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ACCUMULATION, FINANCE AND EFFECTIVE DEMAND IN MARX, KEYNES AND KALECKI

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ACCUMULATION, FINANCE AND EFFECTIVE DEMAND IN MARX, KEYNES AND KALECKI (*)

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I. Introduction

Macroeconomic theory is often portrayed as being divided into two rival camps: those who concentrate on the laws of effective supply (production capacity and output), and those who focus on the dominant role of effective demand. The first group implicitly or explicitly treats aggregate demand as a subsidiary factor, while the second does the same for aggregate supply. In its most extreme form, this difference is presented as the dichotomy between Say's Law (aggregate production generates its own demand) and Keynes' or Kalecki's Law (aggregate demand induces its own supply).

The trouble with such a dichotomy is that Marx's work falls into neither camp. He emphatically rejects Say's Law. [Kenway, 1980; Foley, 1983] But he also rejects the contrary notion that capitalist accumulation is ultimately limited by effective demand. He argues instead that both aggregate supply and demand are themselves regulated by more basic factors. Because capitalist production is fundamentally anarchic, this regulation process is characterized by constant shocks and discrepancies. Nonetheless, the inner mechanisms of the system continue to operate. The end result is a turbulent and erratic pattern in which supply and demand endlessly cycle around an underlying growth trend. [Bleaney, 1976, Ch. 6; Shaikh, 1978, pp. 231-232; Garegnani, 1979, pp. 183-185]

The conventional opposition between Say's Law and Keynes' Law is evidentally inadequate to Marx's approach. A third alternative is required. It is the aim of this paper to show that such an alternative can be constructed. In this regard, the critical element turns out to be the linkage between accumulation expenditures and the finance needed to carry them out. This enables us to identify a central weakness in Keynes' and Kalecki's treatments of effective demand, since they typically sidestep the implications of this crucial issue. More importantly, the feedback between deficit finance, debt and accumulation provides a natural foundation for a macroeconomic model of cyclical growth in which <u>both</u> the cycle and the growth trend are intrinsic to the system. The resulting scenario is very similar to Marx's description: supply and demand fluctuate erratically around a cyclical growth path whose trend is itself ultimately regulated by the profitability of the system. By contrast, conventional theories of effective demand generally find the system to be inherently static except in the fortuitous presence of exogeneous growth factors such as technical change, population growth, and bursts of innovation. [Mullineaux (1984), pp. 87-89.]

It is important to note that our present analysis is concerned solely with the relation between effective demand and accumulation <u>in the absence of any changes in technology or potential profitability</u>. This is exactly the domain occupied by Marx's schemes of reproduction, by Keynes's theory of output, employment and effective demand, and by Kalecki's theory of effective demand and cycles. Such a focus is a necessary prelude to the analysis of the factors which change the underlying trends of the economy and ultimately transform normal growth into a general crisis. Thus, while the theory of business cycles falls within the scope of this paper, the theory of long waves and depressions lies outside of it.

The remainder of this paper is divided into two parts. In Part I we develop a general framework within which we can compare Marx, Keynes, and Kalecki on effective demand, finance and investment. The framework which we will use is grounded in Marx's schemes of reproduction, in Chipman's (1951) illuminating treatment of Keynesian flows, and in the pioneering elaborations of the Marxian schema by Dumenil (1977) and Foley (1983). This will enable us to track not only commodity flows and their money counterparts, but also the crucial linkages between accumulation expenditures, finance and bank credit.

Our general framework reveals two crucial differences between Marx and Keynes/Kalecki. The first of these has to do with the mode by which effective demand is assumed to be financed. When Marx introduces the question of the reproduction of social capital, he quite explicitly abstracts from credit, so that both personal and business expenditures are assumed to be financed out of current revenue. This is why we do not encounter a multiplier effect even when he is tracing out the consequences of an increase in the level of aggregate investment expenditures.@ [Bleaney (1976), pp. 106-107]. Keynes and Kalecki, on the other hand, assume that while personal expenditures are limited by current revenue, business expenditures can be maintained at any desired level through borrowing. But while this latter assumption of unconstrained deficit finance is crucial to the "independence of investment from savings" and its associated multiplier effect@ [Asimakopulos (1983)], neither Keynes nor Kalecki ever really connect borrowing to any subsequent debt service payments. In an important paper, Asimakopulos (1983) demonstrates that they tend instead to shift attention to the determination of the interest rate. Though we cannot pursue it here, it can be shown that the Keynesian and Kaleckian scenarios are actually predicated on the stringent assumption that business expenditures are completely unaffected by a growing burden of debt service payments. Debt, it turns out, is the dark side of Keynesian economics.

But the question of credit is only one half of the story, because there is an equally important difference concerning the analysis of accumulation. Marx portrays individual capitals as aggressive and expansive, impelled ever upward by the hope of greater profits and the threat of competitors. Growth is therefore an inherent aspect of production plans, and depending on the balance of supply and demand, it is the trend of production which is adjusted upward or downward. In contrast to this, both Keynes and Kalecki implicitly assume static production plans, so that any imbalance between supply and demand feed back on the level (rather than the trend) of production. This difference in the treatment of production plans is at least as important as the one in the treatment of finance.

Part II of the paper utilizes our general schema to develop a

simple but powerful macroeconomic model of cyclical accumulation. In effect, the feedback between deficit finance, debt and accumulation binds both aggregate supply and demand to a growth path which is itself ultimately regulated by the intrinsic profitability of the system. At the same time, the shocks and discrepancies generated by the anarchy of capitalist production keep the system constantly off this growth path. The end result is a series of erratic fluctuations whose center of gravity cycles endlessly around the growth path. The structure of this model is relatively new, since it combines Marx's emphasis on profitability as the engine of growth with Frisch's work on shocks as the generator of cycles.@ [Mullineaux (1984), pp. 8-9; Kalecki, 1939, p. 148, footnote 1] Its results are also relatively new, because until recently it has not been possible to derive both cycles and growth from the same set of relations.@ [Mullineaux (1984), pp. 87-89.]

I. Aggregate Demand, Supply, and Finance.

In order to highlight the underlying structure of the relations involved, we will abstract from all except the most essential relations. Thus, we will assume that both workers and capitalist consume out of their respective current personal incomes (wages and dividends), but that only capitalists save; that the unit of time is the same as the average period of production; and that all funds borrowed in a particular period must be paid back, with interest, in the next period. More importantly, in order to make the comparison between Marx and Keynes/Kalecki as strict as possible, we will take the wage rate, the interest rate, and all prices as constant even in the face of persistent imbalances in demand and supply in the respective markets. This locates us squarely within the domain of modern effective demand theories, in which the burden of adjustment to discrepancies between supply and demand is shifted entirely onto changes in output and employment and as we shall see, implicitly onto liquidity.

