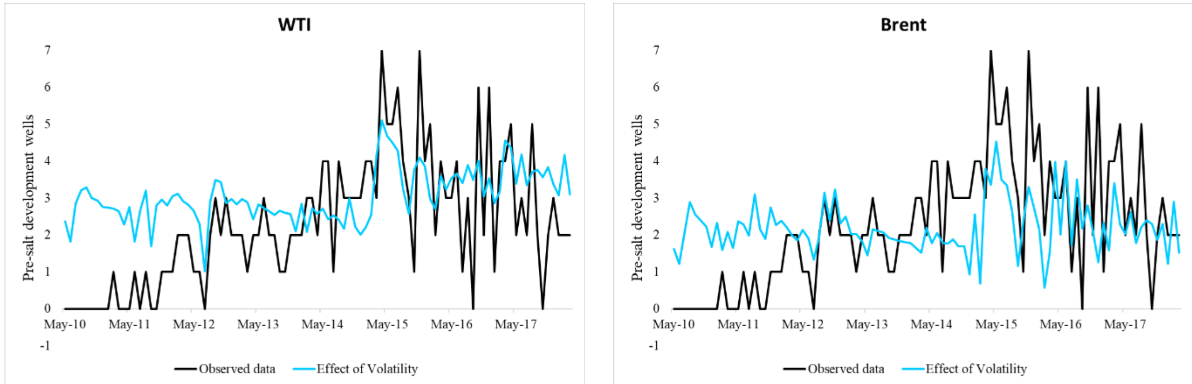

Figure A9: Determinants of pre-salt development wells for WTI and Brent. The effect of volatility (light blue line).

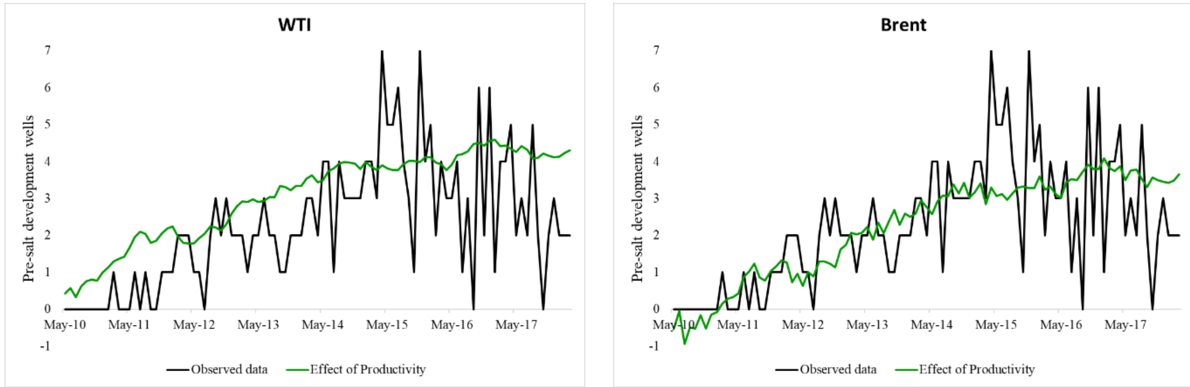

Figure A10: Determinants of pre-salt development wells for WTI and Brent. The effect of average well productivity (green line).

