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Portfolio Management with Semi-Parametric Bootstrapping

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Abstract

Estimation risk is an important topic within the area of risk management. Uncertanties on the parameter estimates carry on to the final statistical product, for example to the investment strategies, and need to be estimated and accounted for. Unless the exact expressions for the estimators' variances are known, the product's variability will be assessed through bootstrap techniques. We address this issue in this paper and propose a semi-parametric bootstrap method for reproducing the data, a method which parametrically takes care of all marginal characteristics of the returns data, and also takes care of the dependence structure existing in the data in a very simple and clever non-parametric way. The technique is applied to the problem of assessing variability of the Markowitz efficient frontier. Simulation experiments are conducted to assess the out-of-sample forecasting usefulness of the semi-parametric bootstrap methodology.

Key Words: Resampling, Efficient Frontier, Portfolio Management, Estimation Risk, Bootstrap.

1 Introduction

Markowitz[1] mean-variance (MV) optimization has been the standard tool for efficient portfolio allocations and diversification for the last 50 years. It is implemented

in probably all commercial portfolio optimizers for asset allocation and equity portfolio management. The Markowitz model, for a given level of expected return, obtains the portfolio's composition with the lowest risk (standard deviation). It is a quadratic optimization problem, whose inputs are just estimates of the population mean μ and covariance matrix Σ.

However, the inputs $(\widehat{\mu}, \widehat{\Sigma})$ are estimated with errors, and the optimization routines are known as errors maximization algorithms. The errors carry on to the weights with magnified variability, resulting in unstable portfolios, extreme weights, and poor out-of-sample performance. The problem is how to assess this variability. Unless the exact expressions for the estimators variances are known, estimates variability are typically assessed through bootstrap techniques (Efron and Tibshirani[2]), which are computer intensive techniques for resampling the data at hand.

Jorion[3] introduced in the finance literature the basic idea of resampling the efficient frontier. Since then, the discussions on the topic usually take place on the statistical validity of the resampled curves, on the role of the average resampled portfolio (Michaud[4] patented the use of the average resampled portfolios), or on how to obtain the average portfolio, either ranks based as in Michaud[4] or based on same risk/return trade-off.

Little effort has been put, however, on the appropriate selection of a resampling methodology. The existing methods are basically two: a non-parametric and a parametric bootstrap. The non-parametric bootstrap is the simplest one, where the bootstrapped observations are sampled from the original data with replacement. The parametric bootstrap assumes usually the multivariate normal distribution for the data (more generally, an elliptical distribution), and then simulate from the assumed distribution. Under the parametric approach, either $(\widehat{\mu}, \widehat{\Sigma})$ are assumed as being the true parameter values and a new data set is simulated, or a new set of inputs is simulated from the distribution assumed for $(\widehat{\mu}, \widehat{\Sigma})$ (see Scherer and Martin[5]). To deal with serial dependences in the data, moving block bootstrap was introduced by Carlstein[6] and Künch[7] .

To be faithful to the data at hand and to correctly obtain a set of replications of the efficient frontier containing the same information brought in by the original

one plus the nature provided and uncontrolled variability, one must be as close as possible to the true multivariate distribution generating the data. We address this question in this paper and propose a semi-parametric bootstrap for reproducing the data, a method which parametrically takes care of all marginal characteristics of the returns data, and also takes care of the dependence structure existing in the data in a very simple and clever non-parametric way. Our semi-parametric bootstrap approach to risk estimation is an alternative to the confidence regions of Jobson[8] and to the cited bootstrap methods.

In summary, the semi-parametric bootstrap methodology finds the margins' distribution by fitting unconditional or conditional models, and finds the dependence structure linking the margins by using the data ranks. The first step may be as simple as fitting a Normal distribution to a return series, and the second step just uses the empirical distribution. Alternative sophisticated univariate time series models for the mean and volatility may be used and usually provide good fits, specially for daily returns.

In Section 2 we explain the semi-parametric bootstrap. In Section 3 we provide a real data illustration of its usefulness when assessing the variability of the efficient frontier and optimal weights. In Section 4 we conduct several simulation experiments designed to assess the forecasting ability of the semi-parametric bootstrap method. In Section 5 we give our concluding remarks.