What then remains is the core of the framework needed to encompass the differing treatments of supply and demand in Marx, Keynes and Kalecki. And this core can in turn be reduced to two essential relations: one between current supply and demand for aggregate output, as summarized by the current level of (positive or negative) excess demand E_t; and the other, between anticipated uses and sources of funds, as summarized by the current level of (positive or negative) borrowing B_t.

1. Aggregate Supply

Because the unit of time is the same as the average production period, the currently available supply Q_t is the product of inputs used in the previous period. Following Marx, we will assume that the inputs used in the last period are the same as those purchased in that period. This really amounts to saying that the tie between planned and actual production is strong enough for us to safely ignore the small discrepancies between purchases and use of inputs in any one period.

Given that M_{t-1} is the aggregate money value of materials purchased in the previous period, W_{t-1} is the corresponding expenditure on wages, and DEP_t is the current depreciation allowance, then current potential profit P_t is simply the difference between current aggregate supply and these costs. This potential profit P_t is simply the money form of aggregate surplus value, and actual profit will equal it only when sales equal output.@ [For a discussion of the relation of money profits to aggregate surplus value, see Shaikh, 1984] In any case, we can write aggregate supply as

I.1 $Q_t = M_{t-1} + W_{t-1} + DEP_t + P_t$

2. Aggregate Finance

In what follows, we will treat all borrowing and lending (saving) as mediated by the banking sector. What is more, we will assume that this sector is willing and able to fulfill the needs of businesses and private individuals without having to change the interest rate. This replicates the Keynesian and Kaleckian assumption that finance be freely gotten (or lending be freely accepted) at some given rate of interest below the potential rate or profit. As Asimakopulos points out, it is this assumed lack financial constraint upon planned investment which is the real secret of their notion that investment is independent from savings. @ [Asimakopulos (1983), p. 222. See also pp. 223-227]

Over any period t, the sum of expected revenues of the business sector is simply the aggregate output offered for sale (planned supply) Q_t, plus any reflux of principal and interest receipts on past lending. However, since we shall be explicitly treating the payment of finance charges as a use of funds, it is more convenient to count financial receipts as a negative finance charge. On this basis, the total expected source of funds is simply the total output Q_t.

On the other hand, over the same period the total planned uses of funds must encompass five basic categories. First, circulating capital expenditures for materials M_t and wages W_t to be purchased in this period (in order to produce output for sale and/or for inventory adjustment in the next period). Second, fixed capital expenditures for gross investment in plant and equipment IG_t. Third, the payment of finance charges F_t, which represent currently due principal and interest charges on past borrowing (or when negative, the current receipt of principal and interest revenues on past lending). Fourth, disbursements of dividends R_t to capitalists, for whom they will serve as current income. And fifth, planned changes in money reserve levels CMR_t.

I.2 $B_t = M_t + W_t + IG_t + F_t + R_t + CMR_t - Q_t$

Planned output can be destined for sales or for retention in final goods inventories. Since current circulating capital expenditures $M_t + W_t$ are the precursors of planned output, they already embody planned adjustments to inventory stocks. Planned adjustments to money stocks are different, however, because their determinants are not subsumed under those of any of the other disbursments. This is why CMR_t appears as a separate category above. Finally, it is most important to recognize that expenditure plans must first be financed if they are to be carried out. This means that whenever internal sources of funds Q_t do not appear sufficient to cover the uses planned, it will be necessary to fill the gap by prior borrowing. Borrowing therefore precedes expenditure.

Combining equations I.1 and I.2, we get

$$B_t = M_t + W_t + IG_t + F_t + R_t + CMR_t - Q_t$$
$$= M_t + W_t + IG_t + F_t + R_t + CMR_t - (M_{t-1} + W_{t-1} + DEP_t + P_t)$$
$$= (M_t - M_{t-1}) + (W_t - W_{t-1}) + (IG_t - DEP_t) + F_t + CMR_t - P_t$$

I.3 $B_t = A_t + I_t + F_t + R_t + CMR_t - P_t$

where

$A_t = (M_t - M_{t-1}) + (W_t - W_{t-1})$ = accumulation of circulating capital

$I_t = IG_t - DEP_t$ = net accumulation of fixed capital = net fixed investment@ [strictly speaking, net investment is the difference between gross fixed investment and current retirements IR_t. But we shall abstract here from the difference between current retirements and current depreciation allowances DEP_t, so that I_t can stand for net investment]

F_t = Finances charges = principal due + interest due

R_t = Dividend payments = personal income of capitalists

CMR_t = Desired changes in money reserves

Equation I.3 is our fundamental equation of finance. It tells us that the business sector as a whole will borrow from the banks whenever its need for funds exceed its anticipated revenues. The money disbursements F_t and CMR_t play a particularly important role here, because they reflect the feedback of past events on current financial need. In the case of finance charges F_t, this is because past borrowing shows up in the present as finance charges currently due; and in the case of the desired change in money reserves CMR_t, because the need to make up any unwelcome declines in past money reserves inevitably puts a burden on current funds.

3. Aggregate Demand

Current aggregate demand D_t is composed of the demand for materials M_t, the demand for plant and equipment (gross fixed investment) IG_t, and workers' and capitalists' consumption demands $CONW_t$ and $CONR_t$ respectively.

$$I.4 \quad D_t = M_t + IG_t + CONW_t + CONR_t$$

4. Excess Demand

The excess demand E_t in any period is defined as the difference between current aggregate demand D_t and the current aggregate supply of output Q_t.

$$I.5 \quad E_t = D_t - Q_t$$

Combining the expressions for aggregate supply (I.1), aggregate demand (I.4), and aggregate excess demand (I.5), and recalling that workers' consumption $CONW_t$ equals their wages W_t, we get

$$E_t = D_t - Q_t$$

$$= M_t + IG_t + CONW_t + CONR_t - (M_{t-1} + W_{t-1} + DEP_t + P_t)$$

$$= [(M_t - M_{t-1}) + (W_t - W_{t-1})] + (IG_t - DEP_t) + CONR_t - P_t$$

$$I.6 \quad E_t = A_t + I_t + CONR_t - P_t$$

where A_t and I_t are circulating and fixed capital investment, as described earlier under equation I.3

Equation I.6 is our fundamental equation of effective demand. It tells us that the level of excess demand in any period is given by the difference between the sum of total capital expenditures (investment in circulating and fixed capital), capitalist consumption expenditures, and the potential profits contained in current output. Note that when this excess demand is positive (negative), realized profits will be greater (smaller) than potential profits.