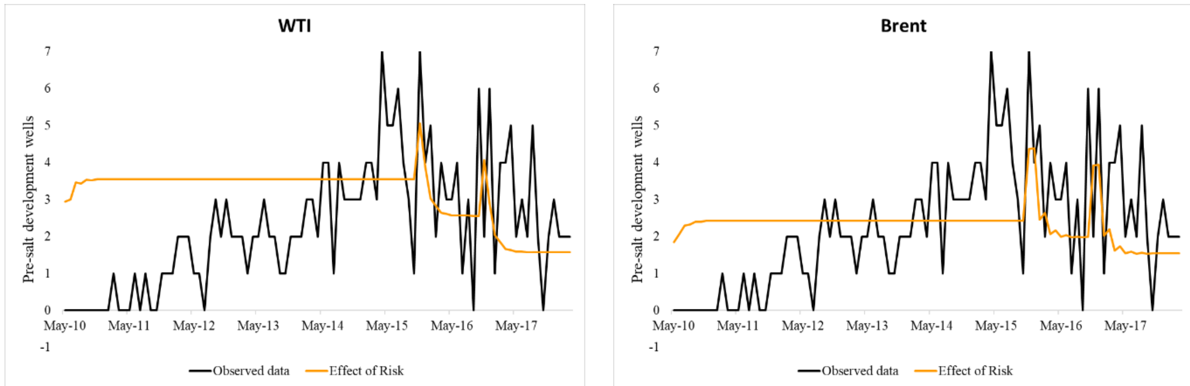

Figure A11: Determinants of pre-salt development wells for WTI and Brent. The effect of risk (orange line).

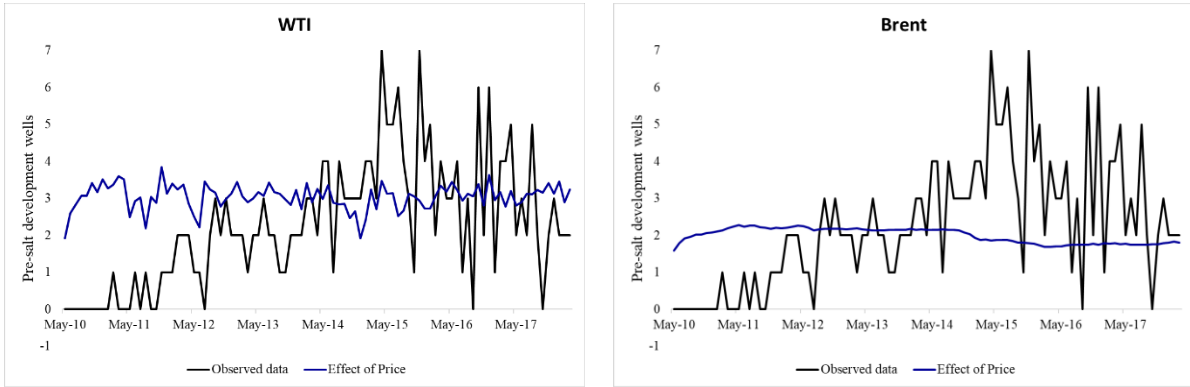

Figure A12: Determinants of pre-salt development wells for WTI and Brent. The effect of prices (blue line).

Appendix B - Mathematical formulation

B.1. Logistic function

This thesis fits the creaming curve to a logistic function using the function *SSlogis* in the RStudio software. Equation A.1 represents the logistic function.

$$Y = \frac{Asym}{(1 + \exp(\frac{(xmid - x)}{scal}))} \tag{B.1}$$

Where *Asym* is a numeric parameter representing the asymptote, *xmid* represents the x value at the inflection point of the curve and *scal* is a numeric scale parameter on the input axis (RDocumentation, 2019).

B.2. Gaussian function

The Gaussian function is defined as follows:

$$Y = asym * 0.5 * \left[1 + erf(\frac{x - mi}{\sigma\sqrt{2}})\right] \tag{B.2}$$

Where *asym* is a scale factor representing the asymptote, *mi* is the mean, the deviation is *sigma* (σ), and the Gauss error function is *erf*.

B.3. Gompertz function

The Gompertz function is defined as follows:

$$Y = alpha\ e^{-beta\ e^{-kx}} \tag{B.3}$$

Where *alpha* is a numeric parameter representing the asymptote, *e* is the Euler number; *beta* is a positive number that sets the displacement along the abscissa-axis, *k* is a positive number that sets the growth rate.

