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Choosing an optimal investment strategy: The role of robust pair-copulas based portfolios

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Abstract

This paper is concerned with the efficient allocation of a set of financial assets and its successful management. Efficient diversification of investments is achieved by inputing robust pair-copulas based estimates of the expected return and covariances in the mean-variance analysis of Markowitz. Although the whole point of diversifying a portfolio is to avoid rebalancing, very often one needs to rebalance to restore the portfolio to its original balance or target. But when and why to rebalance is a critical issue, and this paper investigates several managers' strategies to keep the allocations optimal. Findings for an emerging market target return and minimum risk investments are highly significant and convincing. Although the best strategy depends on the investor risk profile, it is empirically shown that the proposed robust portfolios always outperform the classical versions based on the sample estimates, yielding higher gains in the long run and requiring a smaller number of updates. We found that the pair-copulas based robust minimum risk portfolio monitored by a manager which checks its composition twice a year provides the best long run investment.

Keywords: Pair-copulas; Optimal financial portfolios; Robust estimation; Rebalancing.

JEL Classifications: $C01$, $C19$,$C32$, $C51$, $C58$, $G11$.

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1 Introduction

Financial institutions and portfolio managers are primarily concerned with the efficient allocation and monitoring of sets of financial assets. Periodic portfolio rebalancing, aiming to restore the investment back to its desired target risk and return, is a crucial step in the process of controlling risk. Commonly asked questions are how often a portfolio should be rebalanced, and which would be the best indicators of changes in the global economy or in the balance among the component assets.

Efficient diversification of investments based on the mean-variance analysis of Markowitz (1952) is widely used by institutional investors. Statistically, the resulting efficient frontier just relies on the estimates of the expected return and covariance matrix, and the sample estimates are the usual inputs. However, the statistical good properties of the sample estimates are attached to the highly improbable assumption of multivariate normality.

A better characterization of the data underlying multivariate distribution will provide more reliable estimation of the efficient frontier. This means we must know not just the marginal univariate series behavior and their correlations, but their whole d-dimensional probability distribution. This may be accomplished by modeling the data through pair-copulas (Frigessi and Bakken (2007); Min and Czado (2008); Berg and Aas (2008); Fischer et al. (2008)).

A pair-copula construction is just a hierarchical decomposition of a multivariate copula into a cascade of bivariate copulas. Since an appropriate copula function can be found for any type of association — linear, nonlinear, ranging from perfect negative to perfect positive dependence — one can expect the model to truthfully represent the data at hand. Estimation takes place at the level of the two-dimensional data, therefore avoiding the famous *curse of dimensionality*.

The analysis of financial data from emerging markets poses some specific challenges. Atypical points in transaction prices (from non-confirmed unexpected news, market manipulation, and so on) distort classical statistical inference, corrupting the inputs to the mean-variance algorithm. A distorted correlation matrix and inflated risk estimates will provide misleading allocations. To handle deviations from the true underlying distribution, robust methodologies are called for. We suggest to apply the robust estimates for pair-copulas models, initially proposed in Mendes et al. (2007). For each parametric copula family there exist a robust *weighted minimum distance* or a *weighted maximum likelihood* estimator providing accurate estimates under contaminations. The robust portfolios are obtained by inserting the robust pair-copulas based mean and covariance estimates in the mean-variance Markowitz procedure.

Robust methods typically detect the pattern implied by the vast majority of the data, providing more stable estimates. Robust allocations are resistant to unjustified sudden fluctuations of the market, which are identified by the robust estimates as point contaminations. Therefore, robust portfolios are primarily designed for long run investments. We note that the notion of "long run" may vary across markets and to account for changes in the economy, some periodic rebalancing of the portfolio may still be needed. It is expected from a robust investment to yield higher gains in the long run and to require a smaller number of updates.

There is no universally accepted best strategy for portfolio management. Best strategy will change with investor risk aversion, portfolio target return or standard deviation. Among many others, we consider the popular strategy followed by institutional investors that monitors a portfolios at an annual (or monthly) frequency and then rebalances only if the allocation to an asset shifts more than some threshold (5%, 1%). We do not consider additional factors when implementing the rebalancing strategies, such as trading costs or cost of time spent which would reduce the return of the portfolio. However we keep track of the rebalancing frequency of each manager and are able to draw some conclusions based on their number of rebalancings.

