DESIGN OPTIMIZATION OF OILFIELD SUBSEA INFRASTRUCTURES
WITH MANIFOLD PLACEMENT AND PIPELINE LAYOUT

Vinícius Ramos Rosa

Tese de Doutorado apresentada ao Programa de Pós-graduação em Engenharia de Produção, COPPE, da Universidade Federal do Rio de Janeiro, como parte dos requisitos necessários à obtenção do título de Doutor em Engenharia de Produção.

Orientadores: Virgílio José Martins Ferreira Filho
Eduardo Camponogara

Rio de Janeiro
Abril de 2017
DESIGN OPTIMIZATION OF OILFIELD SUBSEA INFRASTRUCTURES WITH MANIFOLD PLACEMENT AND PIPELINE LAYOUT

Vinícius Ramos Rosa

TESE SUBMETIDA AO CORPO DOCENTE DO INSTITUTO ALBERTO LUIZ COIMBRA DE PÓS-GRADUAÇÃO E PESQUISA DE ENGENHARIA (COPPE) DA UNIVERSIDADE FEDERAL DO RIO DE JANEIRO COMO PARTE DOS REQUISITOS NECESSÁRIOS PARA A OBTENÇÃO DO GRAU DE DOUTOR EM CIÊNCIAS EM ENGENHARIA DE PRODUÇÃO.

Examinada por:

Prof. Virgílio José Martins Ferreira Filho, D.Sc.

Prof. Eduardo Camponogara, Ph.D.

Prof. Laura Silvia Bahiense da Silva Leite, D.Sc.

Prof. Argimiro Resende Secchi, D.Sc.

Eng. Mário Cesar Mello Massa de Campos, Dr.

RIO DE JANEIRO, RJ – BRASIL
ABRIL DE 2017
Rosa, Vinícius Ramos


XI, 67 p.: il.; 29,7cm.
Orientadores: Virgílio José Martins Ferreira Filho Eduardo Camponogara


A Gina, amor e luz da minha vida.
A Luísa, amor em forma de sorriso.
E ao bebê que irá nascer.
Acknowledgment

I wish to express my sincere gratitude and appreciation to:

- Dr. Virgílio José Martins Ferreira Filho for his wise words, enthusiasm and advises;
- Dr. Eduardo Camponogara for his excellent supervision and active participation throughout my research. It was a honor and a great experience to work with him;
- Dr. Thiago Lima Silva for his unconditional help on GAMS code for piecewise-linear approximation;
- Dr. Geraldo Afonso Spinelli Martins Ribeiro, my boss, for his support and encouragement to my research;
- Eng. Renato Brum, my former boss, for his support on the fist stages of my doctorate program;
- Eng. Alex Furtado for his support and for providing me opportunities to meet researches in optimization like Eduardo Camponogara, Bjarn Foss and Sthener Campos;
- Petrobras, the company I work for, proud of their employees, proud of Brazil;
- My wife Gina and my daughter Luísa for patience, support and strength;
- My parents Moysés and Conceição for providing me with education leading me to be an engineer;
- Mary, the Most Blessed Mother and his son Our Lord Jesus Christ.
Resumo da Tese apresentada à COPPE/UFRJ como parte dos requisitos necessários para a obtenção do grau de Doutor em Ciências (D.Sc.)

OTIMIZAÇÃO EM PROJETOS DE INFRA-ESTRUTURAS SUBMARINAS DE CAMPOS PETROLÍFEROS COM POSICIONAMENTO DE MANIFOLDS E CONFIGURAÇÃO DE DUTOS

Vinicius Ramos Rosa

Abril/2017

Orientadores: Virgílio José Martins Ferreira Filho
            Eduardo Camponogara

Programa: Engenharia de Produção

Este trabalho apresenta um método de otimização prático e eficaz para o projeto de redes de produção submarinas em campos de petróleo offshore, o que compreende o número de coletores, sejam manifolds ou plataformas, sua localização, atribuição de poços a esses coletores e diâmetro de dutos que interligam todos os elementos da rede. Ele traz uma solução rápida que pode ser facilmente implementada como uma ferramenta para otimização de layout e de estudos baseados em simulação. O modelo proposto compreende a dinâmica do reservatório e fluxo multifásico em dutos, baseando-se na linearização multidimensional por partes para formular o problema de otimização de layout como programação inteira linear mista. Além da validação da solução ótima obtida pelo método, a simulação de reservatórios define limites para as variáveis e parâmetros do modelo que caracterizam a perda de carga, a dinâmica do reservatório e a produção de óleo dos poços ao longo do tempo. A perda de carga nas tubulações é modelada por funções lineares por partes que aproximam resultados obtidos pelos simuladores de fluxo multifásicos. O modelo de otimização foi aplicado a um verdadeiro campo de petróleo offshore com o objetivo de avaliar sua efetividade.
Abstract of Thesis presented to COPPE/UFRJ as a partial fulfillment of the requirements for the degree of Doctor of Science (D.Sc.)

DESIGN OPTIMIZATION OF OILFIELD SUBSEA INFRASTRUCTURES
WITH MANIFOLD PLACEMENT AND PIPELINE LAYOUT

Vinícius Ramos Rosa

April/2017

Advisors: Virgílio José Martins Ferreira Filho
Eduardo Camponogara

Department: Production Engineering

This work presents a practical and effective optimization method to design subsea production networks, which accounts for the number of manifolds and platforms, their location, well assignment to these gathering systems, and pipeline diameter. It brings a fast solution that can be easily implemented as a tool for layout design optimization and simulation-based analysis. The proposed model comprises reservoir dynamics and multiphase flow, relying on multidimensional piecewise linearization to formulate the layout design problem as a MILP. Besides design validation, reservoir simulation serves the purpose of defining boundaries for optimization variables and parameters that characterize pressure decrease, reservoir dynamics and well production over time. Pressure drop in pipelines are modeled by piecewise-linear functions that approximate multiphase flow simulators. The resulting optimization model and approximation methodology were applied to a real oilfield with the aim of assessing their effectiveness.
# Contents

List of Figures ........................................... ix

List of Tables .......................................... xi

1 Introduction ........................................... 1
   1.1 Objective ............................................ 6
   1.2 Thesis Outline ....................................... 7

2 Literature Review ..................................... 8
   2.1 Review ................................................ 8
   2.2 Discussion .......................................... 16

3 Problem Presentation ................................. 18
   3.1 Problem Statement, Model Premises and Limitations . 18
   3.2 Thesis Contribution .................................. 22

4 Model .................................................... 23
   4.1 Sets, indexes, parameters and variables ............... 23
   4.2 Mathematical Programming Model ...................... 28

5 Application of Methodology to a Representative Oilfield 38
   5.1 Case Study: Ipanema Field ........................... 38
      5.1.1 Layout Optimization ............................ 43

6 Conclusions, Discussion and Future Work ............... 52
   6.1 Conclusions and Discussion ........................ 52
   6.2 Future work .......................................... 53

Bibliography ............................................. 55

A Detailed results for all wells ....................... 60
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Offshore production system, adopted from GUNNERUD [1].</td>
<td>2</td>
</tr>
<tr>
<td>1.2</td>
<td>Scheme of layout with subsea manifolds, adopted from UMOFIA and KOLIOS [2].</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Subsea manifold to be installed, from LIMA et al. [3].</td>
<td>3</td>
</tr>
<tr>
<td>1.4</td>
<td>Scheme of a subsea manifold, from BAI and BAI [4].</td>
<td>4</td>
</tr>
<tr>
<td>1.5</td>
<td>Network production, from BEGGS [5].</td>
<td>4</td>
</tr>
<tr>
<td>1.6</td>
<td>Flowrate as a function of the pressure at concentrator A, from BEGGS [5].</td>
<td>5</td>
</tr>
<tr>
<td>5.1</td>
<td>Ipanema oil saturation - top view. Red = oil, blue = water (aquifer).</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>Producer wells - black names, water injector wells - blue names.</td>
<td></td>
</tr>
<tr>
<td>5.2</td>
<td>Ipanema original layout - all flowlines have 4&quot; diameter, green dotted lines. The yellow traces are the anchorage system.</td>
<td>40</td>
</tr>
<tr>
<td>5.3</td>
<td>All possible connections of subsea layout to Ipanema field.</td>
<td>41</td>
</tr>
<tr>
<td>5.4</td>
<td>Pressure weighted by pore volume after 10 years of production.</td>
<td>42</td>
</tr>
<tr>
<td>5.5</td>
<td>Reservoir partitioned in regions of similar pressure behavior.</td>
<td>42</td>
</tr>
<tr>
<td>5.6</td>
<td>Pressure envelopes for tanks.</td>
<td>44</td>
</tr>
<tr>
<td>5.7</td>
<td>Envelopes and average parameters for wells - PROD6</td>
<td>45</td>
</tr>
<tr>
<td>5.8</td>
<td>Layout that maximizes NPV.</td>
<td>47</td>
</tr>
<tr>
<td>5.9</td>
<td>Pressure and cumulative production of tanks 01, 02, 04 and 06. Comparison among boundaries, optimization solution and ECLISPE simulation of solution.</td>
<td>48</td>
</tr>
<tr>
<td>5.10</td>
<td>Whole reservoir cumulative production. Comparison between optimization solution and ECLISPE simulation of solution.</td>
<td>49</td>
</tr>
<tr>
<td>5.11</td>
<td>Well PROD6 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.</td>
<td>50</td>
</tr>
<tr>
<td>5.12</td>
<td>NPV, capex and cumulative production - comparison among optimal solution and the four solutions obtained for definition of pressure boundaries.</td>
<td>51</td>
</tr>
</tbody>
</table>
5.13 Liquid rates for all wells satellite with 6” diameter flow line and riser. PROD1, the dark blue line, is shut down after roughly 6 months of production. .............................. 51

A.1 Well PROD1 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution. .............................. 60

A.2 Well PROD2 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution. .............................. 61

A.3 Well PROD3 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution. .............................. 62

A.4 Well PROD4 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution. .............................. 63

A.5 Well PROD5 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution. .............................. 64

A.6 Well PROD6 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution. .............................. 65

A.7 Well PROD7 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution. .............................. 66

A.8 Well PROD8 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution. .............................. 67
# List of Tables

5.1 Flexible Line Costs .............................................. 44  
5.2 Manifold Cost ...................................................... 44  
5.3 Oil Profit and Interest Rate ................................. 46  
5.4 Model Results and Computational Time .................. 47
Chapter 1

Introduction

At the early stages of an oilfield development plan, after bottom-hole and wellhead locations have been defined, production engineers design the subsea pipeline network to bring production from wells to facilities for processing.

The development of an offshore oil field is always a long, complex and extremely expensive task. It is strongly intensive in capital, thereby motivating initiatives of development plan optimization.

The following is a list of important decisions to be made in the development of a field, related to the design of facilities and those associated with their operation:

1. Project stage
   - Number and location of producer and injector wells;
   - Number, capacity and location of production facilities (platforms and manifolds);
   - Designation of wells to manifolds and platforms;
   - Definition of production offloading.

2. Design and operation stage
   - Programming of the start-up of production facilities;
   - Programming of drilling and completion, as well as the first oil from the producers;
   - Moment of abandonment of wells and platforms.

3. Operation stage
   - Production and injection rates over the field concession period;
   - Secondary recovery method and artificial lift.
Although there are technological and economic constraints limiting most of all possible combinations of these parameters, it is difficult to choose the best possible configuration. In this context, mathematical programming methods are applicable for defining the optimal set of parameters leading to maximization of the project net present value. Without the approach by mathematical optimization, project teams tend to study only a few cases without guarantee of optimum or very close to optimal configuration.

Figure 1 shows a schematic of a field with more than one reservoir. There are three main components: the reservoirs, the wells and the subsea network. The subsea network comprises:

- Well heads, called christmas trees, which are a set of safety and control valves;
- Subsea manifolds, equipment on the seabed which concentrate and control the production from wells linked to them;
- Surface Manifolds, which concentrate the production of wells directly linked to the platform before starting the processing;
- The processing plant, which receives the production of submarine and surface manifolds, and finally;
- Subsea pipelines that interconnect wells, subsea manifolds and platform.

Subsea manifolds organize the wells into clusters as Figure 1.2 presents. It is a set of valves in the seabed that functions as a catchment substation, a hub of oil
produced by several wells, which is sent to a platform by a single pipe. The objective of using manifolds is to reduce the number of pipes connecting wells to platforms, reducing the overall length used and thus allowing greater flexibility in the field operation project [4]. Similar to platforms, manifolds also have limitations on the maximum number of interconnected wells, depth, operating pressure and flowrate.

Figure 1.2: Scheme of layout with subsea manifolds, adopted from UMOFIA and KOLIOS [2].

Figure 1.3: Subsea manifold to be installed, from LIMA et al. [3].

The interaction among reservoirs, wells and subsea network strongly affects the wells production and the field accumulated production.

It is needed to model the reservoir responses to changes in parameters of wells. At design stage, the changes that have to be predicted are often related to alterations in the subsea network configuration.