Appendix C - Statistics of the three functions fitting for 1P and 3P reserves

Table C1: Results of model fitting to a logistic function - 1P reserves

Formula: y ~ SSlogis(x, *Asym*, *xmid*, *scal*)					
	Estimate (MMbbl)	**Std. Error (MMbbl)**	**t value**	**Pr(>|t|)**	
Basin	**Campos**				
Asym	17992.2	295.014	60.99	<2e-16	***
xmid	351.733	5.126	68.62	<2e-16	***
scal	86.395	5.122	16.87	<2e-16	***
Residual standard error: 740.2 on 43 degrees of freedom					
Basin	**Recôncavo**				
Asym	1688.769	16.18	104.38	<2e-16	***
xmid	131.334	5.29	24.83	<2e-16	***
scal	68.441	4.021	17.02	<2e-16	***
Residual standard error: 99.6 on 75 degrees of freedom					
Basin	**Potiguar**				
Asym	1099.2	12.51	87.86	<2e-16	***
xmid	252.283	7.716	32.7	<2e-16	***
scal	81.18	5.655	14.36	<2e-16	***
Residual standard error: 53.8 on 41 degrees of freedom					
Basin	**Santos**				
Asym	8534.66	292.47	29.18	<2e-16	***
xmid	202.68	3.994	50.75	<2e-16	***
scal	26.169	2.592	10.1	6.43E-13	***
Residual standard error: 391.8 on 43 degrees of freedom					

***** indicates significance on the 0.001 level*

Table C2: Results of model fitting to a logistic function - 3P reserves

Formula: y ~ SSlogis(x, *Asym*, *xmid*, *scal*)					
	Estimate (MMbbl)	**Std. Error (MMbbl)**	**t value**	**Pr(>|t|)**	
Basin	**Campos**				
Asym	20649.75	338.589	60.99	<2e-16	***
xmid	351.733	5.126	68.62	<2e-16	***
scal	86.395	5.122	16.87	<2e-16	***
Residual standard error: 849.6 on 43 degrees of freedom					
Basin	**Recôncavo**				
Asym	1760.244	16.864	104.38	<2e-16	***
xmid	131.334	5.29	24.83	<2e-16	***
scal	68.441	4.021	17.02	<2e-16	***
Residual standard error: 103.8 on 75 degrees of freedom					
Basin	**Potiguar**				
Asym	1182.971	13.464	87.86	<2e-16	***
xmid	252.283	7.716	32.7	<2e-16	***
scal	81.18	5.655	14.36	<2e-16	***
Residual standard error: 57.9 on 41 degrees of freedom					
Basin	**Santos**				
Asym	16710.99	572.66	29.18	<2e-16	***
xmid	202.68	3.994	50.75	<2e-16	***
scal	26.169	2.592	10.1	6.43E-13	***
Residual standard error: 767.1 on 43 degrees of freedom					

**** indicates significance on the 0.001 level*

Table C3: Results of model fitting to a Gaussian function - 1P reserves

Formula: y ~ *asym* * 0.5 * (1 + *erf*((x - *mi*)/(*sigma* * sqrt(2))))					
	Estimate (MMbbl)	**Std. Error (MMbbl)**	**t value**	**Pr(>|t|)**	
Basin	**Campos**				
asym	17795.72	294.7	60.39	<2e-16	***
mi	349.023	5.342	65.33	<2e-16	***
sigma	143.357	8.416	17.03	<2e-16	***
Residual standard error: 773 on 43 degrees of freedom					
Basin	**Recôncavo**				
asym	1681.22	1.58E+01	106.38	<2e-16	***
mi	131.341	5.19E+00	25.32	<2e-16	***
sigma	113.02	6.307	17.92	<2e-16	***
Residual standard error: 100.8 on 75 degrees of freedom					
Basin	**Potiguar**				
asym	1092.412	11.796	92.61	<2e-16	***
mi	249.137	7.532	33.08	<2e-16	***
sigma	137.363	8.602	15.97	<2e-16	***
Residual standard error: 53.3 on 41 degrees of freedom					
Basin	**Santos**				
asym	8413.614	286.424	29.38	<2e-16	***
mi	201.348	3.934	51.17	<2e-16	***
sigma	43.497	4.22E+00	10.32	3.31E-13	***
Residual standard error: 382.2 on 43 degrees of freedom					