Summarizing, in this paper we address both problems of composing and managing portfolios, given a set of financial instruments. We do not address the issue of choosing the component assets. We robustly estimate the data multivariate distribution using pair-copulas obtaining the inputs which will define the robust efficient frontier. The trajectory of a target return and the minimum risk portfolios will be managed by twelve managers during a 2-years period of out-of-sample investigation. We use data from emerging markets because of the higher volatility of these stock markets and their greater potential for interdependence with the major markets. More specifically, we use six-dimensional contemporaneous daily log-returns from the most traded Brazilian stocks, due to Brazil's important position among emerging equity markets. The robust portfolios are compared to their classical version based on the sample empirical estimates,

The contributions of this paper are three fold: (i) we introduce and investigate the performance of pair-copulas based robust portfolios; (ii) we investigate 12 managing strategies aiming to keep (or restore) the portfolio target, to guarantee the same risk aversion level; (iii) we illustrate the ideas using Brazilian data.

Findings in the paper are striking and convincing. We found that despite the investment type, the robust methodology always outperform the classical version. We are able to determine the best rule for restoring the portfolio to its original balance and keep the allocations optimal. We show that the best strategy depends on the investor risk profile, and that pair-copulas based robust minimum risk portfolios monitored by a manager which checks its composition twice a year provides the best long run investment. Additional exercises are not provided because the aim of the paper is to find best portfolio composition along with best long run managing strategy for a *given* set of financial instruments.

The remaining of this paper is organized as follows. In Section 2 we briefly consider the classical mean-variance methodology for obtaining efficient portfolios, and briefly review the definitions of pair-copulas and robust estimates. In Section 3 we define 12 strategies for portfolio monitoring. In Section 4 we analyze two 6-dimensional data sets and assess the performance of classical and robust target and minimum risk portfolios. In Section 5 we discuss and summarize the results. Section 6 provides the references.

2 Statistical Methodologies

Derived from simple mathematical terms relating the expected return and risk of a portfolio, Markowitz's optimization procedure (Markowitz, 1952) for obtaining the efficient frontier can be considered the most widely used result of modern economics. Based on the idea that one should diversify to reduce risk, for a given expected return, the portfolio theory minimizes risk. To this end the theory considers not only the means and variances of a set of securities, but also their covariances. However, the true data generating process is unknown and the inputs for the mean-variance algorithm must be estimated.

2.1 Classical allocations

Markowitz mean-variance optimization is a quadratic optimization problem, whose classical inputs are the sample mean and the sample covariance matrix, the maximum likelihood estimates (MLE) under multivariate normality. We call the portfolios obtained using the classical inputs as *classical portfolios.*

One of the most important causes of limitation of the method in practice is the lack of optimality presented by classical estimates in the financial environment. This problem was studied by Klein and Bawa (1976), Jobson and Korkie (1980, 1981), Canela and Collazo (2007), among others. Following Markowitz work, a large part the literature was devoted to obtain reliable alternatives for the sample mean and the sample covariance matrix.

2.2 Robust pair-copulas based allocations

In the last decade, copulas have been widely used in finance (see for example, Embrechts et al. (1999); Demarta and McNeil (2005); and Gatzert et al. (2008)). The reason for this popularity is the inadequacy of the multivariate normal distribution when modeling financial data. Any multivariate distribution may be decomposed on its marginal univariate distributions and a copula, which contains all information about the dependence structures linking the margins (Nelsen (2007); Joe (1997)). Therefore, modeling the data through copulas allows one to obtain estimates of any characteristic of the distribution, including Markowitz inputs, the mean vector and the covariance matrix. We call these estimates as copulas- (or later pair-copulas-) -based estimates.

Consider a continuous random vector $X_1, \ldots, X_d$ with joint cumulative distribution function (c.d.f.) $H(x_1, \ldots, x_d)$ and marginal distributions $F_1, \ldots, F_d$. Sklar's theorem (Sklar, 1959) ensures that there exists a d-copula C such that for all $(x_1, \ldots, x_d) \in [-\infty, \infty]^d$

$$H(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)). \tag{1}$$

Conversely, if C is a d-copula and $F_1, \ldots, F_d$ are c.d.f.s, the function H defined by (1) is a d-dimensional distribution function with margins $F_1, \ldots, F_d$. Furthermore, if all marginal c.d.f.s are continuous, C is unique. A d-dimensional copula C is a d-dimensional c.d.f. on $[0, 1]^d$ with standard uniform marginal distributions.