2 Replicating the Efficient Frontier

Let $\mathbf{r}$ represent a d-dimensional vector of financial returns from a distribution F with mean μ and (positive definite) covariance matrix Σ, and let $\mathbf{w}= (w_1, w_2, \cdots, w_d)$ be a vector of portfolio weights. The MV algorithm looks for a portfolio allocation that maximizes the expected utility $\mathbf{w}'\mu - \lambda\mathbf{w}'\Sigma\mathbf{w}$ over $\mathbf{w}\in \Re^d$, subject to $\mathbf{w}'\mathbf{1} = 1$ and $w_i \geq 0$, $i = 1, \cdots, d$ (long only), and where λ represents the investor risk aversion, the rate between portfolio return and standard deviation. The smaller the λ, the higher the risk aversion. For example, an extremely risk averse investor will choose the portfolio with the smallest possible standard deviation, the so-called "minimum

variance portfolio", which has a λ equal to zero.

Assuming that stock returns are jointly normally distributed implies that investors are using an expected utility given by the quadratic form. Even though normality seldom holds for financial returns, many theoretical results are obtained under this assumption. For instance, under normality, weights are unbiased and their variance may be obtained (see Britten-Jones[9] and Fusai and Meucci[10]). Michaud and Michaud[11] reviews and summarizes recent research and new developments in estimation error and MV portfolio optimization.

Despite the method used to estimate μ and Σ when it comes to replicating the efficient frontier what really matters is how well the data is replicated. To this end it is necessary to understand the data generating process (DGP). The following picture schematically represents the returns DGP.

Suppose we are working with *daily* d-dimensional log-returns $\mathbf{r} = (r_1, r_2, \cdots, r_d)$, and consider a period of length T. Every business day t, $t = 1, 2, \cdots, T$, nature generates a d-dimensional vector of errors $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \cdots, \epsilon_d)$ according to some zero mean and unit variance marginal distributions $F_1, F_2, \cdots, F_d$ not necessarily the same. Linking these marginal distributions there is a dependence structure (in the picture the oval box), which creates all relationships between the marginal errors, including the inter-relationships caused by economic, political and geographic factors and market macro-structure. At each day t, the errors vector passes through a black box holding information on market micro-structure, occasional (bad) news, seasonalities, and so on. At the end of the period T, the output is the observable return data, which may possess serial correlations, and other characteristics besides a mean vector μ and a $d \times d$ covariance matrix Σ.

Under our approach, in order to properly identify the models, and to obtain

faithful replications of the data, the analyst needs to identify the squared (black) box and the oval (black) box. He/she sees the oval box as a d-dimensional joint distribution F containing all marginal and joint information, in particular, the errors covariance matrix of the sequence of independent and identically distributed (i.i.d.) random vectors $\boldsymbol{\epsilon}_1$, $\boldsymbol{\epsilon}_2$, $\cdots$,$\boldsymbol{\epsilon}_T$. To model the information provided by the squared black box, the analyst uses the sequence of T (not i.i.d. anymore) d-dimensional returns. He chooses and fits univariate time series models, able to capture the temporal dynamics in the mean and in the volatility, for example, some ARFIMA(p, q) and FIEGARCH(r, d, s) type models.

In practice, this is done in the reverse order. The identification of the squared box is parametric. Using available computing facilities nowadays we are able to obtain excellent univariate (conditional or unconditional) fits tailored for each series of log-returns. We note that, in particular, the ARFIMA(p, q)-FIEGARCH(r, d, s) models may have $p = q = r = d = s = 0$ and therefore we are back to *unconditional models*, with just *a few* parameters to estimate, such as the mean, standard deviation, asymmetry and kurtosis. In this case a skew-t (Hansen[12]) distribution is a powerful and flexible option for univariate fitting.

From the d models fitted (say, M_1, M_2, $\cdots$, M_d) we obtain d series of standardized (zero mean and unit variance) residuals and identify their marginal distributions $(\widehat{F}_1, \widehat{F}_2, \cdots, \widehat{F}_d)$. The filtered data still contain the information about the oval box, and to *non-parametrically* get this information we substitute the $T \times d$ matrix of standardized residuals by their *matrix of ranks* $\mathbf{R}$. That is, we substitute each column of residuals by their ranks, numbers between 1 and T. For example, suppose $T = 100$. Row j of $\mathbf{R}$ may be $(9, 65, 34, 78)$. The closer the numbers, the stronger the dependence structure.