5. The Link Between Excess Demand, Borrowing, and Loanable Funds

In the preceding sections we have derived two fundamental aggregate equations, one for borrowing B_t and the other for excess demand E_t. But if we now go back to the former, we see that it can be tied back into the latter. Thus from equation I.3

$$B_t = A_t + I_t + F_t + R_t + CMR_t - P_t$$

In this expression, the term R_t represents the dividend payments made by firms, which is at the same time the personal income (revenue) of the capitalist class. Decomposing this into consumption $CONR_t$ and savings $SAVR_t$

$$I.7 \quad R_t = CONR_t + SAVR_t$$

we can then substitute into the borrowing equation to get

$$B_t = (A_t + I_t + CONR_t - P_t) + F_t + SAVR_t + CMR_t$$

$$= F_t + SAVR_t + CMR_t + E_t$$

$$I.8 \quad E_t = B_t - (F_t + SAVR_t + CMR_t)$$

The terms in parentheses on the left hand side are all anticipated leakages of funds from the circulation of the business sector as a whole: F_t is the direct leakage to the banking sector, in the form of planned finance payments; capitalist saving $SAVR_t$ is a net leakage to the household sector, because it is only that portion of wage and dividend payments which is saved (by the capitalists) which does not return back in the form of consumption expenditures; and CMR_t is the anticipated leakage of funds from circulation to money reserves. [If CMR is negative, this means that either the release of reserves or the availability of some other external source of funds (such as the direct production of gold) is anticipated.] With this in mind, we can see that <u>aggregate demand will be greater than aggregate supply ($E_t \geq 0$) only when the current injection of business credit B_t is greater than the sum of business and personal leakages</u>. But because current excess demand depletes money reserves, and because current borrowing implies future finance charges, there is also an intrinsic link between current net injections of credit and future leakages. In effect, episodes of excess demand carry the seeds of their own negation, because the net injections of credit which fuel them also carry over into the future as accelerated leakages. We will see that this feedback will play a vital role in stabilizing the growth of the system.

Finally, it is interesting to note that equation I.8 above can also be interpreted in terms of the supply and demand for loanable funds. To begin with, the total inflow of loanable funds into the banks (from the private sector alone, since we have not yet introduced a state sector) is simply the sum of finance charges being paid by businesses, funds earmarked for addition to (or subtraction from) to business money reserves, and personal savings from capitalists.

$$I.9 \quad \text{Total Inflow of Loanable Funds} = F_t + CMR_t + SAVR_t$$

On the other hand, the total outflow of loanable funds is simply business borrowing (since households as a whole are net lenders).

I.10 Total Outflow of Loanable Funds = B_t

The difference between total outflow and total inflow is a measure of the private sector's excess demand for bank funds.

$$\text{Excess Demand for Bank Funds} = B_t - (F_t + CMR_t + SAVR_t)$$

But from equation I.8, we know that the above expression is exactly the same as aggregate excess demand E_t.

I.11 Excess Demand for Bank Funds = E_t

The preceding result tells us that <u>the excess demand for commodities is at the same time an excess demand for credit</u>. A boom period thus tends to automatically raise the rate of interest, other things being equal, which in turn feeds back on the boom, ultimately choking it off and even reversing it.

It is interesting to note that this important mechanism regulating capitalist growth was essentially overlooked by both Keynes and Kalecki when they formulated their respective theories of effective demand. What is more, while their critics forced them to recognize that they had implicitly relied on 'credit inflation' (Kalecki) or increased bank 'finance' (Keynes) as the crucial foundation of their explanation of increases in economic activity.@ [Asimakopulos (1983), pp. 223, 226), neither author ever really integrated this knowledge into their arguments. Instead, both chose to rely on the premise that any excess demand for loanable funds would be accomodated without altering the interest rate, just as they also assumed that the excess demand for commodities and labor would leave prices and wages unchanged. The net effect was to insulate accumulation from all price feedbacks, and shift the burden of adjustment entirely onto changes in quantities.

The great strength of this emphasis on effective demand is that it directed attention to a previously underrepresented aspect of the adjustment process. But it also carried with it a particularly severe penalty: internally generated growth became impossible to explain. Thus we get the two characteristic branches of modern effective demand theories. The Harrodian branch, in which any balanced growth path is a knife-edge which wanders off explosively in either direction. The explanation of sustained growth then requires the assumption of some externally determined and growing upper and lower bounds positioned around the warranted growth path, so as to contain its instability and channel it in an upward direction. Alternatively, we have the Kaleckian branch, in which the system is stable but incapable of generating growth on its own. Once again, it is exogenous factors such as waves of new products, new markets, and new methods of production which used to explain bursts of growth.@ [Mullineaux (1984), Ch 2]

We have already noted that neither Kalecki nor Keynes formally account for the relation between the deficit finance of acumulation and the consequent patterns of debt. As a result, the only way left for financial variables to condition the rate of accumulation seemed to be through the effect of the <u>rate of interest</u> on investment decisions. This was the approach taken by Keynes himself. Recently, this same route has been taken by several authors (Taylor, 1985; Foley, 1985) as a means of breaking out of the impasse generated by the apparent instability of growth within theories of effective demand. But while the influence of interest rate movements is clearly important, it is not necessarily the central factor regulating accumulation. I would argue that the feedback between finance, debt and accumulation is a more fundamental mechanism. Thus even when the interest rate is assumed to be held constant, say through some "appropriate" set of state policies, this latter mechanism will turn out to be sufficient to stabilize accumulation. The resulting theory of effective demand is very much in the Classical Marxian tradition, with the internal profitability of the system driving accumulation and the consequent debt burden regulating it. It also vitiates all claims that there is an inherent contradiction between theories of effective demand and Classical theories of growth. The next section will therefore develop a simple model embodying the above principles.

II. A Macroeconomic Model of Internally Generated Cyclical Growth

To begin with, let us reproduce our previously developed fundamental equations for aggregate excess demand E_t and aggregate borrowing B_t.