When C is absolutely continuous, taking partial derivatives of (1) one obtains

$$h(x_1, \cdots, x_d) = c(F_1(x_1), \cdots, F_d(x_d)) \prod_{i=1}^{d} f_i(x_i) \tag{2}$$

for some d-dimensional copula density c. This expression is the basis for the usual two steps inference approach, where firstly the marginal estimates are obtained, and then the copula parameters are estimated (Joe and Xu (1996), Joe (1997)).

The equation (2) allows for tailored marginal modeling considering all characteristics of each F_i, including the mean, standard deviation, skewness, kurtosis and any type of short and long memory serial dependence, plus a search for the best fit for the dependence structure through a large number of copula families that may be considered. This results in flexible multivariate distributions well fitted to the data.

A copula function is invariant under monotone increasing transformations of $\mathbf{X}$, making copula-based dependence measures interesting scale-free tools for studying dependence. An important copula-based dependence concept is the *coefficient of upper tail dependence*, λ_U, defined as

$$\lambda_U = \lim_{\alpha \to 0^+} \lambda_U(\alpha) = \lim_{\alpha \to 0^+} Pr\{X_1 > F_1^{-1}(1-\alpha) | X_2 > F_2^{-1}(1-\alpha)\} \ ,$$

provided a limit $\lambda_U \in [0, 1]$ exists. If $\lambda_U \in (0, 1]$, then X_1 and X_2 are said to be *asymptotically dependent* in the upper tail. If $\lambda_U = 0$, they are *asymptotically independent*.

Similarly, the *lower tail dependence coefficient*, λ_L, is given by

$$\lambda_L = \lim_{\alpha \to 0^+} \lambda_L(\alpha) = \lim_{\alpha \to 0^+} Pr\{X_1 < F_1^{-1}(\alpha) | X_2 < F_2^{-1}(\alpha)\} ,$$

provided a limit $\lambda_L \in [0, 1]$ exists. It follows that

$$\lambda_U = \lim_{u \uparrow 1} \frac{\overline{C}(u, u)}{1 - u}, \text{ where } \overline{C}(u_1, u_2) = Pr\{U_1 > u_1, U_2 > u_2\} \text{ and } \lambda_L = \lim_{u \downarrow 0} \frac{C(u, u)}{u} .$$

The coefficient of tail dependence measures the amount of dependence in the upper (lower) quadrant tail of a bivariate distribution. In finance, it is related to the strength of association during extreme events. The copula derived from the multivariate normal distribution does not have tail dependence. Therefore, if this copula is assumed for modeling log-returns, for many pairs of variables it will underestimate joint risks.

More flexibility may be gainned by considering pair-copulas models. Pair-copulas is an hierarquichal decomposition of a d-copula into a cascade of of potentially different bivariate copulas. It was originally proposed by Joe (1997), and later discussed in detail by Bedford and Cooke (2001, 2002), Kurowicka and Cooke (2006) and Aas et al. (2007). The composing bivariate copulas may vary freely, with respect to choice of the parametric families and parameter values. Therefore, all types and strengths of dependence can be covered.

For large d, the number of possible pair-copula constructions is very large. Bedford and Cooke (2001) introduced a systematic way to obtain the decompositions, the so called *regular vines*. These graphical models help understanding the conditional specifications made for the joint distribution. Two special cases are the canonical vines (C-vines) and the D-vines. Again, the success of estimation procedure starts with good marginal fits (see Frahm, Junker, and Schmidt, 2004), which typically pose no difficulties. MLE estimates may be computed at both steps.

However, occasional atypical points may occur in finance, and they may corrupt the classical estimates of the dependence structure. Mendes and Accioly (2011) proposed to robustly estimate pair-copula models using the Weighted Minimum Distance (WMDE) and the Weighted Maximum Likelihood (WMLE) estimates. They are based on either a redescending weight function or on a hard rejection rule. Robust estimation of a pair-copula model is also performed at the level of bivariate data, and details about the robust estimates found in Mendes et al. (2007).

The WMDE minimize some selected weighted distance from the empirical copula, a goodness of fit statistics. The WMLE result from a two-step procedure. In the first step, outlying data points are identified by a robust covariance estimator and receive zero weights, and in the second step the copula MLE are computed for the reduced data.

There are many high breakdown point estimators of multivariate location and scatter that could be used in this preliminary phase. We use the robust Stahel-Donoho (SD) estimator based on projections (Stahel, 1981 and Donoho, 1982) which is implemented in the free R software. This affine equivariant estimator possesses high breakdown point, a bounded influence function, all good robustness properties which are expected to carry over the weights of the portfolios.