A reader familiarized with copulas may identify the rank matrix $\mathbf{R}$ with the support set for the empirical copula. Genest and Favre[13] argue that among all data functions which are invariant under monotone increasing transformations, the data ranks in $\mathbf{R}$ are the statistics retaining the greatest amount of information about the data dependence structure (Oakes[14]).

In summary, to obtain the proposed semi-parametric replications of the data one

should:

1. Fit conditional or unconditional models $(M_1, M_2, \cdots, M_d)$ to the univariate series. In the case of conditional modeling, obtain the standardized residuals and their estimated distributions $\widehat{F}_1, \widehat{F}_2, \cdots, \widehat{F}_d$. Derive the rank matrix $\mathbf{R}$.

2. For $k = 1, \cdots, B$, B large,

   2.1 Generate T i.i.d. random values from each $\widehat{F}_i$, $i = 1, 2, \cdots, d$, forming the $T \times d$ matrix of i.i.d. innovations, $\mathbf{Z}^{(k)}$.

   2.2 Order each column of $\mathbf{Z}^{(k)}$ according to the corresponding column in $\mathbf{R}$.

   2.3 Take each ordered column j as a new set of innovations and apply the corresponding model M_j, obtaining a new $T \times d$ data matrix $\mathbf{X}^{(k)}$ in the original scale.

Once the data replications are available, they may be used to compute any quantity of interest. Questions related to the size B are discussed in Efron and Tibshirani[2]. Each one out of the B new data sets $\mathbf{X}^{(k)}$, $k = 1, \cdots, B$ contains the same marginal (dynamic or not) information found in the original data, as well as their dependence structure. Using the rank matrix $\mathbf{R}$ in step 1 to re-order the innovations and reproducing the dependence structure in step 2.2 are the novelties. The idea of resampling using ranks have been inspired by the work in Mendes[15]. The method is amazingly simple and when fitting unconditional models it becomes even simpler *not* requiring any sophisticated computer software.

3 Illustration

To illustrate we selected a 6-dimensional data set that could represent a moderate risk profile investor willing to invest in emerging and developed markets. The data are 1629 contemporaneous daily log-returns from January, 2, 2002 to October, 20, 2008, on: (V1) a Brazilian Hedge Fund index, the Arsenal Composite Index (ACI); (V2) a Brazilian index for inflation indexed treasury bonds, the IMA-C; (V3) a Brazilian market value weighed Stock Index, the IBrX; (V4) an index of Large World

Companies (WLDLg, by MSCI Barra); (V5) an index of Small World Companies (WLDSm, by MSCI Barra); (V6) a Total Return US T-Bonds Index (LBTBond, by Lehman Bros.).

We compute the classical sample estimates and obtain the long only constraint classical efficient frontier. We note that any estimation method for the inputs could have been used. We apply the proposed semi-parametric bootstrap and for each replicated data set $\mathbf{X}^{(k)}$ we compute the classical inputs $(\widehat{\mu}^{(k)}, \widehat{\Sigma}^{(k)})$ obtainning the replications of the efficient frontier. For the sake of comparisons we also compute replications based on the parametric bootstrap assuming multivariate normality (as in Scherer and Martin[5]).

When replicating the efficient frontier what really matters is the weights stability across replications. To investigate this issue we examine in Figure 1 the box-plots of weights distribution for portfolios ranked numbers 2, 8 and 15 (out of 20). The original weights for each variable are signed with arrows. The box-plots corresponding to the parametric bootstrap based on normality are marked with a "-N". It is very impressive how the weights distribution based on our semi-parametric approach is much more concentrated. This is crucial to reduce portfolio rebalancing costs.

We observe that in many situations the parametric bootstrap weights distribution is shifted with respect to the original weight. For example, for portfolio 8 and variables 1 and 2, the original weight values are approximately located at the 0.75- and the 0.25-quantiles of the weights' distribution. For the same portfolio and variables, the arrows point to the center of the distribution provided by the new method. This means that in the d-dimensional space, the distance between the original portfolio's weights and the center of the distribution of weights from the replications are smaller for the new method. It is possible to measure that.