II.1 $E_t = A_t + I_t + CONR_t - P_t$

II.2 $B_t = F_t + SAVR_t + CMR_t + E_t$

In what follows, we will abstract altogether from the payments of dividends by firms, and hence from rentier income. This is done merely to highlight the fact that the central relations within our model lie squarely within the circuit of capital itself, and not within any subsidiary circuits of personal revenue. Though we will not pursue the matter here, it is possible to show that the inclusion of personal savings does not change our results in any substantial way.@ [The real secret of treating personal savings turns out to lies in keeping track of the relationship the annual savings flow, the corresponding stock of financial assets which it feeds, and the interest income which it earns] In any case, once this assumption is made, it follows that both capitalist personal consumption $CONR_t$ and personal savings $SAVR_t$ are zero, so that equations II.1-II.2 above reduce to

II.3 $E_t = A_t + I_t - P_t$

II.4 $B_t = F_t + CMR_t + E_t$

1. Stocks and Flows of Money Reserves

The next step in our analysis is to specify the determinants of the desired changes in money reserves CMR_t. The simplest way to do this is to assume that firms wish to maintain constant money reserve levels, and that in each period the desired changes in these levels is the amount necessary to bring them up to the proper level. This is the most basic form of stock-flows adjustment mechanism, and is sufficient for our purposes. It should be noted, however, that an assumption such as a constant ratio of desired money reserves to output will only strengthen our results because it will increase the debt burden and thus further stabilize the system.

In any case, given a constant target level of money reserves, the desired addition to money reserves in any period will equal the unexpected depletion of money reserves in the last period. But since borrowing in any period has been assumed to be already predicated on maintainance of desired levels of money reserves in the face of expected receipts (as indicated by the presence of the term CMR in the fundamental finance equation I.3 above), any unexpected run down of reserves can only if actual receipts are below expected ones - i.e. if demand turns out to be less than supply. This means that the desired change in money reserves in any period is simply the negative of the excess demand in the last period.

II.6 $CMR_t = - E_{t-1}$

If we now substitute the above relation into our fundamental equation of finance, we can write

II.7 $B_t = F_t + E_t - E_{t-1}$

2. Borrowing and Debt Service

Once again, we will deal with the issue in the simplest possible way, which is to assume that all debt must be paid back, with interest, in one period. This means that the finance charges in any period can be written as

II.8 $F_t = (1+i)B_{t-1}$

where i is the interest rate on both borrowing ($B_{t-1} > 0$) or lending ($B_{t-1} < 0$) by firms. If we substitute this result into the borrowing equation II.7, we get

II.9 $B_t = (1+i)B_{t-1} + E_t - E_{t-1}$

3. Circulating Capital Investment and the Expansion of Output

We noted in section I.1 above that current output Q_t is the result of input purchases M_{t-1} and W_{t-1} made in the previous year. If we define the profit margin on these prime costs as m, then

II.10 $m = P_t/(M_{t-1} + W_{t-1})$

so that with a constant profit margin

$$P_t - P_{t-1} = m\,[(M_{t-1} - M_{t-2}) + (W_{t-1} - W_{t-2})]$$

But from the definition of accumulation in circulating capital (equation I.3), the above term in brackets is simply A_{t-1}.

II.11 $P_t - P_{t-1} = m\,A_{t-1}$

We therefore find that the rate of change of potential profit (surplus value) in any given period is proportional to the level of circulating capital investment in the preceding period.

4. Accumulation and the availability of Internal Finance

We have seen that businesses must finance their expenditure plans prior to carrying them out, and that it is the resulting expenditures which in turn form the basis for the actual sales in that period. Because excess demand E_t is the difference between actual sales and output offered, it is also the difference between actual aggregate profits P^*_t and potential profits P_t, so that

II.13 $P^*_t = P_t + E_t$

Now, when we consider the internal finance actually available for expenditure in this same period, it is clear that actual profit cannot form any part of this fund <u>because it is itself the result of already financed expenditures</u>.③ [Kalecki (1965), pp. 45-46)] It follows from this that only the profits realized in the <u>previous</u> period are available as a component of internal finance in this period.

Debt follows a similar pattern, in that it is the funds actually borrowed in the last period which reappear in the present as a component of finance internally available for accumulation. But here, the impact is a strictly negative one, since past borrowing imposes a current obligation to pay not only the principal but also interest. As such, the very borrowing that helped realize past profits must now take its toll on present internal funds.

The net result of the above considerations is that the total fund internally available for accumulation at the beginning of period t (end of period t-1) is the actual profit realized in the past period, minus that portion which is already obligated to debt service.

II.14 $X_{t-1} = P^*_{t-1} - (1+i)B_{t-1} = E_{t-1} + P_{t-1} - (1+i)B_{t-1}$

As we can see from the above equation, when past supply and demand balance ($E_{t-1} = 0$) and there is no borrowing ($B_{t-1} = 0$), then the internally available funds in this period are simply equal to past potential profit (surplus value). This is of course the Marx's basic result.

Now, as Marx so frequently emphasized, capitalists accumulate because the production of surplus value is the very aim of the circuit of capital, and because every successful negotiation of this circuit lands them in new territory in which the lure of ever greater profit and the threat of ever hungry competitors combine to propel them forward. From this point of view, the capitalist system is inherently expansive, in the specific sense that capitalists always try to accumulate. Moreover, since individual firms and even whole industries can expand at the expense of others, there is always an incentive to accumulate as long as aggregate profits exist. Thus it is profit, both actual and anticipated, which drives the system forward.

The strength of the tendency to accumulate can be expressed as the ratio of current circulating capital expenditures to current potential profits (surplus value). This ratio will generally be determined by several factors, ranging from the level and trends of past profits to various expected gains. We need to make no specific assumption about these factors. Instead, it is sufficient to assume that this ratio will increase or decrease whenever the corresponding ratio of total internally available funds X to potential profit increases or decreases.

$$A_t/P_t = A_{t-1}/P_{t-1} + k\,[(X_{t-1} - P_{t-1})/P_{t-1}]$$

which from II.14 yields

II.15 $A_t/P_t = A_{t-1}/P_{t-1} + k\,[(E_{t-1} - (1+i)B_{t-1})/P_{t-1}]$

Equation II.15 is our accumulation reaction function.