3 Portfolio Managing (Rebalancing: when and why)

Rebalancing an efficient portfolio means to update the data and re-evaluate the efficient frontier obtaining a new set of weights defining the assets' new allocations according to the preferred risk exposure. Although involving costs, the task is performed because one wants to bring the assets' allocations back to the investor's level of risk tolerance and expected return.

The frequency of rebalancing is usually pre-determined and follows some rule. Many rebalancing strategies may be formulated. We define 12 rebalancing strategies through a list of explicit rules to follow, aiming to represent the various degrees of managers' risk tolerance, some expressing a very rational line of thinking, others representing some subjective approach. They determine how frequently the portfolio is monitored and also which risk measure is being controlled. We do not address two issues. One, if a portfolio should be rebalanced to its target or to a new allocation suggested by the economy. Two, we do not consider rebalancing costs (transaction costs, taxes, time, labor, and so on), which are difficult to quantify, but we do keep track of the number of rebalancing events. The rules for rebalancing are:

Manager 1: Does nothing. Keeps the portfolio's allocations unchanged.

Manager 2: Rebalances the portfolio weekly (5 business days).

Manager 3: Rebalances the portfolio monthly (21 business days).

Manager 4: Rebalances the portfolio every three months (63 business days).

Manager 5: Rebalances the portfolio every semester (126 business days).

Manager 6: Follows the Drawdown. If the portfolio's Drawdown duration reaches three days, he/she rebalances.

Manager 7: Follows the Drawdown. If the portfolio's Drawdown duration reaches six days, he/she rebalances.

Manager 8: Follows the Drawdown. If the portfolio's Drawdown value is equal to or less than the 0.25 quantile of the portfolio's Drawdown empirical distribution, he/she rebalances.

Manager 9: Follows the Value-at-Risk (VaR). He/she inspects the portfolio monthly (each 21 days) and rebalances if there were at least three returns smaller that the current $\text{VaR}_{0.01}$ in the last 100 days, or if the current $\text{VaR}_{0.01}$ was not violated during the last 300 days.

Manager 10: Re-evaluates the allocations monthly (each 21 days) and rebalances if for any asset its updated allocation differs from its current allocation by 5% or more.

Manager 11: Re-evaluates the allocations monthly (each 21 days) and rebalances if for any asset its updated allocation differs from its current allocation by 10% or more.

Manager 12: Re-evaluates the allocations every semester (each 126 days) and rebalances if for any asset its updated allocation differs from its current allocation by 10% or more.

In the analysis of Section 4 we investigate the performance of the managers when monitoring a minimum risk and a fixed target portfolio. The goal is to find the best rebalancing strategy which would provide the higher accumulated gain along a testing period, while maintaining the main characteristic of each investment, namely, to possess minimum risk, and to attain some target return, respectively. We assume that the assets chosen to form the portfolio will be available during the validation period, and that they will not be exchanged.

At each portfolio update, managers 6, 7, 8, and 9 also update the corresponding risk measure. The $\text{VaR}_{0.01}$ is computed as the 0.01-quantile of the empirical distribution of the portfolio. The Drawdown is computed as the sum of losses occurring in a sequence of negative daily returns. In other words, the drawdown is defined as the percentual accumulated loss due to a sequence of drops in the price of an investment (Grossman and Zhou (1993); Chekhlov et al. (2003)). It is a flexible risk measure collected over non-fixed time intervals and provides a different perception of the risk and price flow of this investment. Thus, for managers 6, 7 and 8 there is no periodicity in his/her behavior.

Rule followed by Manager 9 was somehow suggested by the data analyzed in Section 4, which presented a period of high turbulence at the end of the estimation period, yielding more extreme risk measures. So the rule tried to detect if the economy has changed either back to a less volatile period, or even increasing to a more volatile one. We note that the type I error probability in both cases are approximately 6% and 5%.

Managers 10, 11, and 12 follow rules which are popular among private investors and also managers in financial institutions.

4 Empirical Analyzes

As stated in the Introduction, our objective in this empirical investigation is two-fold. For a given data set, firstly we want to compare the performance of the robust and classical portfolios (as defined in Section 2). Secondly, we want to assess the performance of the twelve managers, finding the best rule for each portfolio type.