**** indicates significance on the 0.001 level*

Table C4: Results of model fitting to a Gaussian function - 3P reserves

Formula: y ~ *asym* * 0.5 * (1 + *erf*((x - *mi*)/(*sigma* * sqrt(2))))					
	Estimate (MMbbl)	**Std. Error (MMbbl)**	**t value**	**Pr(>|t|)**	
Basin	**Campos**				
asym	20424.27	338.231	60.39	<2e-16	***
mi	349.023	5.343	65.33	<2e-16	***
sigma	143.357	8.416	17.03	<2e-16	***
Residual standard error: 887.1 on 43 degrees of freedom					
Basin	**Recôncavo**				
asym	1752.376	16.473	106.38	<2e-16	***
mi	131.341	5.187	25.32	<2e-16	***
sigma	113.013	6.307	17.92	<2e-16	***
Residual standard error: 105.1 on 75 degrees of freedom					
Basin	**Potiguar**				
asym	1175.666	12.695	92.61	<2e-16	***
mi	249.137	7.532	33.08	<2e-16	***
sigma	137.363	8.602	15.97	<2e-16	***
Residual standard error: 57.36 on 41 degrees of freedom					
Basin	**Santos**				
asym	16474.01	560.831	29.37	<2e-16	***
mi	201.348	3.935	51.17	<2e-16	***
sigma	43.497	4.216	10.32	3.31E-13	***
Residual standard error: 748.3 on 43 degrees of freedom					

**** indicates significance on the 0.001 level*

Table C5: Results of model fitting to a Gompertz function - 1P reserves

Formula: y ~ *alpha* * exp(-*beta* * exp(-*k* * x))					
	Estimate (MMbbl)	**Std. Error (MMbbl)**	**t value**	**Pr(>|t|)**	
Basin	**Campos**				
alpha	1.89E+04	5.02E+02	37.69	<2e-16	***
beta	1.04E+01	1.98E+00	5.23	4.76E-06	***
k	7.59E-03	6.35E-04	11.95	2.94E-15	***
Residual standard error: 848.4 on 43 degrees of freedom					
Basin	**Recôncavo**				
alpha	1.71E+03	1.51E+01	113.25	<2e-16	***
beta	2.35E+00	9.93E-02	23.67	<2e-16	***
k	9.89E-03	4.77E-04	20.72	<2e-16	***
Residual standard error: 81.15 on 75 degrees of freedom					
Basin	**Potiguar**				
alpha	1.13E+03	1.53E+01	73.837	<2e-16	***
beta	4.22E+00	4.66E-01	9.075	2.34E-11	***
k	7.57E-03	4.97E-04	15.244	<2e-16	***
Residual standard error: 50.46 on 41 degrees of freedom					
Basin	**Santos**				
alpha	9.45E+03	5.88E+02	16.078	<2e-16	***
beta	5.36E+01	2.44E+01	2.197	0.0334	*
k	2.10E-02	2.82E-03	7.467	2.73E-9	***
Residual standard error: 378.9 on 43 degrees of freedom					

**** indicates significance on the 0.001 level*

Table C6: Results of model fitting to a Gompertz function - 3P reserves

Formula: y ~ *alpha* * exp(-*beta* * exp(-*k* * x))					
	Estimate (MMbbl)	**Std. Error (MMbbl)**	**t value**	**Pr(>|t|)**	
Basin	**Campos**				
alpha	2.17E+04	5.76E+02	37.68	<2e-16	***
beta	1.04E+01	1.98E+00	5.23	4.75E-6	***
k	7.59E-03	6.35E-04	11.95	2.94E-15	***
Residual standard error: 973.7 on 43 degrees of freedom					
Basin	**Recôncavo**				
alpha	1.79E+03	1.58E+01	113.25	<2e-16	***
beta	2.35E+00	9.93E-02	23.67	<2e-16	***
k	9.89E-03	4.77E-04	20.72	<2e-16	***
Residual standard error: 105.1 on 75 degrees of freedom					
Basin	**Potiguar**				
alpha	1.22E+03	1.65E+01	73.837	<2e-16	***
beta	4.23E+00	4.66E-01	9.075	2.34E-11	***
k	7.57E-03	4.97E-04	15.244	<2e-16	***
Residual standard error: 54.31 on 41 degrees of freedom					
Basin	**Santos**				
alpha	1.85E+04	1.15E+03	16.078	<2e-16	***
beta	5.36E+01	2.44E+01	2.197	0.0334	*
k	2.10E-02	2.82E-03	7.467	2.73E-9	***
Residual standard error: 742 on 43 degrees of freedom					