4. Differential Equation Form

Before we proceed further, it is useful to consolidate the preceding results, and to translate them into differential equation form so as to facilitate the proofs of model to be developed. Gathering the basic relations derived so far, we get

II.3 $E_t = A_t + I_t - P_t$

II.9 $B_t = (1+i)B_{t-1} + E_t - E_{t-1}$

II.10 $P_t - P_{t-1} = m\,A_{t-1}$

II.15 $A_t/P_t = A_{t-1}/P_{t-1} + k\,[(E_{t-1} - (1+i)B_{t-1})/P_{t-1}]$

Next, we express the same results in differential equation form, where a prime is used to indicate the time derivative of a variable, and the time index of a variable is implicit.

II.16 $E = A + I - P$

II.17 $B' = cB + E'$

II.18 $P' = mA$.

II.19 $(A/P)' = k[(E - (1+c)B)/P]$

where $c = \exp(i) - 1$ = the interest factor, which is approximately equal to the interest rate i.

5. The Medium Run

As the first step in exploring the properties of our model, let us initially assume that net fixed investment is zero. Within conventional effective demand theories, such an assumption would immediately imply a stationary state, because it is this level of net investment which essentially determines the level of output and employment. But in our case, the result is very different, because even in the absence of fixed investment the system converges cyclically onto a balanced growth path. To see this, it is useful to express all variables as a proportion of surplus value P, and then reduce the system to two basic equations.

Defining $e = E/P$, $b = B/P$, and $a = A/P$, equations II.16, 18, and 19 become (for zero fixed investment)

II.20 $e = a - 1$

II.21 $P'/P = ma$

II.22 $a' = k[e - (1+c)b]$

From II.20, $e' = a'$, so that II.22 becomes

II.23 $e' = k[e - (1+c)b]$

In the case of the finance equation II.17, since $b = B/P$

$$
\begin{aligned}
b'/b &= B'/B - P'/P = c + (E'/E)E/B - P'/P \\
&= c + (E'/E - P'/P)e/b + (P'/P)e/b - P'/P \\
&= c + (e'/e)e/b + ma(e/b - 1), \quad \text{since from II.22 } P'/P = ma \\
&= c + e'/b + m(e + 1)(e/b - 1), \quad \text{since } a = e + 1 \text{ from II.21}
\end{aligned}
$$

so that

II.24 $b' = cb + e' + me(e - b) + m(e - b)$

Equations II.23-24 summarize the essential structure of our model as a system of two nonlinear differential equations. However, the mathematical structure of this can be simplified even further by defining z = e -b and rewriting II.23-24 to get

$$e' = k[e(1+c) - ce + (1+c)b] = -kce + k(1+c)z$$

$$-z' = b' - e' = cb - ce + ce + mez + mz = +ce + (m-c)z + mez$$

This gives us our final version.

Such a system has several remarkable properties. First of all, it has a simple equilibrium point for which e=0 and z=0 (so that b=0).@ [Sanchez, 1968, pp. 84-87] Secondly, for any single disturbance the system is (generally cyclically) convergent around this equilibrium point for all positive values of the reaction coefficient k, as long as the interest factor c is less than the rate on return on circulating capital m (see Appendix A). Thirdly, the fact that the balance point contains e=0 implies that that the system automatically converges around balanced growth (at a rate of growth equal to the rate of profit since we have abstracted from both capitalist consumption and fixed investment). Fourthly, the fact that the balance point contains b=0 implies that accumulation is self-financing on average, with periods of excess demand for commodities and for bank funds being regularly succeeded by periods of excess supply in both. Finally, when we introduce random shocks to represent the anarchy of capitalist production, the system cycles endlessly around the basic growth path. The overall picture, in other words, is one in which the fluctuations of demand and supply serve to keep the system oscillating around (maximum) expanded reproduction.

Figure 1 illustrates the typical phase diagram response of e and z to a single shock, Figure 2 shows their corresponding movements over time, and Figure 3 shows the parallel responses of the level of circulating capital accumulation A and of potential profit (surplus value) P. This latter diagram makes it particularly clear that the system is stable around the balanced growth path. Figures 4-5 then show the cyclical behavior of the system over successive intervals as it is subjected to random pertubations representing the anarchy of capitalist production.

We now reintroduce fixed investment expenditures I. Going back to the original system in equations II.16-19, we define the proportion of fixed investment expenditures in potential profit (surplus value) as g, and the portion of surplus value in excess of fixed investment as $P^{\#}$. Thus

$$g = I/P, \quad \text{and} \quad P^{\#} = P - I = P(1-g)$$

Then the whole equation set can be rewritten in terms of $P^{\#}$.

II.27 $E = A - P^{\#}$

II.28 $B' = cB + E'$

II.29 $P^{\#\prime} = m^{\#}A$, where $m^{\#} = m(1-g)$

II.30 $(A/P^{\#})' = k[(E - (1+c)B)/P^{\#}]$

The above system is structurally identical to the earlier pure circulating capital model, with the exception that the stability condition now becomes i < m(1-g). However, this condition is really equivalent to the earlier one, since it simply requires that the rate of interest be less than the rate of return on total (fixed and circulating) capital advanced.

If we can assume that the fixed investment share g is relatively slow to change in comparison to the fluctuations in the circulating capital share a, then over the "medium run" defined by any given level of the investment share g, the system will oscillate around e=0 and b=0 just as before. It should be noted, incidentally, that the assumption of the robustness of the investment share is merely a dynamic version of the equivalent static Keynesian about the investment level in the short run.

Most interestingly of all, the fact that excess demand tends to fluctuate around zero implies that the sum of the circulating capital and fixed capital investment shares tends to center around 1 (see equation II.16). I.E.

II.31 $a + g = 1$, on average

Now suppose the level of capacity utilization is above normal, and the fixed investment share g rises in response to this. Then, even though this may initially set off a boom, the circulating capital share (a) will eventually be lowered by a corresponding amount. The same factors which accelerate the growth of capacity will therefore decelerate the growth of actual production, so that the capacity utilization level will tend to fall back toward normal (or even past it). This tendency is in striking contrast to the knife-edge instability usually found in convention effective demand models. It is, on the other hand, an implicit result in most Classical and Marxian analyses of accumulation. The next section will demonstrate the importance of this point.