There is a correlation between well productivities and the distances between wells and platforms. Long distances require higher pressures available in the wells
to achieve profitable oil flow. Short distances provide lower pressure drop along pipelines, leading to higher throughputs.

In addition, locating a subsea manifold also means choosing which wells will be connected to it. Figure 1.5 from BEGGS [5] illustrates a production network where wells 1, 2 and 3 are connected to a concentrator A, which can play the role of a manifold in an subsea layout, wells 4 and 5 connected to a concentrator B, the concentrators A and B are connected with concentrator C and this to D.

![Diagram showing network production](image)

**Figure 1.5: Network production, from BEGGS [5].**

Let’s analyze concentrator A. Figure 1.6 shows the flowrate curve versus pressure at point A. To trace it, it was taken the sum of rates from wells 1, 2, and 3, keeping the individual well pressure values. Then, the curve of concentrator A is the sum of the curves of each interconnected well [5]. Suppose that concentrator A could receive a maximum of three wells. If there were more wells that could be connected to A
and this concentrator could only receive three connections, we would have different possibilities of curves that characterize the flow as a function of the pressure at concentrator A.

![Figure 1.6: Flowrate as a function of the pressure at concentrator A, from BEGGS [5].](image)

In this way, a conclusion that arises is that each different combination of well connection in manifolds leads to different production flowrates. Therefore, it is reasonable to assume that there must be an optimal combination of wells interconnecting to manifolds according to some optimization criteria.

It is also important to note that the characteristic curves of wells 01, 02 and 03 in Figure 1.6, besides being influenced by the pressure drop between each well head and the concentrator A, are also a function of reservoir behavior. These curves are not static, they vary over time in function of several variables related to flow in porous medium and the adopted policy of reservoir management.

It is straightforward to see that choosing the best combination of connections to wells and manifolds or platforms has a combinatorial nature. Hence, their optimization must employ mathematical programming techniques combined with multiphase flow modeling and flow in porous medium.

Thus, in this problem not only the minimization of distances must be taken into account, but also the oil production revenues, which are a function of the flow conditions in the subsea pipelines. Instead of optimizing only investments in the project, by capex minimization, optimization must have as objective function the maximization of its net present value.
The point is how to structure the subsea pipeline network in a cost effective manner leading to production maximization.

The traditional approach to this problem is to select a group of experts from several related technical areas to study a few realistic scenarios. Although this approach may provide reasonable solutions to the problem, it cannot guarantee that the mix of options will result in the minimum investment.

On the other hand, mathematical programming methods are widely applied to model and solve investment minimization or net present value maximization problems considering major design restrictions. However, the lack of optimization expertise by technical project teams allied with the distance of the models to real world situations discourage engineers from applying optimization techniques.

To that end, this work presents a practical and effective method to design a subsea production network which accounts for the number of manifolds and platforms to be installed, their location, well assignment to them and pipeline diameters. It brings a fast solution which can be easily implemented as a design optimization tool and a scenario-study simulator. The proposed model comprises reservoir dynamics, multiphase flow, multidimensional piecewise linearization and integer programming. A MILP formulation was developed for optimal design of the subsea infrastructure on piecewise-linear approximation of pressure drop in pipelines.

In many past works, the behavior of the reservoir pressure is modeled as a function of cumulative production. However, the curve of pressure decay along cumulative production is dependent on the production infrastructure. This work also proposes a practical way to overcome this point based on reservoir simulation, performed apart and in advance to define feasible bounds for variables in MILP equations.

To validate the proposal, a case study was performed in a real offshore field that here received a fictitious name of Ipanema.

1.1 Objective

This work aims to optimize subsea layout design by using integer programming to couple both models of flow in porous media and multiphase flow in pipelines.

Therefore, this thesis proposes a workflow that is based on the statements:

a) pressure drop along pipes can be piecewise-linear approximated;

b) variables that represent reservoir behavior can have bounds, obtained from reservoir simulation of subsea layouts which leads to large or small cumulative production over time;
c) feasible solutions in layout design can be provided by a system of equations which combines pressure drop piecewise-linear approximated and bounds for reservoir variables;

d) those equations can be gathered in a optimization model, based on mixed integer linear programming, to maximize net present value of an oilfield development plan.

1.2 Thesis Outline

This thesis is organized in six chapters. This chapter presented the context of our work, the motivation for this research and its objective.

Chapter 2 discusses the previous works which dealt with oilfield layout optimization. A discussion is made about many publications, pointing their contributions and limitations. Then, all related works are classified into two categories: those which are concerned in minimizing project capital expenditure and those which maximizes the project net present value.

Chapter 3 presents the problem in detail. All premises are pointed and discussed, limitations are pointed out and considerations are explained. The proposed methodology is detailed in sentences, not in equations, to a better understanding of this work.

Chapter 4 comes up with the mathematical programming model developed to achieve the research objectives.

Chapter 5 brings an application of methodology to a representative oilfield, using models of real wells and reservoir. Results and limitations are presented and discussed.

Chapter 6 is dedicated to conclusion, final remarks, thesis contributions and some proposal of future works.
Chapter 2

Literature Review

Subsea layout optimization is often treated as part of field development design optimization. For this reason, most of the papers do not develop layout optimization in details. Some papers address pipeline routing, diameter specification and manifold placing, but these design issues are not handled simultaneously, combined with reservoir pressure updates. Further, the existing literature on layout design optimization is typically targeted at capex minimization or net present value maximization. The latter works usually rely on a simple reservoir model which in most of the cases is too modest to capture the key reservoir dynamics. On the other hand, the former works do not take into account the influence of pipeline pressure drop and reservoir response on well production.

2.1 Review

From the literature that treats layout design optimization only as capex minimization, the first notable work is by DEVINE and LESSO [6]. They proposed a heuristic based on the $p$-median problem to optimize well trajectories and their allocation to fixed platforms, considering coordinates in reservoir to be reached by drilling. The decision variables are the number, capacity and location of the platforms, the designation of the drilling objectives to the platforms. The objective function is to minimize development costs. The work considers fixed platforms and directional wells with dry christmas tree. The drilling objectives in the reservoir have their locations in a continuous space. The procedure for the proposed solution is a heuristic and was referred to as an algorithm for location-allocation alternatives. This routine is an iterative process consisting of the following steps: (1) the targets in the reservoir are assigned to each platform arbitrarily; (2) the location of each platform is defined to reduce the costs of directional drilling; (3) the targets are then reassigned to the platforms to further reduce drilling costs; Steps (2) and (3) are repeated until there is no change in the total cost. Although the optimal number of platforms is one
of the decision variables, the procedure does not determine the number of platforms to be used. The authors suggest varying the number of platforms such as sensitivity analysis. In addition, the optimal solution of an allocation problem depends on the efficiency of the algorithm proposed in testing all possibilities of platform location and the allocation of objectives to the platforms. The proposed methodology is based on the problem of localization of $p$-medians, of NP-complete complexity, which is currently solved by modern heuristics such as the genetic algorithm.

GRIMMETT and STARTZMAN [7] proposed a model that evaluates previously some pointed locations for drilling wells and installation of a platform. To obtain the optimal solution, the authors use integer linear programming solved by branch and bound. Simultaneously is selected the type, size, quantity, location of the platforms and the allocation of the drilling to these platforms. The objective function is to minimize investments. Capacity and technological constraints are imposed, such as the maximum distance between the platform and the target to be achieved in the reservoir. Pre-processing is performed to eliminate non-viable configurations in order to reduce computational time.

HANSEN et al. [8] proposed the problem of planning multi-capacity facilities as a means of defining the location of platforms, whose objective is the minimization of investments. The model considers directional wells drilled from fixed platforms, with pre-defined capacities and continuous space for platform location. They stated an NP-complete problem, an integer-mixed programming model which was solved by tabu search heuristic.

FAMPA [9] was the first to consider subsea production manifolds in layout optimization. The model considers manifold location and well allocation as a covering problem solved by cutting-plane generation and Balas’s algorithm. The work dealt with subsea layout optimization as a minimization problem solved in three steps. The first one corresponds to definition of manifold location on the seabed and its attribution to the wells, which starts from the coordinates of the drilling objective points in reservoir. The second step optimizes a model to define the location of wellheads. The third step is concerned with the definition of platform location. As a model parameter, there is a radius considered as the largest possible distance between the drilling objective in the reservoir and a manifold or platform. Thus, it obtains the geometry of the directional wells and subsea lines. When projecting in the horizontal the radius from drilling objectives in the reservoir, circles can be traced. Combining all circles from every well, some circles can intercept. The intersections between these circles form a discrete set of places to install a manifold. The authors formulated this first step of the solution as a covering problem where the coverage matrix contains the possible objective interconnections. The location of wellheads is the least distance solution between the drilling objectives in the
reservoir and the manifold to which the well interconnects, respecting technological restrictions. The location of the platform, which would be the third step, had the solution suggested as a future work.

DING and STARTZMAN [10] proposed a model for minimization of investment in oil field development. The problem to solve is very similar to GRIMMETT and STARTZMAN [7]. The number of wells to be drilled and the respective drilling targets in the reservoir are known in advance. The need is to decide for each well the christmas tree position, the well geometry and the number of platforms. All wells were directional, without subsea flexible lines. For the solution of the example problem, a grid of 10x30 cells on the top view of the reservoir was traced. As a simplification to reduce computational time, the candidate positions to receive a platform are previously pointed out. The model consists of the minimization of a cost function, not taking into account flow in the porous media in reservoir and in tubing in the wells. The cost functions are linear but constrained by the number of wells, the capabilities of the available platform models, and the physical aspects of well construction. Two solutions are developed for the model: integer programming by pure branch & bound and branch & bound with Lagrangian relaxation. The entire programming is based on the branch and bound procedure with pre-processing to simplify the model by prior elimination of solutions that do not satisfy the constraints, employing addition of inequalities, addition of logical constraints, fixation of variables and other devices. The Lagrangian relaxation solution is applied to the model for performance comparison with the solution by pure branch & bound approach. They were the first to apply Lagrangian relaxation in subsea layout problems, which produced better results compared to traditional branch & bound due to less computational time.


CORTES [12] brings about a multi-objective and interactive approach that yields a set of layout options to be evaluated by the project team. It considers that subsea layout design comprises several and distinct objectives and in many cases they could be conflicting. The multi-objective and interactive approach takes into account the preferences of the decision maker expressed by weights for each objective. The model addresses three objectives: a) minimizing investment of wells and platforms, b) minimizing cost of connecting wells to these platforms, c) maximizing oil production in a given region. Costs minimization is similar to the traditional approach adopted in many facility location problems, like in the Weber problem. The maximization of oil production takes as input the estimates of production associated with each
possible position of the wells.

GARCÍA-DIAZ et al. [13] model capex minimization with graphs, whose arcs represent possible links between objectives in the reservoir and whose nodes correspond to candidate locations for platforms. It is defined a set of candidate sites to receive a platform and another set of targets in the reservoir to be reached by directional wells. The allocation of the wells to the platforms is modeled by a network where the arches correspond to the wells to be drilled. The model is similar to GRIMMETT and STARTZMAN [7] but solved by branch and bound and Lagrangian relaxation.

NADALETTI [14] points out that the project team can reach better solutions by applying decision tools, in order to define the number and location of subsea manifolds which play a part in capex minimization. The author divides the problem of subsea design into four subproblems: grouping of wells, association of wells and platforms, and inclusion of manifolds. The model assigns the design engineer the choice of the most appropriate algorithm to be applied in each subproblem and the configuration of algorithm parameters. Yet, the project team has to decide which models (spanning tree, minimax, or \( k \)-means) and tools are best suited for layout design.

XIAO et al. [15] study optimization of network pipeline design on a Middle-East onshore field. The problem was divided into two parts: first, manifold location; then, pipeline routing of the connections from wells to the collecting station. A gradient-based solution is applied to define the station coordinates, considering Euclidean distances and a non capacited \( p \)-median problem. The objective function minimized the sum of product flow per distance for all wells. Thereby, the authors consider to include pressure drop effects, favoring the proximity of the station to wells of high oil rates. To trace the pipeline network, the problem was modeled as a capacitated minimum spanning tree solved by Prim’s algorithm. Oil rates from the wells are known in advance by reservoir simulation and these values are restrictions to be honored by the optimization model.

GARCÍÁ et al. [16] represent subsea layout design by a graph where vertices represent decisions and the arcs model relations between decisions. The main idea of their proposal is to decompose this graph into subgraphs, to obtain solutions for each subgraph, finally, to include the experience of the project team in recomposing a solution to the whole layout. The model parameters, in other words, data that are defined in advance, are location of drilling objectives in the reservoir, the number and capacity of manifolds and platforms and the seabed with obstacles, represented by a 2D grid. The proposed partition of problem involves four subgraphs: clustering of wells by the \( k \)-means algorithm; hierarchical clustering of wells in the case of using manifolds; on the results of the clusterings, placement of wellheads, platforms and
manifolds; and finally, the pipeline tracing according to dynamic programming to obtain the smallest routes. The project team has a reconciling role in design to promote feasible solutions.