For any fixed portfolio, we measure the distance between the center of its weight distribution and the original weights using the squared Euclidean distance. For example consider portfolio ranked position 8, let $\mathbf{w}^*$ represent the original weights, and let $\bar{\mathbf{w}} \in \Re^d$ represent the distribution center, $\bar{\mathbf{w}} = \frac{1}{B}\sum_{k=1}^{B} \mathbf{w}_k$, where $\mathbf{w}_k$ is the optimal set of weights for portfolio 8 for the k-th simulation. The Euclidean distance is $D = (\bar{\mathbf{w}} - \mathbf{w}^*)'(\bar{\mathbf{w}} - \mathbf{w}^*)$. For portfolio 8 and for the new semi-parametric bootstrap

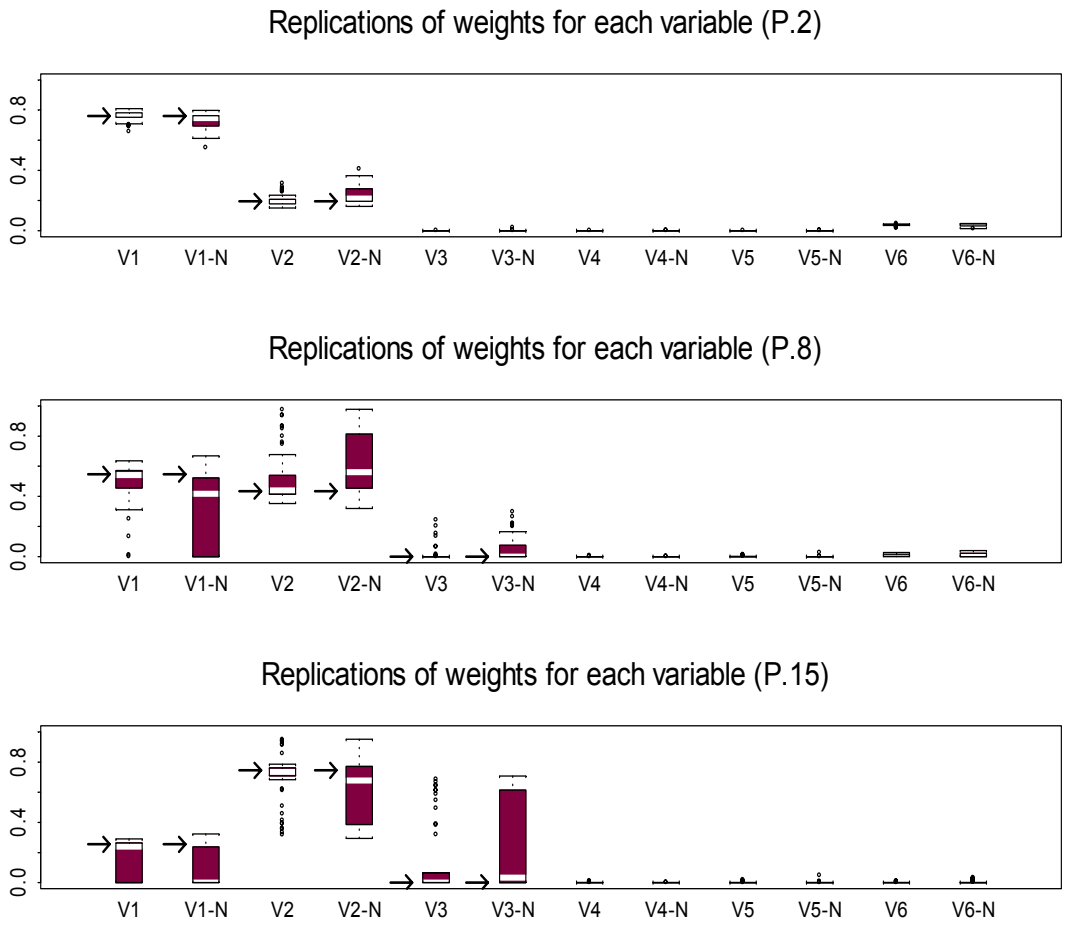

Figure 1: *Weights distributions for portfolios ranked numbers 2 (P.2), 8 (P.8), and 15 (P.15), from the semi-parametric method and the parametric (Normal) method.*

we computed $D = 0.016460$, whereas for the multivariate normal assumption $D = 0.1009245$, a distance 6.13 times greater.