6. The Long Run

We have previously defined the medium run as the period over which the fixed capital investment share g is relatively stable in the face of variations in supply and demand. On this basis, we found that the circulating capital investment share will then adjust itself to a level defined by g (equation II.31): a = 1 - g

We now define the long run as the period over which the fixed capital investment share changes in response to discrepancies between the actual and normal levels of capacity utilization. Defining the normal capacity utilization as 100%, we write the fixed capital reaction function as

II.33 $g'/g = h(u - 1)$

where u = the actual level of capacity utilization (a variable)
h = the reaction coefficient (a positive constant)

To complete this system, we note that capacity utilization u is defined as

$$u = Q/N = (Q/P)(Kf/N)(P/Kf)$$

where Q = actual output
N = output at normal capacity utilization
Kf = the stock of fixed capital
P = potential profit (surplus value), as before

But output $Q = Kc + P$, where Kc = input costs, so that $Q/P = (Kc + P)/P = Kc/P + 1 = 1/m + 1$, where m = profit margin on costs. We have already assumed a constant profit margin on costs (equation II.10). In addition, since we are abstracting from technical change, the capital/normal output ratio $Kf/N = v$ is constant. Then

II.33 $u = n\,(P/Kf)$, where $n = (1/m + 1)v$

$$u'/u = P'/P - Kf'/Kf$$

But $P'/P = ma$ from equation II.29, $a = 1-g$ from II.31, $g = I/P$, $Kf' = I$ = net investment, and $P/Kf = u/n$ from II.33, so that

$$u'/u = ma - I/Kf = m(1-g) - (I/P)P/Kf$$

II.34 $u'/u = m - mg - gu/n$

Equations II.32 and II.34 form a complete long run system.

II.33 $g'/g = -\,h \;+\; hu$

II.35 $u'/u = \;m \;-\; mg \;-\; gu/n$

Such a system has the remarkable property that it is stable around the point $u = 1$ (see Appendix B). This means that for any single displacement, the system tends to return to the normal capacity utilization. More importantly, in the face of random shocks representing a multitude of concrete factors and disturbances, <u>the system tends to cycle endlessly, alternately overshooting and undershooting the normal level of capacity utilization</u> (see Figure 6). Note that since the adjustement of the fixed investment share is a long run process, the period of the present cycle is longer than that corresponding to the adjustment of the circulating capital share. In other words, we have located the basis for both a medium run and a long run cycle.

Taken together, our medium run and long run systems constitute a full model of the interaction of effective demand, accumulation and finance. They provide theoretical foundation for generating two distinct types of business cycles. And most importantly, they provide a natural basis for connecting the Marxian focus on production and potential profitability with the modern emphasis on effective demand and realized profits. In this sense, they provide an alternative to the Keynesian and Kaleckian approaches to the question of effective demand.

Many aspects of this approach remain to be developed. For instance, it can be shown that if one keeps proper track of the relation between savings flows, stocks, and interest income, the consideration of personal savings does not change our basic results. Conversely, the introduction of sustained government deficit spending introduces a new factor, in that it gives rise to a corresponding sustained excess demand. This seems to provide a formal basis for the Classical link between deficit spending and inflation. Lastly, a falling potential rate of profit introduces yet another set of patterns, in which after a certain point the system seems to switch from stable cycles to unstable ones. This too is a very suggestive of Classical Marxist arguments.

Needless to say, the particular formalizations presented here are by no means the only ones which can generate models of a cyclically growing economy. The important thing is that this class of models seems to provide a very fruitful way of integrating effective demand, cycles and growth.

APPENDIX A: STABILITY OF THE MEDIUM RUN SYSTEM

The basic system is given by

II.25 $e' = -kce + k(1+c)z$

II.26 $z' = -ce - (m-c)z - mez$

in which the Jacobian is

$$J = \begin{pmatrix} -kc & k(1+c) \\ -c - mz & -(m-c) - me \end{pmatrix}$$

In spite of its nonlinear character, a system of the above form has only one equilibrium point 0,0 whose stability can be deduced from the Jacobian evaluated near this point (Sanchez, 1968, pp. 84-87). Let this Jacobian be J_0. Then,

$$J_0 = \begin{matrix} -kc & k(1+c) \\ -c & -(m-c) \end{matrix}$$

and its trace and determinants are

$Tr = -[kc + (m-c)] < 0$ for any positive k and $m > c > 0$ (rate of return on circulating capital investment > rate of interest)

$Det = kc(m-c) + kc(1+c) = kc(1+m) > 0$

The system is therefore stable around the origin $e=0$, $z=0$ (sink). What is more, it can be shown that for small and large values of k the system is smoothly convergent, but for intermediate values its convergence is oscillatory.