WANG et al. [17] propose to model the subsea layout design as a covering problem. A partition of the set of wells into subsets is made by a heuristic, where each set is interconnected to a manifold. Only proximity between the wells are considered as parameters in partition. The number of manifolds and their capacity of connections are parameters, which must be known previously to run their model. The objective is the minimization of total interconnection distances between wells and manifolds, so as to minimize capex. This paper does not mention the location of the platform, modeling only manifold placement.

WANG et al. [18] resume their previous work comprising the installation of pipeline end manifold (PLEM) in subsea layout. PLEM can be considered as a simplified type of manifold with less interconnection capacity. This work addresses only the layout configuration in piggy back, in other words, wells connected in series sharing the same production line. In the optimization literature, it is the first approach considering this type of layout configuration. Layout optimization is concerned with minimizing a cost function, related to the distances of the interconnections. The model runs in two steps, initially the set of wells is partitioned into subsets, as previously described in WANG et al. [17], where each subset is associated to a manifold. The piggy back interconnections are drawn between manifolds. The second step evaluates whether there is a reduction of the total distance if PLEM is used. Then, an exhaustive method is adopted to verify if there is a reduction in total distance due to the installation of $n$ PLEMs in the project. The authors argue that the proposed model differs from the known minimum generating tree and traveling salesman models, being classified as a MINLP and, therefore, should have its solution by a heuristic.

It is important to highlight that almost all works that consider well allocation to platforms, in capex minimization, assume that the potential places to install platforms and the wellhead positions are known in advance.

These works extended the previous work of DEVINE and LESSO [6] by introducing a production decline curve for the whole field as the reservoir model.

FRAIR and DEVINE [19] made the first attempt to maximize the project NPV by considering reservoir dynamics in the subsea layout optimization. They extended the original work of DEVINE and LESSO [6] by introducing a production decline curve for the whole field as the reservoir model, and moreover by considering the scheduling of well drilling and the construction of platforms. This problem was represented as a MINLP whose objective was to maximize the NPV. The proposed solution is based on the algorithm of DEVINE and LESSO [6] for well location and
allocation to gathering centers. Influences of well pipe lengths on the total production was not taken into account, only the volume of the reservoir which is likely to be drained by the wells. Then, the problem was divided into two independent sub-problems. One is the location of platforms and allocation of drilling objectives to platforms. The other sub-problem is the well drilling schedule. In the two sub-problems, the reservoir decline curves define average oil rates over time. In the well drilling scheduling, the NPV maximization occurs by anticipating production according drilling prioritization of wells with the highest initial oil flow. FRAIR and DEVINE [19] suggest varying the number of platforms such as sensitivity analysis in problem solving.

The study of Iyer et al. IYER and GROSSMANN [20] is one of the most important studies in this category. They used piecewise linear approximations of reservoir pressure and GOR versus cumulative oil production and modeled several important decisions in their MILP model including: reservoir selection for drilling, installation planning, platform sizing, and production planning. They proposed a sequential decomposition algorithm to obtain an upper (lower) bound by aggregating (disaggregating) wells in each reservoir and time step. Although comprehensive, their model uses several simplifying assumptions such as linear pressure drop vs. flow relation for pipes, constant productivity index for each well throughout the planning horizon, non-interacting and independent wells, uniform fluid pressure and composition throughout the reservoir.

IYER and GROSSMANN [20] formulated the problem of subsea layout design considering non-linear behavior of the reservoir pressure as a function of cumulative production over time. However, they also consider that in each reservoir the wells have the same reservoir pressure. The work also models the scheduling of drilling and starting of platforms with their respective capabilities. There are only directional wells, whose wellheads are located in a template-manifold, a subsea structure to place a drilling rig in shallow waters, which can be considered as a manifold. Candidate sites for drilling wells, sites for placing manifold and places for installing platforms are known a priori. The project team should also know the productivity indexes of every well up to the last year of production. Pressure drop along pipelines is modeled by a simple approximated linear function. An important aspect was included in the model, the interference between wells when they share a same structure in flow to the platforms: pressure constraints are defined at the platforms. However, the restriction imposes an oil rate reduction to wells that arrive with greater pressure instead of choosing an optimal combination of wells to join the respective gathering centers. The formulation of the problem as a MILP with multiperiods leads to a large number of binary variables, due to which the authors classify the problem as intractable for real world applications. Then, a decomposition strategy is proposed to obtain an
upper (lower) bound by aggregating (disaggregating) wells in each reservoir and time step, without a guarantee of global optimum. Nevertheless, according to the authors, it is possible to significantly narrow the difference between the upper and lower search limits. The relaxation to obtain the upper limit involves the aggregation of time periods, the aggregation of wells from the same reservoir into a single well and the adoption of the convex closure in the piecewise-linear functions by parts. These linear functions are approximations of non linear curves for reservoir pressure as a function of cumulative production. Disaggregation is done in parts, solving problems individually as they were independent, generating a feasible solution to be compared with the upper limit defined in the previous step of the algorithm. It is a two step algorithm: a) the location of the platforms; and b) the allocation of the wells and solving simultaneously well drilling scheduling. This study is one of the most important of layout design and was followed by several researchers.

Based on IYER and GROSSMANN [20], VAN DEN HEEVER and GROSSMANN [21], DEN HEEVER et al. [22, 23] proposed a multiperiod generalized disjunctive programming model for oilfield infrastructure planning, for which they developed a bilevel decomposition method. They extended the work of IYER and GROSSMANN [20] in order to model to other design and planning decisions such as the number of platforms, inter-platform connections, platform capacities, investment time, production profiles, and gas compression for exportation. The model fitted an exponential function to describe reservoir pressure versus cumulative oil production and quadratic functions to model gas oil ratio (GOR), also as a function of cumulative oil production. Moreover, the authors consider royalties, tariffs and taxes that are usually neglected because of the added difficulty that they bring to the model solution. A heuristic based on Lagrange decomposition was applied to the problem thereof.

ASEERI et al. [24] introduced oil price and productivity indexes uncertainties in the deterministic model of IYER and GROSSMANN [20] leading to different solutions for layout design related to risks mitigation. This is the first work to consider risk and uncertainties modeling in subsea layout design optimization. Risk modeling was made by the Sample Average Approximation Method, which allowed a non prohibitive computational time to accomplish uncertainty evaluation.


CARVALHO and PINTO [27, 28] revisit the work of IYER and GROSSMANN
[20] by applying the model to a more realistic offshore scenario, but taking the same simplifying assumptions.

BARNES et al. [29] use decline curves to predict reservoir performance and approximate pressure drop in pipelines as a non-linear function. Optimization is carried out in two steps. The authors suggested an MILP followed by an MINLP to address the design and operation of oil and gas fields. Whereas the MILP makes the design decisions such as locations, capacities of platforms and drilling centers, the MINLP makes the well operation decisions such as the oil rates per time step.

ROSA [30] developed an exhaustive search model with the objective of maximizing the NPV by linking concepts of graph theory and pressure gradient in multiphase flow. The model does not consider manifolds and comprises a unique platform. Reservoir dynamics and well productivity index are modeled as given parameters obtained previously from the reservoir project team. ROSA and FERREIRA FILHO [31] extended this work as a MINLP problem by including the candidate places to install manifolds and their interconnections with wells as decision variables. The number of platforms and their location must be known in advance. Pressure drop along pipelines are modeled with quadratic functions, approximated from multiphase simulation. Reservoir dynamics and productivity indexes remain characterized by parameters, assumed to be known in advance.

TAVALLALI et al. [32] is the first work to include reservoir dynamics in a more complex way. This study is not properly a subsea design optimization, but a well position and drilling scheduling optimization model with the goal of maximizing NPV. The reservoir is modeled in a manner similar to simulators, by discretizing the reservoir in units where average values are assumed for many proprieties. The modeling of flow in porous media is made by partial differential equations, one for each phase (oil, gas, water). These equations correlate rates, fluid properties and rocks at space and time. An approximation by finite differences leads to simple ordinary differential equations, integrated in time. The well-reservoir coupling has the same model of the commercial reservoir simulators. The model also contemplates pressure drop in multiphase flow from bottomhole to surface installations. An empirical multiple nonlinear regression was adjusted to predict pressure drop by varying rate, water fraction, gas-oil ratio with outputs from a commercial simulator. Binary variables define whether a well is in a position in the reservoir or not. The candidate sites are not mentioned a priori, instead they are also variables, however there are technological and distance restrictions between wells and gathering centers that make some positions unfeasible. Thus, a large-scale MINLP model is obtained, which according to the authors, can not be solved with existing solvers. This is the challenge to consider a high fidelity reservoir model such offered by commercial simulators. Because the resulting MINLP could not be solved by commercial solvers,
the problem was split into a NLP primal and a MILP master problem, whereby
the latter is obtained by linearization of the nonlinear restrictions in the MINLP.
Although this work presents a reservoir model similar to simulators, it considers
only 2D reservoir models, which is not appropriated for real world applications.

TAVALLALI et al. [33] readdress their previous work TAVALLALI et al. [32]
by extending the model to the seabed installations project. In addition to defining
the location of the wells in the reservoir, it also defines the position and number of
manifolds and platforms. The candidate places for installing manifolds and plat-
forms and their capacities are known a priori. It is the same model of TAVALLALI
et al. [32], but adding binary variables related to the installation of manifolds and
platforms. There is no drilling schedule, all wells are opened at the same time at
the fist time step. If a well is shut in, it remains closed until the end of the planning
horizon. The pressure drop of multiphase flow along pipelines is also modeled like
in TAVALLALI et al. [32].

TAVALLALI and KARIMI [34] extended the work of TAVALLALI et al. [33] by
including drilling scheduling, possibility of reopening wells which were previously
closed and platform or manifold expansion in capacity to support field revitalization
or production increase. The solution approach remains the same.

2.2 Discussion

From the above literature review, it can be argued that most of the papers consider
the reservoir in a simple way, far from the actual conditions found in real oil fields.
These works do not account for fluid injection, secondary recovery and interactions
among wells. On the other hand, the approaches proposed by TAVALLALI et al.
[32, 33] and TAVALLALI and KARIMI [34] are closer to reality, but hard do apply
in practice due to their simple 2D reservoir models and the complexity of using such
approaches on a daily basis.

Reservoir and subsea infrastructure coupling is a research frontier in petroleum
engineering, being approximately by explicit methods in HOHENDORFF FILHO
[35], BENTO [36] and just recently by implicit methods in PATHAK et al. [37].
Optimization of integrated models is even a step beyond. It is far harder than
traditional optimization of waterflooding in reservoirs, comprising the complexity of
handling subsea infrastructure design.

In view of this complexity, our work brings about a practical and easily applied
approach for subsea layout optimization, which accounts for reservoir dynamics and
multiphase pressure drop. It is a hands-on approach to model reservoir behavior
along production time while considering the static pressure, water cut, gas-oil ratio,
and the productivity index of each well, besides secondary recovery mechanisms and
multiphase pressure drop in pipelines. Our model optimizes the layout by defining the connections between wells to subsea manifolds or platforms. Moreover, it chooses the pipeline diameters, among those under evaluation, that are optimal for the connection of each well to platform, well to manifold, and manifold to platform. Our optimization model is based on the works of SILVA and CAMPONOGARA [38] and CODAS et al. [39], which rely on multidimensional piecewise-linear approximation of pressure drops in pipelines, 0-1 decision variables and reservoir simulation to impose bounds for the proposed MILP model.
Chapter 3

Problem Presentation

In this Chapter the problem is stated and the proposed approach to solve it is described. At the end, it is pointed out the contribution of this thesis by solving the stated problem by the proposed model.

3.1 Problem Statement, Model Premises and Limitations

Let us consider a reservoir at the early stages of development, when the design alternatives are being studied. To the proposed model, there must be given the reservoir model for simulation, well objectives, well trajectories, wellhead positions and a defined secondary recovery method that can be input to the reservoir simulation model.

The design model determines the number of manifolds to be installed, pipeline connections between wells and manifolds, wells and platforms, manifolds and platforms and, finally the diameter of the pipeline in every connection. Options for diameters to be studied must be known in advance, generally taken from flowline supplier catalogs.

Further, possible locations for installing manifolds are previously defined by the project team, as well as pipeline trajectories and their connections to wells, manifolds and platforms. It is a simplification from the work proposed by FAMPA [9]. Candidate locations to place manifolds are planned in advance. All possible flowline trajectories are previously traced. As said before, options of flowline diameters to be evaluated, usually the commercial ones, are defined beforehand. Thus, the project team enumerates options for trajectories, connections and diameters. The optimization model will answer which combination maximizes NPV.

Constraints on manifold and platform capacities can be included, along with bounds on the minimum and maximum number of well connections to manifolds.
Upwards, manifold and platform capacities should be known in advance.

The objective is to maximize the net present value of the project, considering the production curve and related capex for installing the subsea infrastructure according with an optimal layout design.