The daily returns were computed from 10 years of transaction prices on the six most traded Brazilian stocks "Vale do Rio Doce", "Petrobras", "Usiminas", "Banco do Brasil", "Eletrobras" and "Tim" provided by BOVESPA. We split the data in two periods, one for estimation (in-sample 8-years period from June/22/2001 through June/19/2009) and the 2-years out-of-sample period from June/20/2009 through June/24/2011.

We select two portfolio types for the investigations: the Minimum Risk, and a Fixed Target. Since the choice of the target value is subjective, we decided to select a portfolio located approximately at one third along the efficient frontier, for which all component assets typically had a positive weight contribution. Actually, this target portfolio could represent a popular choice for an investment in emerging markets, due to its low risk when compared to the Maximum Risk portfolio, and to its interesting daily target return of 0.1040% corresponding to an annualized log-return of almost 30%. We note that the in-sample average daily return for the six stocks are respectively $(0.1164, 0.0935, 0.1600, 0.1159, 0.0442, -0.0097)$.

The portfolios computed at the end of the estimation period will be called *baseline* portfolios. To obtain the *classical* portfolios' allocations we compute the mean-variance inputs using the classical sample estimates. To obtain the allocations of the *robust* portfolios we carry on the two-steps estimation method, as follows.

Initially the unconditional distributions of each series is estimated. We fit by maximum likelihood a skew-t distribution (Hansen, 1994) to the six 8-years series of log-returns, and from the estimated c.d.f.s we obtain the pseudo uniform$(0, 1)$ data. Estimates are not shown here, but available to the reader by request. The marginal fits were carefully checked since a poor fit would result in the probability integral transformed values not being standard uniform or *i.i.d.*. As a consequence, any copula model would be misspecified.