The statistical equivalence between any resampled portfolios may be tested using the Mahalanobis distance, an statistical distance based on the weights covariance matrix. The resulting statistics is chi-squared distributed only under normality of the weights distributions. For any fixed (rank) portfolio, one can visually inspect its convex hull associated with the 90% statistically equivalent portfolios, using only the replications for which all d-variables weights were simultaneously inside a 90%

confidence interval.

As pointed out by a referee, "showing that simulations from the model fit have less variability is not the same as showing that the method is more efficient". In order to investigate this issue we carry on simulation experiments in the next section comparing the performance of the semi-parametric method with the parametric and non-parametric alternatives.

4 Out-of-sample Validations

We mimic a real life situation where a trader manages three clients' optimal portfolios and wants to forecast the portfolios' expected utilities for the next period based on the current allocations. The portfolios are defined through three λ values reflecting the clients' different risk preferences, namely $\lambda = 0.5, 1, 2$. The manager takes the available data along with the current allocations, and replicates the data (500 replications) according to his/her resampling bootstrap methodology: non-parametric (NPB), parametric (PB), or the semi-parametric (SPB). For each replication he/she computes the respective expected utilities, finally taking the average values over the 500 replications as the forecast for the next day. The aim here is to determine which resampling methodology should be used by the manager, the methodology resulting in a expected utility closest to the true one.

In a simulation experiment we know the true DGP (as explained in Section 2), and thus the "true" expected utility is known, namely

$$EU_{\lambda,0} = w_0'\mu_0 - \lambda w_0'\Sigma_0 w,$$

where (μ_0, Σ_0) are the true parameters (inputs) which provide the true weights w_0. To measure distance between the expected utilities derived from the resampling methods — $EU_{\lambda,NPB}$, $EU_{\lambda,PB}$ and $EU_{\lambda,SPB}$ — and the true $EU_{\lambda,0}$ we compute the absolute differences.

As in a real data example provided in Chapter 2 of Michaud[4] , and used later by other authors (Markowitz and Usmen[16], Liechty, Harvey, and Liechty[17]), we assume that the data could represent a collection of monthly percent returns from 8 assets (6 equity indexes and 2 bond indexes), over a period of 216 months. For simplicity, we

also take the parameters estimates used in these references as the true parameters values, although any other value could indifferently be used.

The experiments run as follows.

1. Assume some multivariate distribution F and the true parameter values (μ_0, Σ_0).

2. For $i = 1, \cdots, 100$

   (a) Generate an 216×8 data set by independently drawing from the 8-variate distribution assumed in 1. These are the manager's data at period i, used to define the current allocations (three λ-based portfolios).

   (b) For each bootstrap method — NPB, PB, SPB —

      i. Obtain 500 replications of the 216×8 data.

      ii. For each sample compute the sample estimates which combined with the current λ-based portfolios' weights, give rise to the corresponding expected utilities.

      iii. For fixed λ, return the average expected utility over the 500 simulations, denoted by $EU_{i,\lambda,NPB}$, $EU_{i,\lambda,PB}$ and $EU_{i,\lambda,SPB}$. They are used to compute the absolute distances between the average expected utilities and the true one, that is, the absolute values of $EU_{i,\lambda,NPB}$ - $EU_{\lambda,0}$, $EU_{i,\lambda,PB}$ - $EU_{\lambda,0}$, and $EU_{i,\lambda,SPB}$ - $EU_{\lambda,0}$.

The following multivariate distributions were considered in step 1: the Normal; t-student with 5 degrees of freedom (df); a multivariate distribution with Normal univariate margins linked by a t-copula with 5 df; and a multivariate distribution with t-student with 5 df univariate margins linked by a Gaussian-copula.

For each fixed λ we report in Table 1 the proportion of winnings for each bootstrap method over the 100 periods. We also report the utility functions grand mean and standard deviation, and note that the true expected utilities values are $EU_{0.5,0} = 0.0128581$, $EU_{1,0} = 0.0117284$ $EU_{2,0} = 0.0101549$, also given in last row of Table 1. The figures in Table 1 indicate that the semi-parametric bootstrap resampling methodology was able to provide better forecasts under all distributional assumptions.