From the perspective of a production petroleum engineer, it is possible to enumerate the layout configurations that bring the highest and lowest cumulative production during the field lifetime. Among the many possible link combinations of wells to manifolds, wells to platforms and manifolds to platforms, it is expected that, along years of production, the combinations with large diameters will lead to high cumulative production, low well bottomhole pressure and low reservoir pressure. Instead, small diameters will lead to reduced cumulative production, high well bottomhole pressure, and high reservoir pressure. Hence, by applying these extreme layouts to a numeric reservoir simulator, boundaries for reservoir pressure and well bottomhole pressure are identified, allowing to plot their maximum and minimum curves over time. Thus, to any other possible layout configuration, well bottomhole and reservoir pressures will lie between these two boundaries, which define an envelope of possible solutions. Following this perspective, drawdown envelopes for each well can also be found over time.

If there is heterogeneity in reservoir pressure distribution, the multi tank approach can be employed [40–45]. This is not a full application of material balance model, rather, only the partition of the reservoir in regions of similar pressure behavior. Then, along time, for each partition this work adopts a single pressure calculated by average of all cells which belong to the partition.

Thus, all wells that are located at a given region share same reservoir pressure [46].

Since in this proposed model there are pressure envelopes for reservoir pressure, nothing prevents it from being defined individually per well. In consequence, instead of reservoir or tank pressure, it is possible to consider each well static pressure or well surrounding pressure in the reservoir model. The same way that envelopes for reservoir or tank pressure, an envelope for static pressure will be implemented for each well.

The water cut for every well is the average of each water cut calculated from outputs of $n$ reservoir simulations of layouts that lead to high and low cumulative production. The same is adopted for the productivity index and gas-oil ratio. The productivity index can be calculated with tank pressure or static pressure, depending on which one is used. If any of these parameters presents great differences among the outputs from simulation of those layouts, the proposed methodology can not be applied properly.

The pressure drop from wells to manifolds, from wells to platforms, and from
manifolds to platforms are modeled by multiphase flow simulation. Pressure drop are simulated in advance, and then given to the reservoir simulator in the form of tables like vertical lift tables.

For a better understanding of this proposal, suppose that during the optimization process a given layout is being evaluated, according to which all wells are linked to manifolds by a pipeline with the smallest diameter. As an example, let us consider a single well for which we assume some values for variables and parameters. For a given time step, at an iteration in the optimization process, let us consider that:

- bottomhole pressure boundaries are $190 \text{ kgf/cm}^2$ and $210 \text{ kgf/cm}^2$, being the well bottomhole pressure equal to $200 \text{ kgf/cm}^2$;
- tank pressure boundaries are $210 \text{ kgf/cm}^2$ and $240 \text{ kgf/cm}^2$, being the tank pressure equal to $230 \text{ kgf/cm}^2$;
- well drawdown boundaries are $10 \text{ kgf/cm}^2$ and $20 \text{ kgf/cm}^2$, being the drawdown equal to $10 \text{ kgf/cm}^2$;
- productivity index for the well is $50 \text{ (m}^3/\text{d})/(\text{kgf/cm}^2)$;
- then, from drawdown and productivity index, well liquid rate is $500 \text{ m}^3/\text{d} (10 \times 50)$.

If the optimization process changes the layout by choosing larger diameters for all pipelines, then it is expected for well that:

a) with a large diameter pressure drop in pipelines can decrease, which allows a higher production rate;

b) the bottomhole pressure can also decrease so as to raise production;

b) bottomhole pressure decreases constrained by bottomhole pressure boundaries; suppose that the optimization solver evaluates if the well bottomhole pressure can decrease to $190 \text{ kgf/cm}^2$ reaching the lower boundary for this variable;

c) the optimization solver will take the production rate to a higher value calculated by productivity index and drawdown; then, suppose that the solver takes drawdown to the upper boundary of $20 \text{ kgf/cm}^2$ to maximize production, leading the liquid rate to $1000 \text{ m}^3/\text{d}$ due to a productivity index of $50 \text{ (m}^3/\text{d})/(\text{kgf/cm}^2)$.

d) the optimization solver calculates the required pressure for the well to produce $1000 \text{ m}^3/\text{d}$ using the piecewise-linear approximation of pressure drop; and the value of $190 \text{ kgf/cm}^2$ for bottomhole pressure is feasible;
d) to define 190 $kgf/cm^2$ as bottomhole pressure and honor the drawdown upper boundary of 20 $kgf/cm^2$, the tank pressure must decline from 230 $kgf/cm^2$ to 210 $kgf/cm^2$, reaching its lower bound. Then, this is the mechanism that promotes pressure updates in reservoir, based on those bounds obtained from reservoir simulation of layouts that lead to high and low cumulative production.

Instead of 210 $kgf/cm^2$ in the above example, if the tank pressure lower bound was 220 $kgf/cm^2$ the drawdown must be lower than 20 $kgf/cm^2$, leading to a liquid rate lower than 1000 $m^3/d$.

Still on the example, if there was no match between the liquid rate 1000 $m^3/d$ and the required pressure of 190 $kgf/cm^2$ to flow this rate in the piecewise-linear approximation of pressure drop, another combination of liquid rate and bottomhole pressure have to be evaluated.

In other words, by varying layout options, the optimization process chooses the bottomhole pressure that maximizes the net present value of the project.

For higher production rates, the reservoir pressure tends to reach low values; whereas for lower production rates, greater values. Over time, a reservoir pressure curve is an output of the optimization model, constrained by the boundaries obtained previously by reservoir simulation of the extreme layout configurations, which induce the highest and the lowest production. The well bottomhole pressure curve and drawdown are also outputs of the same optimization model, limited by the given boundaries curves.

In addition, there is no need to have a unique curve of pressure decay as a function of cumulative production, as many works in literature do. It is known that, in real oilfield applications, not one but multiple pressure-decay curves are possible, which vary depending on the layout imposed to the reservoir. Instead of reservoir or tank pressure, the proposed approach also allows to account for the static pressure of each individual well, since there is an envelope of possible values for every time step. This is the mechanism proposed to model reservoir dynamics in this study.

Clearly, any secondary recovery mechanism can be represented in our approach because it can be embedded in the numeric simulation, which in turn defines boundaries for the optimization model.

Reservoir management policies are also represented because they are imposed to simulations with layouts that impose for optimization. As an example, if a pressure control policy is implemented in reservoir simulation to avoid undersaturation, it will be reflected in each well by their downhole pressure boundaries.

Artificial lift is not optimized, but rather assumed as a fixed gas-lift injection rate or pump frequency over the production time.
A scheduling of starting production for each well must be given. The model does not comprise scheduling optimization of drilling or production.

Experienced engineers are quick to point out that oilfield design carries a lot of uncertainty in models and specially in data, particularly during the first stages of development. This inherent uncertainty makes it hard to adopt a definitive solution at the beginning of development, just at the time when several key decisions should be taken. In view of these limitations, this work does not aim to give a definitive solution for subsea infrastructure layout design. Instead, it is a practical proposal of foreseen tendencies of the production curve and NPV behavior as a function of the layout to be adopted. More detailed studies should be carried out afterwards.

### 3.2 Thesis Contribution

From the standpoint of production engineering, this work proposes a new and practical approach to the challenging engineering problem of subsea layout design. Differently from other works in the related literature, this thesis brings a practical and simple proposal to optimize subsea layout taking reservoir behavior into account. Possibilities of pipeline routing, diameter definition, manifold placement, and well allocation to gathering systems — manifolds or platforms — are designed by the proposed model in order to maximize NPV, thereby considering the reservoir. This work innovates in subsea layout design by:

- Modeling pressure drop in a realistic way with multidimensional piecewise-linear approximation, like reservoir simulators which model multiphase flow based on lift tables.

- Considering as decision variables not only possibilities of pipeline routing, manifold placement, and well allocation to gathering systems, but also pipeline diameter definition.

- Introducing a simple but practical reservoir model that captures its behavior in different subsea layouts.

- Enabling a fast computational tool to carry out case studies and perform sensitivity analysis of manifold and platform capacities.
Chapter 4

Model

The optimization of subsea layout design is a mixed-integer nonlinear program (MINLP) of considerable computational hardness. The installation of a subsea manifold or platform and the choice of a pipeline diameter among those available commercially defines the problem as noncontinuous. Moreover, pressure drop of multiphase flow in pipelines is highly nonconvex. Then, to combine noncontinuous and nonconvex problems lead to a real challenge to global optimization. An efficient way of solving the optimization of subsea layout design is the conversion of the MINLP to a mixed-integer linear program (MILP) by piecewise-linearizing the pressure drop along pipes.

The sections below details the MILP mathematical model, coupling reservoir, pressure drop and decision variables.

4.1 Sets, indexes, parameters and variables

Sets and indexes

- \( T = \{1, 2, \ldots, t\} \) is the set of time intervals;
- \( W = \{1, \ldots, w\} \) is the set of wells;
- \( \mathcal{M}_{\text{man}} = \{1, \ldots, M_{\text{man}}\} \) is the set of manifolds;
- \( \mathcal{M}_{\text{plat}} = \{1, \ldots, M_{\text{plat}}\} \) is the set of production platforms;
- \( \mathcal{M} = \mathcal{M}_{\text{man}} \cup \mathcal{M}_{\text{plat}} \) is the set of gathering centers;
- \( \mathcal{D}_{m,p} = \{1, 2, \ldots, D_{m,p}\} \) is a set of diameters for the flowline connecting manifold \( m \) to platform \( p \);
- \( \mathcal{D}_{w,m} = \{1, 2, \ldots, D_{w,m}\} \) is a set of configurations for well \( w \) accounting for flowline diameter connecting the well to gathering system \( m \), which can be a manifold or a platform;
\(\mathcal{N} = \{1, 2, \ldots, n\}\) is the set of reservoir simulations of layouts that lead to high and low cumulative production, in order to define boundaries for bottomhole pressure, reservoir (or tank) pressure and drawdown;

\(\mathcal{T}_q = \{1, 2, \ldots, tq\}\) is the set of reservoir tanks (compartments);

\(\mathcal{W}^{tq} \subseteq \mathcal{W}\) is the subset of wells that belong to the reservoir tank \(tq\);

\(\mathcal{H} = \{\text{oil, gas, water}\}\) is the set of phases oil, gas and water;

\(\mathcal{H}^* \subseteq \mathcal{H} = \{\text{oil, water}\}\) is the subset of phases without the phase gas;

\(\mathcal{Q}_{\text{liq}}^d = \{\mathcal{Q}_{\text{liq}}_{w,m,1}^d, \mathcal{Q}_{\text{liq}}_{w,m,2}^d, \ldots\}\) is the set of breakpoints for liquid rate from wells to gathering centers;

\(\mathcal{GOR}^d_{w,m} = \{\mathcal{GOR}^d_{w,m,1}, \mathcal{GOR}^d_{w,m,2}, \ldots\}\) is the set of breakpoints for gas oil ratio from wells to gathering centers;

\(\mathcal{BSW}^d_{w,m} = \{\mathcal{BSW}^d_{w,m,1}, \mathcal{BSW}^d_{w,m,2}, \ldots\}\) is the set of breakpoints for water fraction at liquid rate from wells to gathering centers;

\(\mathcal{M}_m = \{\mathcal{M}_m,1, \mathcal{M}_m,2, \ldots,\}\) is the set of breakpoints for pressure at gathering centers;

\(\mathcal{Q}_{\text{liq}}^d_{m,p} = \{\mathcal{Q}_{\text{liq}}^d_{m,p,1}, \mathcal{Q}_{\text{liq}}^d_{m,p,2}, \ldots\}\) is the set of breakpoints for liquid rate from manifolds to platforms;

\(\mathcal{GLR}^d_{m,p} = \{\mathcal{GLR}^d_{m,p,1}, \mathcal{GLR}^d_{m,p,2}, \ldots\}\) is the set of breakpoints for gas liquid ratio from manifolds to platforms;

\(\mathcal{BSW}^d_{m,p} = \{\mathcal{BSW}^d_{m,p,1}, \mathcal{BSW}^d_{m,p,2}, \ldots\}\) is the set of breakpoints for water fraction at liquid rate from manifolds to platforms;

\(\mathcal{Pr}_p = \{\mathcal{Pr}_p,1, \mathcal{Pr}_p,2, \ldots\}\) is the set of breakpoints for pressure at platforms.