****, * indicate significance on the 0.001, and 0.05 levels*

Appendix D - Cointegration and Error-Correction Model

D.1. Results - GARCH Models

Tests of the residuals from the GARCH (1,1) model suggest that it can be used to proxy the volatility of oil prices. ADF statistics reject the null hypothesis that the residuals for all oil prices (Table D1) contain a stochastic trend, which indicates that the residuals are stationary. Similarly, the ARCH statistic fails to reject the null hypothesis that there are no ARCH-effects for all measures of oil prices, which indicates there is little evidence for conditional heteroscedasticity. The Jarque-Bera tests fail to reject the null hypothesis that skewness and the excess kurtosis are zero for all Brent and WTI prices, which suggests that the residuals from the GARCH model are consistent with a normal distribution. The Ljung-Box Q statistic rejects the null hypothesis that the autocorrelations of residuals are zero for all measures of oil prices. This suggests the residuals from these GARCH models are autocorrelated. This autocorrelation cannot be eliminated by increasing the length of the autocorrelation parameter in the GARCH model.

Table D1: Analysis of residuals from the GARCH models

	ADF	ARCH Statistic	Signif. Level	Jarque-Bera	Signif. Level	Ljung-Box	Q-Statistics
WTI0	-5.923**	0.034	0.853	0.532	0.766	8.283**	0.004
WTI1	-5.937**	0.016	0.898	0.673	0.714	8.911**	0.003
WTI6	-5.799**	0.090	0.764	2.851	0.240	10.550**	0.001
WTI12	-5.590**	0.376	0.541	3.708	0.157	9.676**	0.002
WTI24	-5.372**	0.363	0.548	5.711	0.058+	8.891**	0.003
Brent0	-5.824**	0.187	0.666	3.573	0.168	10.502**	0.001
Brent1	-5.724**	0.195	0.660	3.549	0.170	13.307**	0.000
Brent6	-5.650**	0.229	0.633	2.960	0.228	12.237**	0.000
Brent12	-5.550**	0.223	0.637	2.179	0.336	10.992**	0.001
Brent24	-5.345**	0.078	0.780	0.971	0.615	9.879**	0.002

*Test statistics reject the null hypothesis at the **1%, *5%, +10% level.*

This suggests the residuals from these GARCH models are autocorrelated. Taking this into account, Ansari and Kaufmann (2019) add an autoregressive component to Equation 44, which creates an AR(r)-GARCH (p,q) representation given by Equation D.1.1:

$$Y_t = \alpha + \sum_{r=1}^{R} \beta_r Y_{t-r} + \varepsilon_t \tag{D.1.1}$$

In which Y_t represents the daily return ($\frac{P_t - P_{t-1}}{P_{t-1}}$) to crude oil prices and ε_t comes from a GARCH(p,q) process. After evaluating the best lengths for r, p and q, Ansari and Kaufmann (2019) use an AR(1)-GARCH(1,1) model to generate conditional volatilities of the returns. Such conditional volatilities are very similar to those generated by the GARCH (1,1) model. With that in mind, this thesis proxies the volatility of oil prices by the plain GARCH model – GARCH (1,1) – which is used to generate the results reported throughout this thesis' text.