In summary, the experiments indicate how good would be the long run forecast of an investor that consistently assumes some resampling methodology and forecasts his expected utility according to it. We note that most financial institutions assume the multivariate normal distribution for parametrically resampling the data, and we here suggest to look to the data marginal and joint structures.

Table 1: *Out-of-sample one-step-ahead expected utilities, under different probability distributions and for three risk aversion profiles, $\lambda = 0.5$, 1.0, and 2.0. Numbers in table are the proportions of times the average expected utility under each method (NPB, PB, SPB) was the closest to the true one.*

	$\lambda = 0.5$			$\lambda = 1$			$\lambda = 2$		
	NPB	PBP	SPB	NPB	PBP	SPB	NPB	PBP	SPB
Normal	36%	17%	47%	39%	14%	47%	42%	12%	46%
EU Gr.Mean	0.01563	0.01404	0.01158	0.01750	0.01733	0.01682	0.01557	0.01399	0.01151
EU Std. Dev.	0.00367	0.00342	0.00293	0.00396	0.00399	0.00404	0.00370	0.00344	0.00293
t-student(5)	41%	12%	47%	41%	12%	47%	45%	10%	45%
EU Gr.Mean	0.01629	0.01463	0.01198	0.01814	0.01800	0.01754	0.01615	0.01448	0.01184
EU Std. Dev.	0.00378	0.00351	0.00289	0.00405	0.00412	0.00422	0.00375	0.00350	0.00291
t-cop./Norm.	40%	11%	49%	42%	6%	52%	48%	2%	50%
EU Gr.Mean	0.01624	0.01502	0.01300	0.01752	0.01746	0.01717	0.01620	0.01498	0.01297
EU Std. Dev.	0.00313	0.00299	0.00265	0.00329	0.00331	0.00340	0.00317	0.00304	0.00269
Norm-cop./t-5	33%	19%	48%	32%	20%	48%	38%	21%	41%
EU Gr.Mean	0.01568	0.01338	0.01025	0.01857	0.01820	0.01717	0.01549	0.01323	0.01012
EU Std. Dev.	0.00429	0.00386	0.00309	0.00486	0.00487	0.00483	0.00439	0.00392	0.00311
True EU	0.01286	0.01286	0.01286	0.01173	0.01173	0.01173	0.01016	0.01016	0.01016

5 Concluding Remarks

We proposed a semi-parametric bootstrap method for replicating multivariate data, a method which parametrically takes care of all marginal characteristics of the returns series, including their temporal dependences, and also captures the correlations linking the data in a very simple non-parametric way. The method's flexibility allows one to go beyond the assumption of any particular fixed margins multivariate

distribution. This is very important in finance, since it is well known that returns on financial assets and indexes possess possess specific characteristics following different conditional and unconditional probability distributions. Therefore, no multivariate distribution with fixed margins would successfully model such data sets.

The replication scheme requires the same computational effort required by a parametric bootstrap. It can be implemented in any accounting oriented software, if an unconditional distribution is fitted to the margins, or in any statistical software in the case time series models are chosen for the univariate series. The two-steps semi-parametric bootstrap may be quickly applied to any (high) dimensional data, not suffering from the well known problem of "curse of dimensionality".

The main reason for obtaining replications of a data set is to assess estimation risk. This means obtaining the variability of selected statistics, for example, portfolios' optimal weights or the Value-at-Risk. When constructing efficient frontiers, the resulting portfolios' weights distributions may be used to identify similar portfolios. Statistical tests may be carried on to assess how far one needs to be away from the original set of weights to obtain a statistically different portfolio. This is measured with the concept of distance applied to the resampled weights. The illustration provided showed that the new method has the potential of providing more stable weights distributions for any given portfolio.

We ran out-of-sample simulations to assess the performance of the semi-parametric bootstrap method when forecasting three portfolios' expected utilities. The portfolios reflect the risk preferences of three clients. The experiments indicated that the semi-parametric resampling methodology provided better expected utility forecasts in the long run, as measured by their distances to the true one, under all distributional assumptions made.

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