**Parameters**

- \(\text{bbl}\) is the profit of an oil barrel after operational costs and taxes;
- \(\tau\) is the discount factor;
- \(\Delta t\) is the time length of time interval \(t\);
- \(\text{CLW}^d_{w,m}\) is the cost of a subsea line from well \(w\) to manifold or platform \(m \in \mathcal{M}\) with diameter \(d\);
- $CLM_{m,p}^d$ is the cost of a subsea line between manifold $m$ and platform $p$ with diameter $d$;
- $CM_m$ is the cost of a subsea manifold or platform $m \in \mathcal{M}$;
- $PI_{w,t}$ is the productivity index for well $w$ at time $t$;
- $\overline{PI}_{w,t}$ is the average of PI for well $w$ at time $t$ calculated from outputs of $n \in \mathcal{N}$ reservoir simulations of layouts that lead to high and low cumulative production, in order to define boundaries for bottomhole pressure, tank pressure and drawdown;
- $GOR_{w,t}$ is the gas-oil ratio for well $w$ at time $t$;
- $\overline{GOR}_{w,t}$ is the average of GOR for well $w$ at time $t$ calculated from outputs of $n \in \mathcal{N}$ reservoir simulations of layouts that lead to high and low cumulative production, in order to define boundaries for bottomhole pressure, tank pressure and drawdown;
- $BSW_{w,t}$ is the water fraction for well $w$ at time $t$;
- $\overline{BSW}_{w,t}$ is the average of BSW for well $w$ at time $t$ calculated from outputs of $n \in \mathcal{N}$ reservoir simulations of layouts that lead to high and low cumulative production, in order to define boundaries for bottomhole pressure, tank pressure and drawdown;
- $q_{w,\text{max}}^h$ is the upper bound for phase $h \in \mathcal{H}$ for well $w$;
- $q_{m,\text{max}}^h$ is the upper bound for gathering center $m \in \mathcal{M}$;
- $P_{tq,\text{lo}}^t$ is the lower bound for the pressure of tank $tq$ at time $t$, obtained from reservoir simulation of layout that lead to high cumulative production;
- $P_{tq,\text{hi}}^t$ is the upper bound for the pressure of tank $tq$ at time $t$, obtained from reservoir simulation of layout that lead to low cumulative production;
- $D_{w,t}^{\text{lo}}$ is the lower bound for drawdown of well $w$ at time $t$;
- $D_{w,t}^{\text{hi}}$ is the upper bound for drawdown of well $w$ at time $t$;
- $p_{w,\text{hi}}^t$ is the bottomhole pressure lower bound of well $w$ at time $t$, obtained from reservoir simulation of layout that lead to high cumulative production;
- $p_{w,\text{lo}}^t$ is the bottomhole pressure upper bound of well $w$ at time $t$, obtained from reservoir simulation of layout that lead to low cumulative production;
- $\text{bigM}_w$ is a bigM constant for well $w$;
• bigM_m is a bigM constant for gathering center m ∈ M;
• \( q^{GLC}_{w,t} \) is the gas-lift injection rate into well w during period t;
• \( \Delta P_{w,m}^{d,(q,g,b,p)} \) is the matrix of pressure drop from well w to the gathering center m ∈ M obtained from multiphase flow simulation by varying diameter, liquid rate, gas fraction, water fraction and pressure at gathering center;
• \( q \) is representation of a element from the set of liquid rates \( Q_{\text{liq}}^{d,w,m} \) from well w to gathering center m ∈ M or from the set of liquid rates from manifold \( M_{\text{man}} \) to platform p, \( Q_{\text{liq}}^{d,m,p} \);
• \( b \) is representation of a element from the set of water fraction \( BSW_{w,m}^{d} \) from well w to gathering center m ∈ M or from the set of water fraction from manifold \( M_{\text{man}} \) to platform p, \( BSW_{m,p}^{d} \);
• \( g \) is representation of a element from the set of gas ratio \( GOR_{w,m}^{d} \) from well w to gathering center m ∈ M or from the set of gas ratio from manifold \( M_{\text{man}} \) to platform p, \( GOR_{m,p}^{d} \);
• \( p \) is representation of a element from the set of pressure at gathering center \( MP_{m} \) from well w to gathering center m ∈ M;
• \( pr \) is representation of a element from the set of pressure at platform \( Pr_{p} \) from manifold \( M_{\text{man}} \) to platform p;
• \( \Delta P_{m,p}^{d,(q,g,b,pr)} \) is the matrix of pressure drop from manifold \( M_{\text{man}} \) to platform p obtained from multiphase flow simulation by varying diameter, liquid rate, gas fraction, water fraction and pressure at platform;
• \( x_{\max}^{m} \) and \( x_{\min}^{m} \) are respectively the maximum and minimum number of well connections at a gathering center m ∈ M.

Binary Variables
• \( x_{m} \in \{0,1\} \) is a binary variable that indicates whether a gathering system m ∈ M, which can be a manifold or a platform, is installed or not;
• \( x_{w,m} \in \{0,1\} \) indicates whether or not well w is connected to manifold or platform m;
• \( x_{w,m}^{d} \in \{0,1\} \) indicates whether or not well w is connected to manifold or platform m with diameter d;
• \( x_{m,p} \in \{0,1\} \) assumes value 1 if manifold m is installed and connected to platform p;
\[ x_{m,p}^d \in \{0,1\} \] assumes value 1 if manifold \( m \) is installed and connected to platform \( p \) with a flowline of diameter \( d \);

\[ z_{w,t} \in \{0,1\} \] assumes value 1 if well \( w \) is producing during period \( t \) and 0 otherwise;

\[ x_{w,m,t} \in \{0,1\} \] assumes value 1 if well \( w \) is linked to manifold or platform \( m \) and producing during period \( t \);

\[ y_{m,t} \in \{0,1\} \] assumes value 1 if there is flow to manifold \( m \in \mathcal{M}_{\text{man}} \) during period \( t \).

**Continuous Variables**

- \( q_{w,t}^h \) is the flow of phase \( h \) from well \( w \in \mathcal{W} \) at time \( t \in \mathcal{T} \);
- \( q_{p,t}^h \) is the flow of phase \( h \) from platform \( p \) at time \( t \);
- \( p_{w,t} \) is the bottomhole pressure at well \( w \) at time \( t \);
- \( P_{w,t}^R \) is the reservoir pressure at well \( w \) at time \( t \);
- \( NP_{t}^{tq} \) is the cumulative production for tank \( tq \in \mathcal{T}_q \) at time \( t \);
- \( P_{t}^{tq} \) is the pressure for tank \( tq \) at time \( t \);
- \( q_{w,m,t}^h \) is the flow of phase \( h \) from well \( w \) to a gathering system \( m \in \mathcal{M} \) at time \( t \);
- \( q_{w,m,t}^{d,h} \) is the flow of phase \( h \) from well \( w \) to a gathering system \( m \in \mathcal{M} \) at time \( t \) though a pipe of diameter \( d \);
- \( q_{m,p,t}^h \) is the flow of phase \( h \) from manifold \( m \) to a platform \( p \) at time \( t \);
- \( q_{m,p,t}^{d,h} \) is the flow of phase \( h \) from manifold \( m \) to a platform \( p \) at time \( t \) though a pipe of diameter \( d \);
- \( \Delta p_{w,m,t}^d \) is the pressure drop from bottomhole of well \( w \) to gathering center \( m \in \mathcal{M} \), at time \( t \), through a pipe with diameter \( d \);
- \( \Delta p_{w,m,t} \) is the pressure drop from bottomhole of well \( w \) to gathering center \( m \in \mathcal{M} \), at time \( t \);
- \( q_{w,m,t}^{d(q,g,h,p)} \) is the convex combination among breakpoints of liquid rate, gas oil ratio, water fraction and pressure at gathering center \( m \), for a diameter \( d \), at time \( t \).
• $\eta_{d,q}^{w,m,t}$, $\eta_{d,g}^{w,m,t}$, $\eta_{d,b}^{w,m,t}$, $\eta_{d,p}^{w,m,t}$ are variables which form a SOS2 set (special ordered set of type two) for liquid, gas, water and pressure respectively. They are stated for flow between well $w$ to a gathering system $m \in \mathcal{M}$ at time $t$, for a diameter $d$;

• $p_{d,m,t}$ is the pressure at the end of pipe which connects well $w$ to a gathering center $m \in \mathcal{M}$;

• $p_{m,t}$ is the pressure at the gathering center $m \in \mathcal{M}$;

• $\Delta p_{d,m,p,t}$ is the pressure drop from manifold manifold $m \in \mathcal{M}_{\text{man}}$ to a platform $p$, at time $t$, through a pipe with diameter $d$;

• $\Delta p_{m,p,t}$ is the pressure drop from manifold manifold $m \in \mathcal{M}_{\text{man}}$ to a platform $p$, at time $t$;

• $\phi_{d,q}^{m,p,t}$, $\phi_{d,g}^{m,p,t}$, $\phi_{d,b}^{m,p,t}$, $\phi_{d,pr}^{m,p,t}$ are variables which form a SOS2 set (special ordered set of type two) for liquid, gas, water and pressure respectively. They are stated for flow between manifold $m$ to a platform $p$ at time $t$, for a diameter $d$;

• $p_{p,t}$ is the pressure at platform $p$ at time $t$;

• $q_{h,m,t}$ is the flow of phase $h$ at a gathering system $m \in \mathcal{M}$ at time $t$;

4.2 Mathematical Programming Model

The model for layout design optimizes NPV:

$$\max f = \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} q_{w,t}^{\text{oil}} \Delta t (1 + \tau) \text{bbl} - \sum_{w \in \mathcal{W}} \sum_{m \in \mathcal{M}} \sum_{d \in \mathcal{D}_{w,m}} x_{w,m}^{d} CLW_{w,m}^{d} - \sum_{m \in \mathcal{M}} x_{m} CM_{m}$$

$$- \sum_{m \in \mathcal{M}_{\text{man}}} \sum_{p \in \mathcal{M}_{\text{plat}}} \sum_{d \in \mathcal{D}_{m,p}} x_{m,p}^{d} CLM_{m,p}^{d} \quad (4.1)$$

Each well $w \in \mathcal{W}$ must be connected to exactly one gathering system (manifold
or platform) and a given pipeline diameter:

\[
x_{w,m} = \sum_{d \in D_{w,m}} x_{w,m}^d, \forall m \in \mathcal{M},
\]

(4.2)

\[
\sum_{m \in \mathcal{M}} x_{w,m} = 1,
\]

(4.3)

\[
x_{w,m} \leq x_m, \forall m \in \mathcal{M},
\]

(4.4)

with \(x_{w,m} \in \{0,1\}\) being a binary variable that indicates whether or not well \(w\) is connected to a gathering system \(m\).

A manifold \(m\), if installed, must be connected to a platform which has been installed. Therefore, for all \(m \in \mathcal{M}_{\text{man}}\):

\[
x_{m,p} = \sum_{d \in D_{m,p}} x_{m,p}^d, \forall p \in \mathcal{M}_{\text{plat}}.
\]

(4.5)

\[
x_m = \sum_{p \in \mathcal{M}_{\text{plat}}} x_{m,p},
\]

(4.6)

\[
x_{m,p} \leq x_p, \forall p \in \mathcal{M}_{\text{plat}}.
\]

(4.7)

There must be at least one well connected to a given manifold:

\[
x_m \leq \sum_{w \in \mathcal{W}} x_{w,m}, \forall m \in \mathcal{M}_{\text{man}}.
\]

(4.8)

Time is discretized in intervals given by the set \(\mathcal{T} = \{1, 2, \ldots, t\}\).

Well production is given in terms of their productivity index, reservoir and bottom hole pressure for all \(w \in \mathcal{W}\) and \(t \in \mathcal{T}\) as follows:

\[
PI_{w,t}(P_{w,t}^R - p_{w,t}) - q_{w,t}^{\text{liq,max}}(1 - z_{w,t}) \leq q_{w,t}^{\text{liq}}
\]

\[
\leq PI_{w,t}(P_{w,t}^R - p_{w,t}) + q_{w,t}^{\text{liq,max}}(1 - z_{w,t}),
\]

(4.9a)

\[
q_{\text{oil},t} = q_{w,t}^{\text{liq}}(1 - BSW_{w,t}),
\]

(4.9b)

\[
q_{\text{gas},t} = q_{\text{oil},t} \cdot GOR_{w,t},
\]

(4.9c)

\[
q_{\text{water},t} = q_{w,t}^{\text{liq}} \cdot BSW_{w,t},
\]

(4.9d)

\[
BSW_{w,t} = \overline{BSW^n_{w,t}},
\]

(4.9e)

\[
GOR_{w,t} = \overline{GOR^n_{w,t}},
\]

(4.9f)

\[
PI_{w,t} = \overline{PI^n_{w,t}}.
\]

(4.9g)

in which \(GOR_{w,t}\) is the gas-oil ratio, \(\overline{GOR^n_{w,t}}\) is the average of GOR calculated from outputs of \(n\) reservoir simulations of layouts that lead to high and low cumulative
production, in order to define boundaries for bottomhole pressure, tank pressure and
drawdown. $BSW_{w,t}$ is the water fraction, $\bar{BSW}_{w,t}$ is the average of BSW calculated
from outputs of $n$ previous reservoir simulations, as made for GOR. $PI_{w,t}$ is the
productivity index, $\bar{PI}_{w,t}$ is the average of PI calculated from outputs of $n$
previous reservoir simulations, as made for GOR and BSW. $P_{w,t}^R$ is the reservoir pressure
during period $t$, $q_{w,t}^{\text{liq,max}}$ is the bound for liquid production, and $z_{w,t} \in \{0, 1\}$ is a
binary variable which assumes value 1 if well $w$ is producing during period $t$ and 0
otherwise.

A well $w$ is shut if and only if nodal analysis does not have an operational point,
remaining shut thereafter.

Then, when $z_{w,t} \in \{0, 1\}$ assumes value 1, the well liquid rate of a well is cal-
culated as the difference between reservoir and bottomhole pressure, multiplied by
productivity index.