Appendix B: STABILITY OF THE LONG RUN SYSTEM

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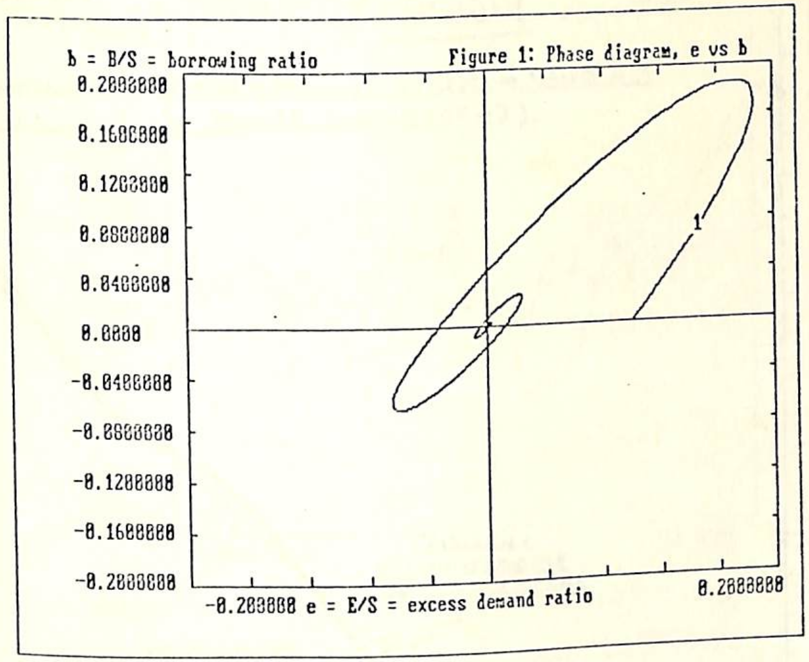
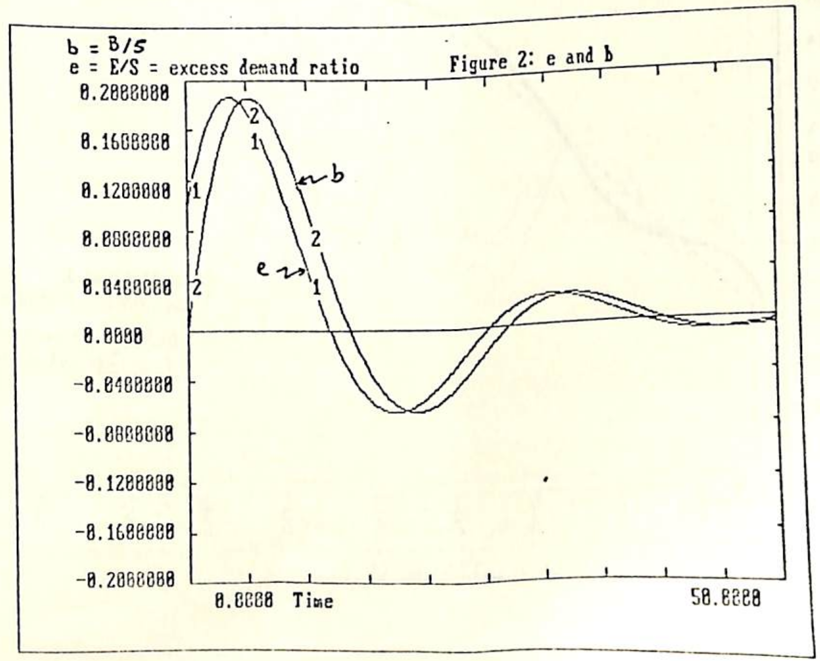
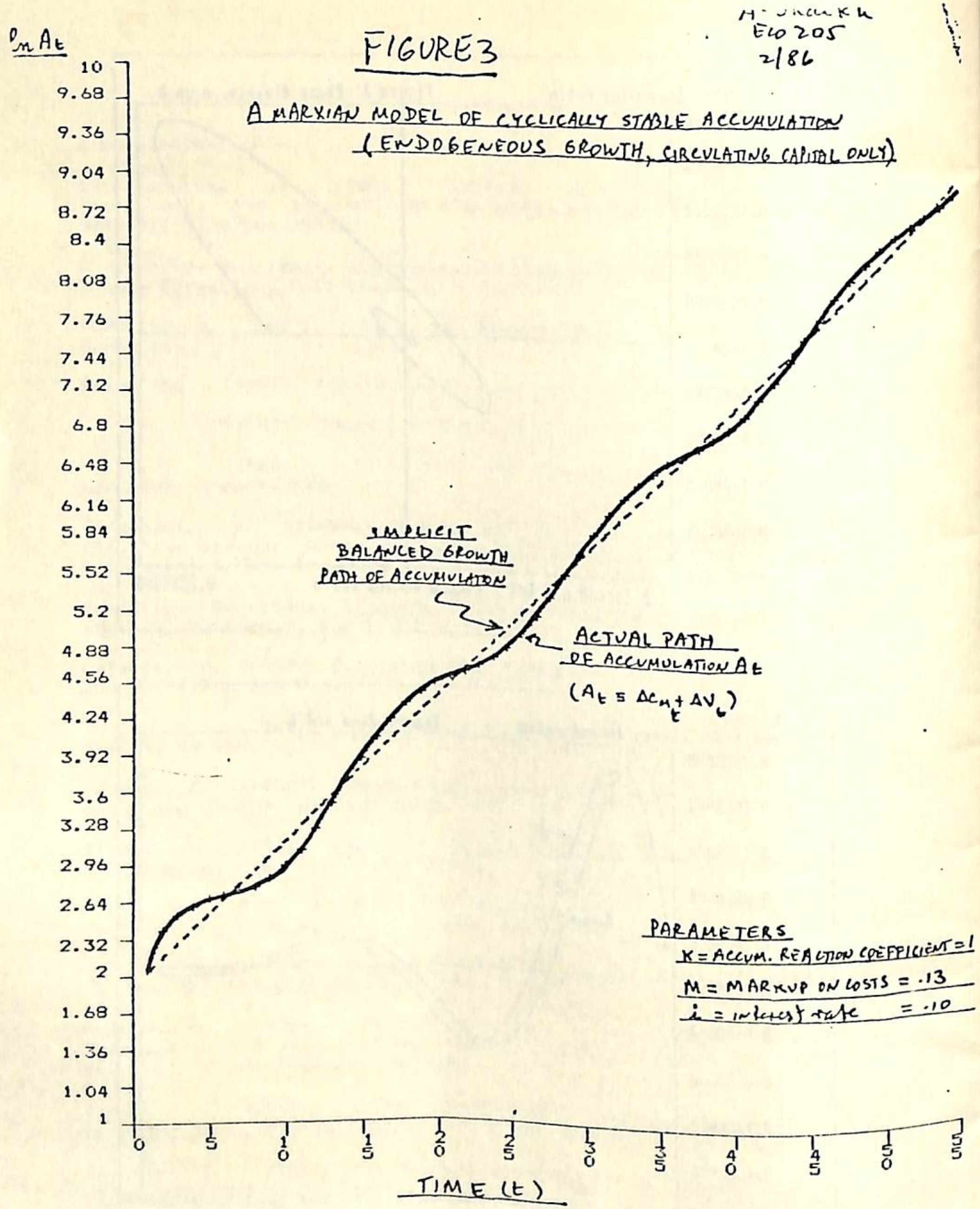
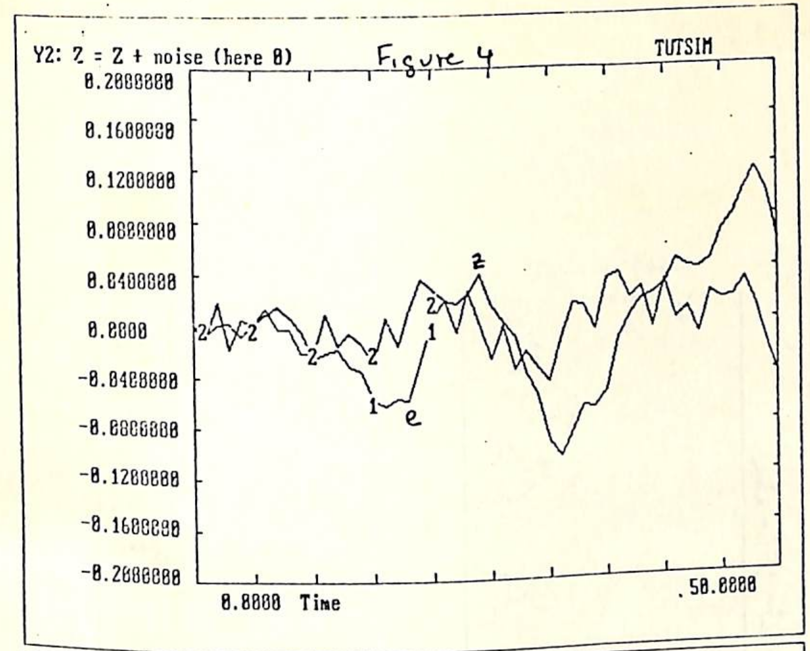