D.2. Illustrating the model

To give the physical intuition of the CVAR model, this study uses the CVAR Brent model (Table 15 in section 4.3.2). It includes three endogenous variables (Well, PerVol and Vol), and four exogenous variables; AWP, Profit, Revenue and Risk. The rank defined by the likelihood-based trace test (Johansen, 1996) is two, i.e., there are two cointegrating relations among the seven variables. The number of cointegrating relations is inferior to the number of endogenous variables, which means the rank of Π is not full. For simplicity, the short-run effects ($A_0 \Delta w_t$, $A_1 \Delta w_{t-1}$, $\Gamma_{11} \Delta x_{t-1}$), and the dummy variables ΘM are set to zero, and the CVAR model becomes Equation D.2.1:

$$\begin{bmatrix} \Delta Well \\ \Delta PerVol \\ \Delta Vol \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} \beta_{11} & \cdots & \beta_{17} \\ \beta_{21} & \cdots & \beta_{27} \end{bmatrix} \begin{bmatrix} Well_{t-1} \\ PerVol_{t-1} \\ Vol_{t-1} \\ \vdots \\ Rev_{t-1} \\ Risk_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \qquad \text{(D.2.1)}$$

The β coefficients in Equation 10 correspond to estimated eigenvectors obtained by solving the reduced rank problem and defined based on the ordering of the eigenvalues. The unrestricted β relations in Equation 10 only exceptionally may have a physical interpretation. To give them a physical interpretation, identifying restrictions are imposed. “Generally, the structure of cointegration relations is identified when it is not possible to take a linear combination of the two cointegrating relations without violating any of the imposed restrictions” (Kaufmann and Juselius, 2013, p. 4).

The model is defined after the restrictions are imposed. Then, the Π matrix is obtained (Equation D.2.2):

$$\begin{bmatrix} \Delta Well \\ \Delta PerVol \\ \Delta Vol \end{bmatrix} = \begin{bmatrix} -0.799 & -0.064 \\ 0.241 & 0.036 \\ 0.093 & 0.101 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.70 & 0 & 0.198 \\ 0 & 1 & -3.596 & 0 & 0 & -1.247 & 0 \end{bmatrix} \begin{bmatrix} Well_{t-1} \\ PerVol_{t-1} \\ Vol_{t-1} \\ \vdots \\ Rev_{t-1} \\ Risk_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \qquad \text{(D.2.2)}$$

Equation D.2.3 presents the error correction model for $Well$, and the two cointegration relations (CR) are presented in Equation D.2.4 and Equation D.2.5.

$$\Delta Well = -0.799CR1 - 0.064CR2 + \varepsilon_{1t} \qquad \text{(D.2.3)}$$
$$CR1 = Well_{t-1} - 0.7AWP_{t-1} + 0.198Risk_{t-1} \qquad \text{(D.2.4)}$$
$$CR2 = PerVol_{t-1} - 3.596Vol_{t-1} - 1.247Rev_{t-1} \qquad \text{(D.2.5)}$$

The error correction term (α_{11}) is -0.799, that is not statistically different from -1. This means the response of $Well$ to the previous period deviation from long-run equilibrium is immediate. To obtain that α_{11} is not statistically different from -1, this study applies the t test: $t = \frac{\mu - \bar{X}}{se} = \frac{-1-(-0.799)}{\frac{\alpha}{t\,stat}} = \frac{-1-(-0.799)}{\frac{-0.799}{-6.938}}$ = -1.745. The t-value at 71 degrees of freedom results in p > 0.085, which does not reject the null hypothesis that there is no difference between -1 and -0.799.

For simplification, the short-run effects were previously set to zero. Indeed, the short-run matrices ($A_0 \Delta w_t$, $A_1 \Delta w_{t-1}$, $\Gamma_{11} \Delta x_{t-1}$) and the dummy variables ΘM derive from the long-run effects. Equations D.2.6, D.2.7 and D.2.8 include only the coefficients in the short-run matrices that rejects the null hypothesis that they equal zero (evaluated against a t distribution).

$$\Gamma_{11} \Delta x_{t-1} = -0.59(\Delta Vol_{t-1}) \tag{D.2.6}$$
$$A_0 \Delta w_t = 0.868(\Delta Risk) \tag{D.2.7}$$
$$A_1 \Delta w_{t-1} = 1.115(\Delta Risk_{t-1}) \tag{D.2.8}$$

The methodology illustrated throughout the Appendix D.2 for the CVAR Brent model (Table 15 in section 4.3.2) is the same used to the CVAR model chosen as the most accurate (Table 14 in section 4.3.2).