Thus, for each $w \in W$:

$$z_{w,t} \geq z_{w,t+1}, t \in \mathcal{T} \setminus \{T\}. \quad (4.10)$$

The reservoir pressure $P_{w,t}^R$ can reference tank pressure or static pressure, that is
the average pressure of well surroundings in reservoir. Wells that are placed at the
same tank have the same reservoir pressure [46].

Let $\mathcal{T}_q$ be the set of reservoir tanks. In order to have cumulative production $NP$
accounted by each tank, for all $t \in \mathcal{T}$, we define $NP_t^{\text{jo}}$ as the cumulative production
for tank $tq \in \mathcal{T}_q$ at time $t \in \mathcal{T}$. For a tank $tq$, the cumulative oil production is given by:

$$NP_t^{\text{jo}} = NP_{t-1}^{\text{jo}} + \sum_{w \in \mathcal{W}_q} q_{w,t}^{\text{oil}} \Delta t, \forall t \in \mathcal{T}, \quad (4.11a)$$

$$NP_0^{\text{jo}} = 0, \quad (4.11b)$$
in which $\mathcal{W}_q \subseteq W$ is the subset of wells that belong to the reservoir tank $tq$. Tank
pressure, and its relation to downhole pressure of its wells, are governed by the
following equations, for all $tq \in \mathcal{T}_q$, $w \in \mathcal{W}_q$, and $t \in \mathcal{T}$:

$$P_{t}^{\text{q,hi}} \leq P_{t}^{\text{q}} \leq P_{t}^{\text{q,lo}}, \quad (4.12a)$$

$$p_{w,t}^{\text{hi}} - (1 - z_{w,t})\text{bigM}_w \leq p_{w,t} \leq p_{w,t}^{\text{lo}} + (1 - z_{w,t})\text{bigM}_w, \quad (4.12b)$$

$$D_{w,t}^{\text{lo}} - (1 - z_{w,t})\text{bigM}_w \leq P_{w,t}^R - p_{w,t} \leq D_{w,t}^{\text{hi}} + (1 - z_{w,t})\text{bigM}_w, \quad (4.12c)$$
in which:

- $P_t^{\text{q}}$ is the pressure of tank $tq$ at time $t$,
- $P_{tq,lo}$ and $P_{tq,hi}$ are the lower and upper bound for the pressure of tank $tq$ respectively.
- $D_{w,t}^{lo}$ and $D_{w,t}^{hi}$ are the lower and respectively upper bound for the drawdown of well $w$ at time $t$.

All of these bounds are obtained from reservoir simulation, as stated before.

In order to enforce the tank pressure as the reservoir pressure to the wells, for each $tq \in T_q$ and $w \in W^{tq}$, the following equation is needed for all time $t \in T$:

$$P^{R}_{w,t} = P_{tq}^t.$$  \hfill (4.13)

Flowrate from each well $w \in W$ must be assigned to a gathering system $m \in M$, subsea or topside, using a pipeline of diameter $d \in D_{w,m}$, for all time $t \in T$ and phase $h \in H$:

$$0 \leq q_{w,m,t}^{d,h} \leq q_{w}^{h,max} x_{w,m},$$  \hfill (4.14a)

$$q_{w,t}^{h} - q_{w}^{h,max} (1 - x_{w,m}^d) \leq q_{w,m,t}^{d,h} \leq q_{w,t}^{h} + q_{w}^{h,max} (1 - x_{w,m}^d),$$  \hfill (4.14b)

where $H = \{\text{oil, gas, water}\}$ is the set of phases, $H^* = \{\text{oil, water}\}$, and $q_{w}^{h,max}$ is a sufficiently large constant to implement the Big-M strategy.

Except for the gas phase, for which the gas-lift flowrate must be taken into account, the inlet flowrate into manifold $m \in M_{man}$ at time $t \in T$ is the sum of the flowrates from the wells that are connected to it:

$$q_{m,t}^{h} = \sum_{w \in W} q_{w,m,t}^{h}, \quad \forall h \in H^*,$$  \hfill (4.15a)

$$q_{m,t}^{gas} = \sum_{w \in W} x_{w,m,t} q_{w,t}^{GLC} + \sum_{w \in W} q_{w,m,t}^{gas},$$  \hfill (4.15b)

in which $q_{w,t}^{GLC}$ is the gas-lift injection rate into well $w$ during period $t$. Due to a particularity of the multiphase flow simulator, the gas-lift rate must be added to the total gas rate handled by the manifold. Generally, lift tables consider only the gas-liquid ratio for pipeline models. For well representation, lift tables must consider not only the gas-liquid ratio but also gas-lift rate or pump frequency.

Logic conditions on well assignment to manifolds and platforms, in relation to whether the well is producing or not, are imposed by the following constraints for
all \( w \in \mathcal{W}, m \in \mathcal{M}, \) and \( t \in \mathcal{T} \):

\[
x_{w,m,t} \leq x_{w,m}, \quad (4.15c)
\]

\[
x_{w,m,t} \leq z_{w,t}, \quad (4.15d)
\]

\[
x_{w,m,t} \geq x_{w,m} + z_{w,t} - 1, \quad (4.15e)
\]

\[
x_{w,m,t} \in \{0,1\}, \quad (4.15f)
\]

and for all \( m \in \mathcal{M}_{\text{man}} \) and \( t \in \mathcal{T} \):

\[
\sum_{w \in \mathcal{W}} x_{w,m,t} \geq y_{m,t}, \quad (4.15g)
\]

in which:

- \( x_{w,m,t} \in \{0,1\} \) is a binary value that assumes value 1 if well \( w \) is linked to manifold or platform \( m \) and producing during period \( t \), and

- \( y_{m,t} \in \{0,1\} \) is also a binary variable that assumes value 1 if there is flow to manifold \( m \in \mathcal{M}_{\text{man}} \) during period \( t \).

The flowrate received by a platform \( p \in \mathcal{M}_{\text{plat}} \) at time \( t \in \mathcal{T} \) is the sum of rates from the linked satellite wells and manifolds:

\[
q_{p,t} = \sum_{w \in \mathcal{W}} q_{w,p,t}^\text{liq} + \sum_{m \in \mathcal{M}_{\text{man}}} q_{m,p,t}^h, \forall h \in \mathcal{H}^*, \quad (4.16a)
\]

\[
q_{p,t}^\text{gas} = \sum_{w \in \mathcal{W}} x_{w,p,t} q_{w,p,t}^{\text{GLC}} + \sum_{w \in \mathcal{W}} q_{w,p,t}^\text{gas} + \sum_{m \in \mathcal{M}_{\text{man}}} q_{m,p,t}^\text{gas}. \quad (4.16b)
\]

The pressure drop from a well \( w \in \mathcal{W} \) to a gathering system \( m \in \mathcal{M} \), in a pipeline of diameter \( d \in \mathcal{D}_{w,m} \), is modeled as a piecewise-linear function given in terms of breakpoints on liquid flowrate, GOR, water cut, and manifold or platform pressure, respectively:

- \( \mathcal{Q}_{\text{liq}}^d = \{ \mathcal{Q}_{\text{liq}}^d_{w,m,1}, \mathcal{Q}_{\text{liq}}^d_{w,m,2}, \ldots \} \)

- \( \mathcal{Q}_{\text{GOR}}^d = \{ \mathcal{Q}_{\text{GOR}}^d_{w,m,1}, \mathcal{Q}_{\text{GOR}}^d_{w,m,2}, \ldots \} \)

- \( \mathcal{Q}_{\text{BSW}}^d = \{ \mathcal{Q}_{\text{BSW}}^d_{w,m,1}, \mathcal{Q}_{\text{BSW}}^d_{w,m,2}, \ldots \} \)

- \( \mathcal{M}_{\text{MP}} = \{ \mathcal{M}_{\text{MP}}_{m,1}, \mathcal{M}_{\text{MP}}_{m,2}, \ldots \} \).

These breakpoints are obtained from multiphase flow simulation of pressure drop by varying liquid rate, GOR, water cut and manifold or platform pressure. For each combination among these parameters there is a correspondent pressure drop, calculated by a multiphase flow simulator.
Given these breakpoints, the pressure drop is approximated with a piecewise-linear function for all \( w \in W, m \in M, d \in D_{w,m}, \) and \( t \in T: \)

\[
\Delta p_{w,m,t}^d = \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \Delta P_{w,m}^{d,(q,g,b,p)} \cdot \delta_{w,m,t}^{d,(q,g,b,p)} - p_{w,m,t}^d
\]

\[
q_{w,m,t}^d = \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)} \cdot q \cdot (1 - b),
\]

\[
q_{w,m,t}^{d,g} = \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)} \cdot q \cdot (1 - b) \cdot g,
\]

\[
q_{w,m,t}^{d,\text{water}} = \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)} \cdot q \cdot b,
\]

\[
p_{w,m,t}^d = \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)} \cdot p,
\]

\[
x_{w,m,t}^d = \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)},
\]

\[
z_{w,t} \geq \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)},
\]

\[
x_{w,m,t} + z_{w,t} - 1 \leq \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)},
\]

\[
\gamma_{w,m,t}^{d,q} = \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)} \cdot \forall q \in \text{Qliq}^d_{w,m},
\]

\[
\gamma_{w,m,t}^{d,g} = \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)} \cdot \forall g \in \text{GOR}^d_{w,m},
\]

\[
\gamma_{w,m,t}^{d,b} = \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)} \cdot \forall b \in \text{BSW}^d_{w,m},
\]

\[
\gamma_{w,m,t}^{d,p} = \sum_{q \in \text{Qliq}^d_{w,m}} \sum_{g \in \text{GOR}^d_{w,m}} \sum_{b \in \text{BSW}^d_{w,m}} \sum_{p \in \text{MP}_m} \delta_{w,m,t}^{d,(q,g,b,p)} \cdot \forall p \in \text{MP}_m,
\]

\[
\delta_{w,m,t}^{d,(q,g,b,p)} \geq 0, \forall q \in \text{Qliq}^d_{w,m}, g \in \text{GOR}^d_{w,m}, b \in \text{BSW}^d_{w,m}, p \in \text{MP}_m,
\]

\[
\sum_{d \in D_{w,m}} p_{w,m,t}^d - \text{BigM}_m (1 - x_{w,m}) - \text{BigM}_m (1 - z_{w,t}) \leq p_{m,t}
\]

\[
\leq \sum_{d \in D_{w,m}} p_{w,m,t}^d + \text{BigM}_m (1 - x_{w,m}) + \text{BigM}_m (1 - z_{w,t}),
\]
\[(\eta_{w,m,t}^{d,q})_{q \in Q_{liq,w,m}} \text{ is SOS2,}\] (4.17a)

\[(\eta_{w,m,t}^{d,g})_{g \in GOR_{w,m}} \text{ is SOS2,}\] (4.17p)

\[(\eta_{w,m,t}^{d,b})_{b \in BSW_{w,m}} \text{ is SOS2,}\] (4.17q)

\[(\eta_{w,m,t}^{d,p})_{p \in MP_{m}} \text{ is SOS2.}\] (4.17r)

Constraints coupling the flows for the possible diameters are combined to obtain the flow from a well to a gathering system, for all \(w \in W, m \in M, t \in T:\)

\[\Delta p_{w,m,t} = \sum_{d \in D_{w,m}} \Delta p_{w,m,t}^{d},\] (4.18a)

\[q_{h,w,m,t} = \sum_{d \in D_{w,m}} q_{h,w,m,t}^{d,h}, \forall h \in H.\] (4.18b)

The pressure drop from manifold \(m \in M_{man}\) to platform \(p \in M_{plat}\), in a pipeline of diameter \(d \in D_{m,p}\), is modeled as a piecewise-linear function with breakpoints on liquid flowrate, gas-liquid ratio (GLR), water-fraction (BSW), and platform operating pressure, respectively:

- \(Q_{liq_{m,p}} = \{Q_{liq_{m,p,1}}, Q_{liq_{m,p,2}}, \ldots\}\),
- \(GLR_{m,p} = \{GLR_{m,p,1}, GLR_{m,p,2}, \ldots\}\), and
- \(BSW_{m,p} = \{BSW_{m,p,1}, BSW_{m,p,2}, \ldots\}\),
- \(Pr_{p} = \{Pr_{p,1}, Pr_{p,2}, \ldots\}\).