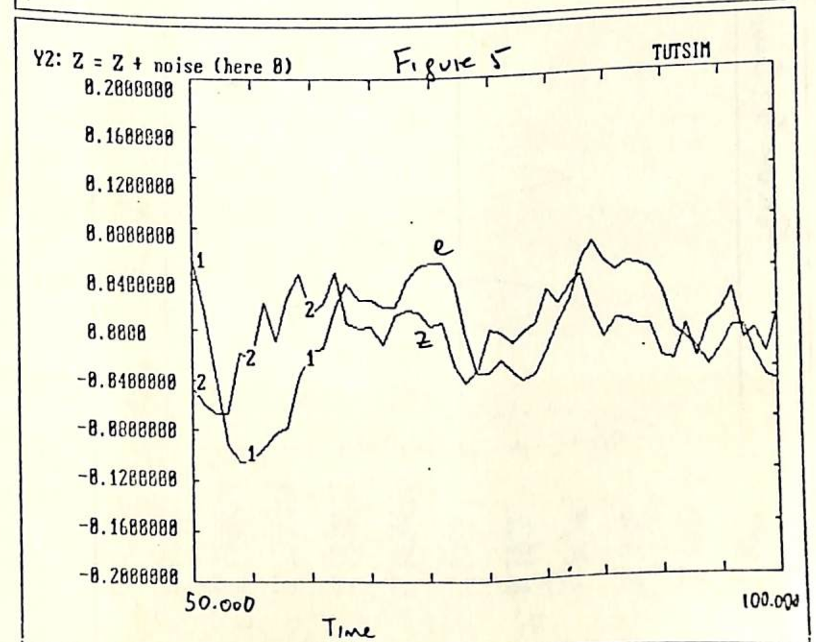




M. ... ECO 205 2/86
FIGURE 3
A MARXIAN MODEL OF CYCLICALLY STABLE ACCUMULATION
(ENDOGENEOUS GROWTH, CIRCULATING CAPITAL ONLY)
ln At
10
9.68
9.36
9.04
8.72
8.4
8.08
7.76
7.44
7.12
6.8
6.48
6.16
5.84
5.52
5.2
4.88
4.56
4.24
3.92
3.6
3.28
2.96
2.64
2.32
2
1.68
1.36
1.04
1
0
5
10
15
20
25
30
35
40
45
50
55
TIME (t)
IMPLICIT BALANCED GROWTH PATH OF ACCUMULATION
ACTUAL PATH OF ACCUMULATION At
(At = ΔCt + ΔVt)
PARAMETERS
K = ACCUM. REACTION COEFFICIENT = 1
M = MARKUP ON COSTS = .13
i = interest rate = .10



Y2: Z = Z + noise (here 0)
Figure 4
TUTSIM
0.2000000
0.1600000
0.1200000
0.0800000
0.0400000
0.0000
-0.0400000
-0.0800000
-0.1200000
-0.1600000
-0.2000000
0.0000 Time
50.0000
1
2
e
z



Y2: Z = Z + noise (here 0)
Figure 5
TUTSIM
0.2000000
0.1600000
0.1200000
0.0800000
0.0400000
0.0000
-0.0400000
-0.0800000
-0.1200000
-0.1600000
-0.2000000
50.000
100.000
Time
1
2
e
z

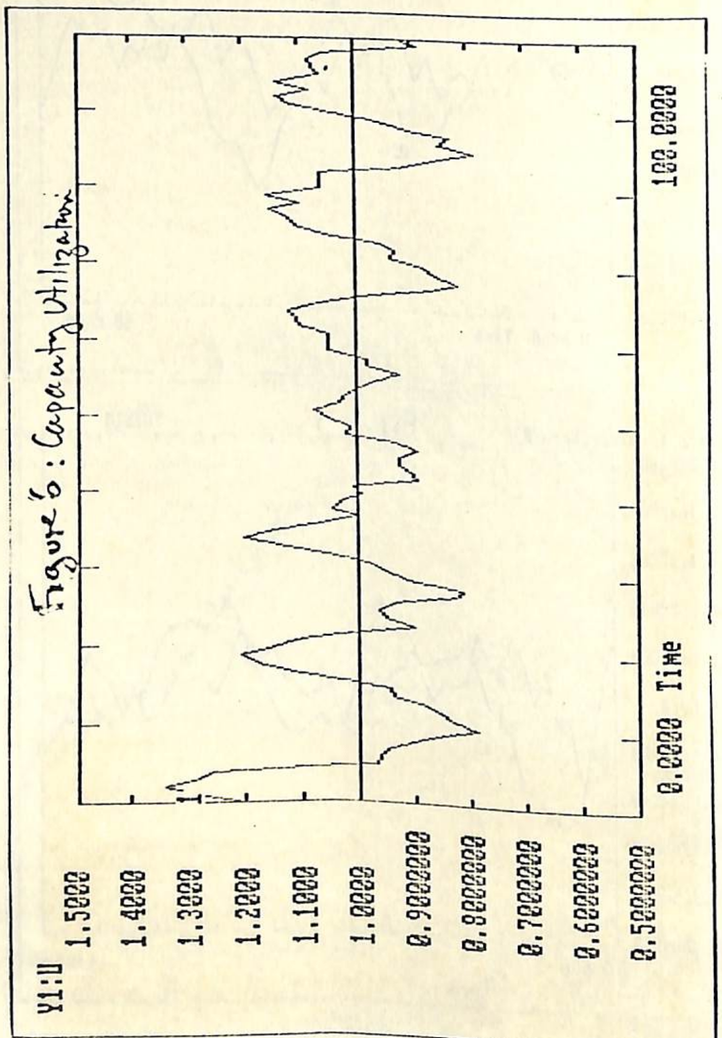


Figure 5: Capacity Utilization

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