Given these breakpoints, the pressure drop in a pipeline is approximated with a piecewise-linear function for all \(m \in M_{man}, p \in M_{plat}, d \in D_{m,p}, t \in T:\)
\[
\Delta p_{m,p,t} = \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d, b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \Delta P_{m,p}^{d,(q,g,b,pr)} \theta_{m,p,t}^{d,(q,g,b,pr)} - p_{m,p,t}^d \\
\]

\[
q_{m,p,t}^d = \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d, b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \theta_{m,p,t}^{d,(q,g,b,pr)} \cdot q \cdot (1 - b), \\
\]

\[
q_{m,p,t}^d = \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d, b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \theta_{m,p,t}^{d,(q,g,b,pr)} \cdot q \cdot g, \\
\]

\[
v_{m,p,t}^d = \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d, b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \theta_{m,p,t}^{d,(q,g,b,pr)} \cdot q \cdot b, \\
\]

\[
x_{m,p,t}^d = \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d, b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \theta_{m,p,t}^{d,(q,g,b,pr)} \cdot p_r, \\
\]

\[
y_{m,t} = \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d, b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \theta_{m,p,t}^{d,(q,g,b,pr)} \cdot q, \\
\]

\[
x_{m,p,t}^d + y_{m,t} - 1 \leq \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d, b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \sum_{q} \theta_{m,p,t}^{d,(q,g,b,pr)} \cdot q, \\
\]

\[
\theta_{m,p,t}^{d,q} = \sum_{g \in GLR_{m,p}^d, b \in BSW_{m,p}^d, p \in Pr} \sum_{t} \theta_{m,p,t}^{d,(q,g,b,pr)} , \forall q \in Q_{\text{liq}}^d, \\
\]

\[
\theta_{m,p,t}^{d,g} = \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d, b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \theta_{m,p,t}^{d,(q,g,b,pr)} , \forall g \in GLR_{m,p}^d, \\
\]

\[
\theta_{m,p,t}^{d,b} = \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d} \sum_{b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \theta_{m,p,t}^{d,(q,g,b,pr)} , \forall b \in BSW_{m,p}^d, \\
\]

\[
\theta_{m,p,t}^{d,pr} = \sum_{q \in Q_{\text{liq}}^d, g \in GLR_{m,p}^d} \sum_{b \in BSW_{m,p}^d} \sum_{p \in Pr} \sum_{t} \theta_{m,p,t}^{d,(q,g,b,pr)} , \forall pr \in Pr, \\
\]

\[
\sum_{d \in D_{m,p}} p_{m,p,t}^d - \text{BigM}_u (1 - x_{m,d}) - \text{BigM}_u (1 - y_{m,t}) \\
\leq p_{p,t} \leq \sum_{d \in D_{m,p}} p_{m,p,t}^d + \text{BigM}_u (1 - x_{m,p}) + \text{BigM}_u (1 - y_{m,t}), \\
\]

35
\[ \theta_{m,p,t}^d, q, g, b, pr \geq 0, \forall q \in Q^{liq}_{m,p}, g \in GLR_{m,p}^d, b \in BSW_{m,p}^d, pr \in Pr_p \]  

(4.19n)

\( (\phi_{m,p,t}^d)_q \in Q^{liq}_{m,p} \) is SOS2,  

(4.19a)

\( (\phi_{m,p,t}^d)_g \in GLR_{m,p}^d \) is SOS2,  

(4.19p)

\( (\phi_{m,p,t}^d)_b \in BSW_{m,p}^d \) is SOS2,  

(4.19q)

\( (\phi_{m,p,t}^d)_pr \in Pr_p \) is SOS2,  

(4.19r)

Constraints are also required for consistency of flows from manifolds to platforms, for all \( m \in M_{man}, t \in T \):

\[ q_{m,p,t}^h = \sum_{d \in D_{m,p}} q_{m,p,t}^{d,h}, \forall p \in M_{plat}, h \in H, \]  

(4.20a)

\[ q_{m,t}^h = \sum_{p \in M_{plat}} q_{m,p,t}^h, \forall h \in H, \]  

(4.20b)

\[ \Delta p_{m,p,t} = \sum_{d \in D_{m,p}} \Delta p_{m,p,t}^d, \forall p \in M_{plat}. \]  

(4.20c)

Platform flow pressure must be greater than the process pressure \( P_{p}^{\text{min}} \):

\[ p_{p,t} \geq P_{p}^{\text{min}}, \forall p \in M_{plat}, t \in T, \]  

(4.21)

with \( p_{p,t} \) being the operating pressure at platform \( p \) during period \( t \).

The pressure balance from wells to gathering systems must be accounted for. At time \( t \in T \), the downhole pressure of well \( w \in W \) must be equal to the pressure of the gathering system \( m \in M \) added to the pressure drop, namely:

\[ p_{m,t} + \Delta p_{w,m,t} - M_m(1 - x_{w,m,t}) \leq p_{w,t} \leq p_{m,t} + \Delta p_{w,m,t} + M_m(1 - x_{w,m,t}), \]  

(4.22)

where \( M_m \) is a sufficiently large constant to implement the Big-M strategy.

The pressure balance between manifolds and platforms is modeled in the same way of wells. For each manifold \( m \in M_{man}, \) platform \( p \in M_{plat}, \) and time period \( t \in T, \) the equations below ensure the pressure balance:

\[ p_{p,t} + \Delta p_{m,p,t} - M_m(1 - y_{m,t}) - M_m(1 - x_{m,p}) \leq p_{m,t} \leq p_{p,t} + \Delta p_{m,p,t} + M_m(1 - y_{m,t}) + M_m(1 - x_{m,p}). \]  

(4.23)

Limits on liquid handling are imposed on the various gathering systems, for all
\( m \in \mathcal{M} \) and \( t \in \mathcal{T} \):

\[
q_{m,t}^{\text{oil}} + q_{m,t}^{\text{water}} \leq q_{m}^{\text{liq, max}},
\]
\[
q_{m,t}^{h} \leq q_{m}^{h, \text{max}}, \forall h \in \mathcal{H}.
\]

(4.24a) \hspace{1cm} (4.24b)

where \( q_{m}^{\text{liq, max}} \) is the bound for the total liquid rate and \( q_{m}^{h, \text{max}} \) is the maximum processing capacity for a phase \( h \).

Manifolds are limited by the number of well connections, such that for all \( m \in \mathcal{M}_{\text{man}} \):

\[
x_{m}^{\min} x_{m} \leq \sum_{w \in \mathcal{W}} x_{w,m} \leq x_{m}^{\max} x_{m},
\]

(4.25)

where \( x_{m}^{\max} \) and \( x_{m}^{\min} \) are respectively the maximum and minimum number of well connections. Similarly, the platforms have limits on the number of connections, so that for all \( p \in \mathcal{M}_{\text{plat}} \):

\[
x_{p}^{\min} x_{p} \leq \sum_{w \in \mathcal{W}} x_{w,p} + \sum_{m \in \mathcal{M}_{\text{man}}} x_{m,p} \leq x_{p}^{\max} x_{p},
\]

(4.26)

in which \( x_{p}^{\min} \) and \( x_{p}^{\max} \) are lower and upper bounds on platform connections.

Putting together all the constraints and objective function, we arrive at the MILP formulation for subsea infrastructure layout design:

\[
P : \max f(s)
\]
\[
\text{s.t.} \quad \text{Eqs. (4.2)} - (4.26)
\]

(4.27a) \hspace{1cm} (4.27b)

where \( s \) is a vector containing all of the decision variables.
Chapter 5

Application of Methodology to a Representative Oilfield

The proposed methodology was applied to a representative oilfield to assess the computational hardness and relevance of the results to infrastructure design. A case study was performed in a real field, which received the fictitious name of Ipanema.

5.1 Case Study: Ipanema Field

The Ipanema field is about 160 km off the coast, average water depth of 565 m. The reservoir is a friable sandstone formation which started production 25 years ago being now in its mature stage. Ipanema had a single platform, namely PLAT1, decommissioned after 21 years of production with a capacity of 50,000 bbl/d. Nevertheless, Ipanema still produces a single well to a platform in a neighbor field.

The reservoir model employed in this study has eight producer wells and two water injection wells. It is a real world model, built in ECLIPSE [47], with all heterogeneity that characterizes the reservoir. The model has two non-communicating reservoirs with same type of oil, namely the north and south wings as can be seen at Figure 5.1. The fluid was modeled as black oil honoring PVT analysis. The original drive mechanism is solution gas, nevertheless, both reservoirs have an aquifer of weak influence on pressure maintenance. In this study, a production control strategy was applied to adjust well production so as to maintain the reservoir average pressure above the bubble point of 240 kgf/cm². Also, a gas oil ratio control was active in every well to avoid values far from the original solubility ratio of 76 sm³/sm³. Both controls apply the same action, that is to raise pressure at well by choking the flow at platform or at manifold. Also, it was considered that water injection control is regulated to respect limits for water rate per injector of 3000 m³/d and fracture pressure of 300 kgf/cm². There are two water injectors, INJE1 that is placed at a
aquifer boundary in north wing and INJE2 is a multi-lateral well in the south wing.

![Ipanema reservoir - oil saturation](image)

Figure 5.1: Ipanema oil saturation - top view. Red = oil, blue = water (aquifer). Producer wells - black names, water injector wells - blue names.

Since in the proposed optimization model there is no drilling scheduling, in reservoir simulation all wells were open for production at same time.

This study intends to evaluate the proposed methodology applied to the original layout configuration of Ipanema. Formerly, it comprised all wells connected directly to the single platform PLAT1, without manifolds, as can be seen at Figure 5.2. As an exercise, keeping PLAT1 as the single platform at its original, the methodology will assess the application of subsea manifolds and definition of flowline diameters.

Then, taking as a starting point the former Ipanema layout, pipeline trajectories between wells and platform were kept without changes. However, due to the absence of manifolds in the former layout, all possible connections between wells to manifolds and manifolds to platform had to be traced. As the methodology advocates, based on the work of FAMPA [9], candidate places for installing a manifold were indicated as wells as all possible connections from manifolds to producers and platform. Finally, the optimization variables were: a) manifold installation at all candidate places and b) diameters of all possible connections.

Producers are models of real wells, from former Ipanema layout, which were modeled in PIPESIM [48] with black oil fluid model, single completion, tubing 5 1/2”, gas lift valve, flowline and riser with 4” diameter. Flexible lines traced
between manifolds and platforms were also built in PIPESIM. The gas-lift injection rate remained constant at 100,000 m$^3$/d for every well along time, not being a variable in the model.

Related to diameter optimization, two possibilities of commercial diameters for flowline and riser were considered in the study: a) 4” and 6” for well flowline/riser; b) 6” and 8” for manifold flowline/riser.

It is needed to define layout configurations that bring the highest and lowest cumulative production during the field lifetime. Then, from the perspective of a production petroleum engineer, to achieve boundaries for optimization, four reservoir simulations were run for the following cases:

a) all wells satellite with flow/riser of 6” diameter;

b) all wells satellite with flow/riser of 4” diameter;

c) all wells linked to a manifold with flowlines of 6” diameter; all manifolds linked to the platform with a flowline/riser of 8” diameter;

d) all wells linked to a manifold with flowline of 4”; all manifolds linked to platform with flowline/riser of 6” diameter.

Figure 5.3 shows all possible layout configurations. Green traces represent well to platform connections, which come from original trajectories in the former Ipanema layout. Numbers in circles represent candidate places for manifold installation. Red and blue traces represent connections related to manifolds, which were not present in the former Ipanema layout. The dotted lines are the small diameters.
For combinations considered to define the model boundaries, well PROD3 is connected to manifold 2.

For every possible connection there is a respective pressure drop table. These tables take part of reservoir simulation and optimization model. Thus, each line is represented by a unique pressure drop table. They were generated as lift tables for reservoir simulators by running the respective models in PIPESIM, covering all possible ranges of liquid, oil and gas rates, and further, the range of pressure for the downstream node, being a given manifold or the platform. As said before, artificial lift was kept constant.

Moreover, tubing diameter evaluation could be considered, combined to flowline lift table such as tubing 5 1/2” and flowline/riser 4”, tubing 6 5/8” and flowline/riser 6”, and so on. However, to carry out this case study, the premise is that wells are previously defined since the cost of drilling wells of bigger diameters was not taken into account.

![Figure 5.3: All possible connections of subsea layout to Ipanema field.](image)

The simulations were run to achieve boundaries for optimization and the outputs were used to partition the reservoir in tanks. In Ipanema field, the simulations allowed to identify wells that exhibited similar reservoir surrounding pressure. These wells were grouped into the same reservoir partition, in other words, the same tank.

Figure 5.4 illustrates pressure distribution after 10 years of simulation, pressure weighted by pore volume. The four layouts considered to achieve bounds for reservoir variables in optimization were respectively represented. The identification of wells with similar pressures in their surroundings lead to group them into six tanks. Oil producer wells PROD1 and PROD2 belong to tank 01; PROD3 and PROD4 to tank 02; PROD8 to tank 04 and PROD5, PROD6 and PROD7 to tank 06. Water injector INJE1 belongs to tank 03 and INJE2 to tank 05. Figure 5.5 shows the reservoir
partition in tanks of similar pressure behavior along simulation time.

Figure 5.4: Pressure weighted by pore volume after 10 years of production.

Figure 5.5: Reservoir partitioned in regions of similar pressure behavior.
After the partition of reservoir, from ECLIPSE it is possible to plot average pressure for regions, namely tanks 01, 02, 04, and 06 as seen in Figure 5.6. Yellow curves are related to all wells linked to manifold with small diameters, whereas blue curves correspond to all wells satellite with small diameters. These are the layouts that lead to the higher reservoir pressure due to small diameters.

Red and green curves are related to all wells satellite with large diameters and all wells linked to manifolds with large diameters, respectively. These are the layouts that lead to low reservoir pressure due to large diameters.

The methodology advocates that any possible layout combination among those simulated will induce tank pressures lying between these boundaries.

Choosing PROD6 to represent all other wells, Figure 5.7 shows envelopes of well bottomhole pressure and drawdown. The methodology also advocates that bottomhole pressure and drawdown are expected to lie between these boundaries for any possible layout combination among those simulated. Figure 5.7 also brings outputs from reservoir simulation of gas oil ratio, productivity index and water cut. These quantities are taken as parameters in optimization, by average of simulation outputs of those four layouts considered, as stated in Equations (4.9). In this case of study, productivity index is calculated with average tank pressure. Liquid rate is not an envelope or average taken as parameter, it appears on the plot only for illustration. The reservoir behavior for a layout combination can take the liquid rate out of the boundaries, therefore boundaries are not imposed on liquid rate, being sufficient to implement pressure bounds in the model.

The rest of the wells have their pressure envelopes and averages for parameters presented in Appendix A.

5.1.1 Layout Optimization

Considering averages for well parameters, pressure boundaries and all lift tables, the optimization model given in Equations (4.1)–(4.26) can have its constraints and equations implemented in the GAMS-CPLEX environment. Due to the high computational time experienced, the time periods were designed to represent a year and consequently the annual averages were taken as the reservoir parameters. Moreover, the case study considered only the first 5 years of production.

Economic parameters are presented in Table 5.1, related to flexible line costs; Table 5.2, cost of manifolds and Table 5.3, interest rate and oil profit after operational costs and taxes.

Lastly, Figure 5.8 brings the layout that optimizes project NPV, the results of optimization which are summarized in Table 5.4 resumes them. If compared to original solution, the proposed solution presents a NPV 18% higher.
Table 5.1: Flexible Line Costs

<table>
<thead>
<tr>
<th>Diameter</th>
<th>$ per meter</th>
</tr>
</thead>
<tbody>
<tr>
<td>4&quot;</td>
<td>1000</td>
</tr>
<tr>
<td>6&quot;</td>
<td>1250</td>
</tr>
<tr>
<td>8&quot;</td>
<td>1500</td>
</tr>
</tbody>
</table>

Manifold 3 is installed, connecting wells PROD1 and PROD8. PROD1 has a line of 4" diameter connecting to manifold 3 and PROD8 has a 6" diameter, the large one. Between manifold 3 and platform PLAT1, the flexible line has 8" diameter, the large one from options evaluated. All other wells are satellite and among them only PROD2 has a flowline/riser with 4" diameter, the small one. All other satellite wells have 6" diameter.

The optimal layout was imposed to the ECLIPSE reservoir simulator in order to verify the obtained results.

Table 5.2: Manifold Cost

<table>
<thead>
<tr>
<th>Manifold number</th>
<th>Cost $ (10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Figure 5.7: Envelopes and average parameters for wells - PROD6

Figures 5.9, 5.10 and 5.11 show the optimization solution. Besides, there are plots of imposing the optimal solution to the reservoir simulator so as to compare results against the established boundaries.

Figure 5.9 compares optimization tank pressure and cumulative production against reservoir simulation of the optimal layout. It also compares optimization results and simulation of the optimal solution against boundaries defined by running simulation of layout that induce the lowest and highest production.

Figure 5.10 compares cumulative production of the whole reservoir, confronting production that came from optimization model and reservoir simulation of optimal solution.
Table 5.3: Oil Profit and Interest Rate

<table>
<thead>
<tr>
<th>Economic Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yearly interest rate (%)</td>
<td>10</td>
</tr>
<tr>
<td>Oil profit ($)</td>
<td>5</td>
</tr>
</tbody>
</table>

Figure 5.11 presents a comparison for wells, equivalent to comparison made by Figure 5.10. Only PROD6 was taken as an example, further plots are available at Appendix A.

These graphics show differences between reservoir responses comparing solution provided by GAMS-CPLEX optimization model and ECIPSE simulation of this optimal solution. However, from the perspective of a petroleum production engineer, which accounts for reservoir tendencies, the proposed model brings feasible solutions and achieve its purpose. Small deviations does not make this optimization model invalid.

These differences are expected due to piecewise linear approximations. Moreover, reservoir simulation of layouts that brings the highest and lowest cumulative production do not comprise all possible reservoir behavior and all relations among wells. If there is some cause and effect relations that is not represented by optimization model and it is noticed in reservoir simulation model, these relations can be modeled by adding constraints.

In order to solve the layout optimization problem in reasonable time some artifices were needed. CPLEX parameter tuning, parameter scaling to avoid numerical instabilities and ill posed matrices, use of multicore parallelism, among others, were included in model. An initially feasible solution was provided to solver before it starts the optimization algorithm, following the technique called warm starting, was the artifice that contributed most to drop computational time. A GAP of 5% between the best solution found and the best bound was considered satisfactory to stop CPLEX optimization so as to testify the optimization model.

Table 5.4 shows the results and computer performance to achieve it, as well as performance to obtain an initial solution to be used as warm starting. GAMS-CPLEX ran in a workstation with 384 GB RAM, 6 cores of 2.4 GHz and 4 TB of hard disk.

CPLEX reported 17,518 single equations, 213,517 single variables, 308 discrete variables and 2,648,357 non zero elements.

This case of study allows to access how the model represents the influence among wells. Figure 5.13 depicts liquid rates for 20 years of simulation in which all wells are satellite with 6” diameter, the largest one. Note that PROD1 is not able to produce after the first year, liquid rate goes to zero. This well shares tank 01 with well PROD2 shown in Figure 5.5.
Table 5.4: Model Results and Computational Time

<table>
<thead>
<tr>
<th></th>
<th>Optimal solution</th>
<th>Warm start</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU Time (s)</td>
<td>44050</td>
<td>2470</td>
</tr>
<tr>
<td>GAP (%)</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>NPV ($ \times 10^6$)</td>
<td>228.4</td>
<td>223.4</td>
</tr>
<tr>
<td>Cumulative Production ($ \times 10^6 m^3$)</td>
<td>10.7</td>
<td>10.5</td>
</tr>
<tr>
<td>Capex ($ \times 10^6$)</td>
<td>24.5</td>
<td>26.6</td>
</tr>
</tbody>
</table>

Figure 5.8: Layout that maximizes NPV.

The opening of all wells at the same time at the beginning of operations lead to a strong depressurization in tank 01. For layouts with the large diameter of 6”, depressurization is even stronger as can be seen in Figure 5.6.

Then, to prevent the bottomhole pressure from getting above the oil bubble point pressure, the control implemented in the reservoir simulator raises the PROD1 surface pressure to also raise the bottomhole pressure. As a consequence, a raise in bottomhole pressure closes the well.

If layout comprises all wells linked to manifolds and bigger diameters, the same phenomena happens to PROD1 and tank 01.

After the time step PROD 1 was closed, since there is no production, the bottom pressure of well 01 is the same as the reservoir surroundings. Thus, in optimization model, lower bound for tank 01 pressure consider that PROD1 is closed after first year of production. These lower boundaries are high enough to prevent the optimization model from finding solutions that keep PROD1 producing, otherwise, constraints would be violated.

A way to keep production of PROD1 over time is by avoiding high depressurization of tank 01. It is important to notice that PROD2 shares tank 01 with PROD1. To keep PROD1 open, a layout for PROD1 and PROD2 must be chosen to avoid
Figure 5.9: Pressure and cumulative production of tanks 01, 02, 04 and 06. Comparison among boundaries, optimization solution and ECLIPSE simulation of solution.

low pressure in tank 01. If it is profitable for NPV maximization to keep PROD1 open, optimization tends to choose the small diameter for PROD1 and PROD2 so as to not reach the pressure lower bound of tank 01.

This explain the solution for PROD2 — small diameter of 4”, satellite. PROD1,
connected to manifold 03 with a small diameter of 4” and manifold 03 connected to PLAT1 with large diameter of 6” performs an intermediate solution between small diameters linked to manifolds and large diameters satellite.

In this way, the proposed optimization model honors simulation conditions.

Further clarifying, the plots in Figure 5.6, it can be seen that depressurization is faster than pressurization through water injection in tank 03 by well INJE1. It is important to remember that there is full communication between tanks 01 and 02, 02 and 03. The partition is not a barrier imposed to fluid flow, it is a logical division to account for averages in these regions. Thus, by reservoir simulation, it is possible for water to flow from tank 03 to tank 01 though tank 02. Water injection from the two injectors can be noticed when average pressure of tank stops to fall, keeping constant or begging to raise. It can be a tricky conclusion if some wells are closed. Shutting in wells also can lead average tank pressure to a near constant value or raise.
Figure 5.11: Well PROD6 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.
Figure 5.12: NPV, capex and cumulative production - comparison among optimal solution and the four solutions obtained for definition of pressure boundaries.

Figure 5.13: Liquid rates for all wells satellite with 6” diameter flow line and riser. PROD1, the dark blue line, is shut down after roughly 6 months of production.
Chapter 6

Conclusions, Discussion and Future Work

6.1 Conclusions and Discussion

The new methodology achieved its purposes by being a simple and practical way to evaluate different layout scenarios.

The proposed method is specially adequate from the perspective of a production petroleum engineer who aims to assess the reservoir tendency and behavior by varying the subsea layout configuration.

Integer programming can couple both models of multiphase flow in pipelines and flow in porous media in a feasible MILP problem.

The reservoir can be modeled according to a production petroleum engineering perspective, as pressure and productivity index, which are bounded and updated along time following previous simulations of extreme cumulative production cases.

Other well parameters that change over time, such as water cut, gas oil ratio and productivity index can be represented by an average at each time step. The average is calculated on outputs of previous simulations of extreme cumulative production cases. If these parameters vary considerably with different layouts, the proposed methodology can not be applied.

Multiphase flow through pipelines can be approximated with piecewise-linear functions to ensure model fidelity for optimization purposes.

The proposal does not achieve the precision of high fidelity models, like implicit coupling [37]. However, the methodology makes it possible to perform a quick analysis of the layout influence on NPV and cumulative production, allowing further analyses to be focused on the promising layouts. It can be useful not only at the early stages of a development plan, but also to evaluate alternatives of layouts for brown fields. In such mature fields, new wells are planned to be drilled as part
of a revitalization program, which may exceed the maximum number of possible connections in the platform and existing subsea manifolds.

Furthermore, in the simplest way, the solution to the optimization model chooses which lift tables lead to the highest NPV. The study presented in the previous section had two lift tables for each possible connection, one with small and the other with large diameter. Thus, the evaluation of artificial-lift alternatives is a matter of including more lift tables, that could be combined with different diameters. Nevertheless, proposed model is able to identify the best artificial-lift method. However, it has difficulties to define precisely the optimum gas-lift rates or pump frequencies for the wells, at every time step. This is due to the need of a lift table for each gas-lift rate or pump frequency, which prevents the model from covering a wide range of lift-gas rates and frequency possibilities. None the less, the methodology can indicate with reasonable precision, which artificial lift fits better to wells, allowing a follow up and a more accurate study to be focused on the promising alternatives. Certainly, difficulties on computational time may arise and for electrical immersible pumps constraints on the free-gas fraction have to be imposed, which can be hard to handle.

The optimization of the number of platforms is straightforward since the model considers them as manifolds. Despite not having platform processing capacity as an optimization variable, a sensitivity study of this parameter can be performed comprising capacity and capex variations.

### 6.2 Future work

As a future work, reservoir can be modeled through upscaling techniques by transforming each well region into a cell. There would be a correspondence between the number of cells and the number of wells, one cell for every well. Then, averages for each well at every time step for water cut, gas oil ratio and productivity index will be not needed, being updated implicitly in model.

Reservoir simulation, including models after upscaling, are based on nonlinear partial differential equations. These equations can be ordered so as to make a system of linear equations. They represent the integration of all the cells in the reservoir simulation grid, having as variables a) pressure of each cell, b) saturation of oil/water/gas from each cell, c) production of each producer well cell and d) injection of each injector well [35].

The solution is obtained through Newton Raphson method, where a rearrangement of the system of equations for a system of residual equations is built to minimize the residue of these equations in an iterative process. The values of primary variables are used to obtain numerically the matrix containing the derivatives of residual
equations for each variable, the Jacobian matrix [35].

The challenge is to solve the Jacobian for each branch & bound iteration. Maybe, it could be expected that, for a very small number of cells, computational time would not be prohibitively high to prevent an optimization solution.
Bibliography


Appendix A

Detailed results for all wells

Comparison among boundaries, optimization solution and ECLISPE simulation of optimal solution for all wells.

Figure A.1: Well PROD1 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.
Figure A.2: Well PROD2 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.
Figure A.3: Well PROD3 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.
Figure A.4: Well PROD4 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.
Figure A.5: Well PROD5 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.
Figure A.6: Well PROD6 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.
Figure A.7: Well PROD7 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.
Figure A.8: Well PROD8 - Comparison among boundaries, optimization solution and ECLISPE simulation of solution.