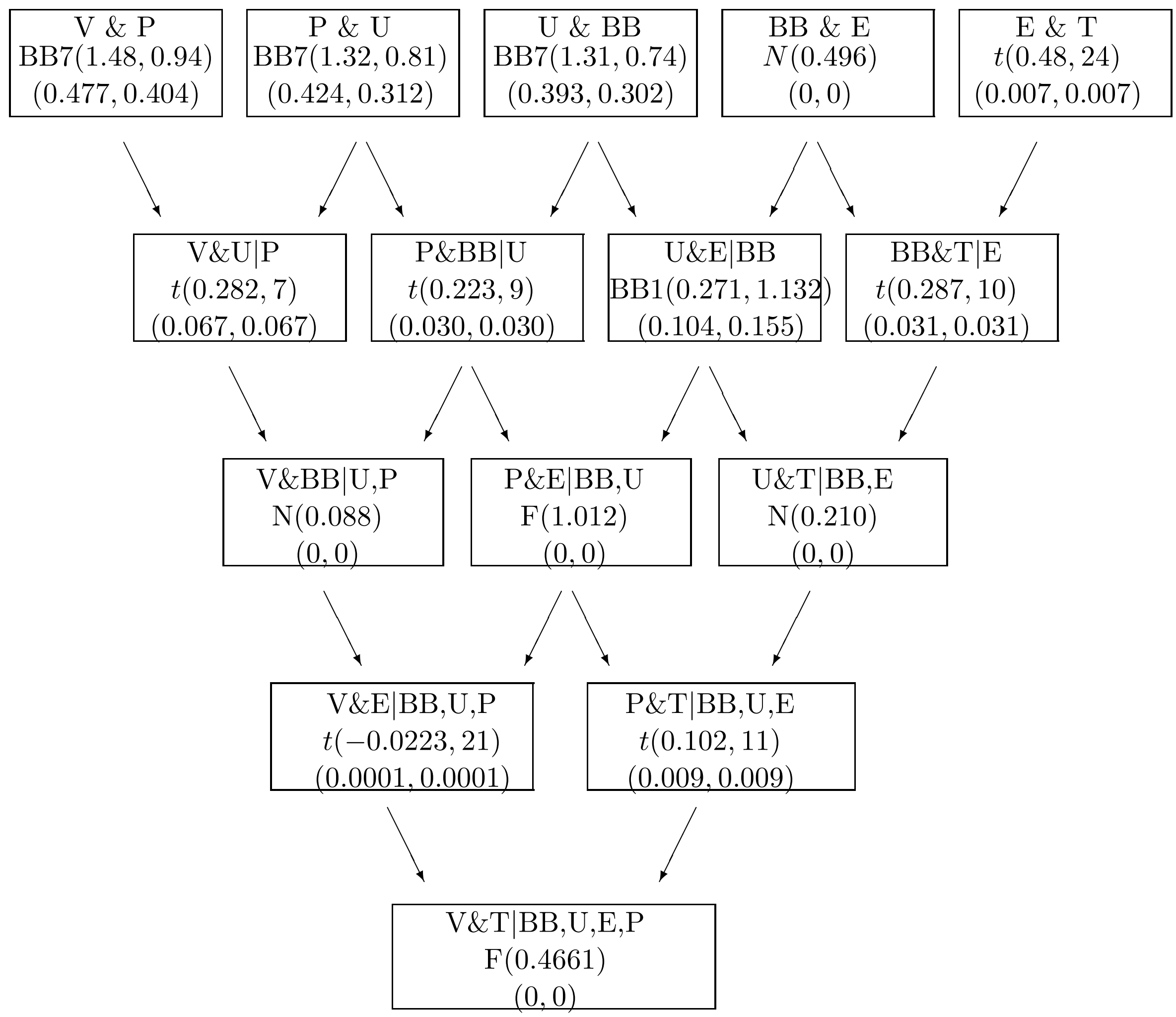

Exhibit 1: D-vine decomposition. Best copula fits and their parameters estimates. The third row inside the boxes gives the tail dependence coefficients (λ_L, λ_U). Notation in figure: Copulas: N: Normal; t: the t-copula; F: Frank. Data: V: Vale; P: Petrobras; U: Usiminas; BB: Banco do Brasil; E: Eletrobras; T: Tim.

Five unconditional and ten conditional bivariate copulas compose the D-vine. The parametric copula families considered included the Normal, t-student, BB7, BB1, Clayton, Gumbel, Tawn, Frank, and the product copula. These copula families cover all possible combinations of values for the lower and upper tail dependence coefficients. In addition, asymmetric dependence may be modeled by the Tawn copula, a non-exchangeable dependence structure. To find the best copula fit, we compared the value of the penalized log-likelihood (AIC), examined the pp-plots based on the estimated and the empirical copula, and computed a GOF test statistic (Genest and Rémillard (2005) and Genest et al. (2007), Berg (2008)).

The D-vine decomposition, along with the robust copula fits with parameter estimates and tail dependence coefficients (λ_L and λ_U), are shown in Exhibit 1. Pairs in the first level are those possessing higher correlations. The robust fits reflect the pattern of the majority of days and are expected to provide better inputs for the mean-variance algorithm and therefore more consistent long-run portfolios.

Likewise in Mendes et al. (2010), we compute the rank correlations provided by the pair-copula decomposition. The pair-copula-based robust estimates along with the skew-t location and scale estimates provide the inputs for the mean-variance algorithm. We run the long-only MV optimization algorithm and construct the classical and the robust pair-copula-based efficient frontiers.

For evaluating the performance of the classical and the robust pair-copula based portfolios we now assume that the four portfolios are managed by the twelve managers along the 2-years validation period. The rules are followed and portfolios are updated according to them.

Firstly, we fix the portfolio type (statistical methodology and investment objective) and find the best rebalancing strategy for this financial instrument. To this end we (a) count the percentage of time its accumulated gain under some rebalancing approach is higher or equal than the other competitors; (b) test if the mean difference between accumulated gains from every pair of managers is statistically different from zero, using a one-sided robust nonparametric t-test (Wilcoxon rank sum test) at the 1% level.

We found that for *Classical Target Portfolio*, blindly rebalancing each 5 days (Manager 2) leads to consistently significantly lower accumulated gains. Keeping the allocations unchanged for two years (Manager 1) is the second worst thing one can do (wins over only Manager 2). For the *Classical Target Portfolio* the best strategy comes from Manager 10, which inspects the allocations each 21 days (every month) and rebalances only if any allocation has changed by 5%. The proportions of times Manager 10 portfolio accumulated gains is higher than the others is typically around 80%, and highly statistically significant, see Figure 1. Figure 1 shows the differences between the accumulated gain from Manager 10 and all other managers. For this data set the resulting portfolio coincides with the one from Manager 3 (which blindly rebalance each month).

For the *Robust Target Portfolio* the worst thing one can do is to blindly frequently rebalance the portfolio each 5 days (Manager 2). This is in line with its long run investment characteristic implied by the robust methodology. However, since the economy changes and there is a target to follow, some type of re-allocations are needed, and the best manager

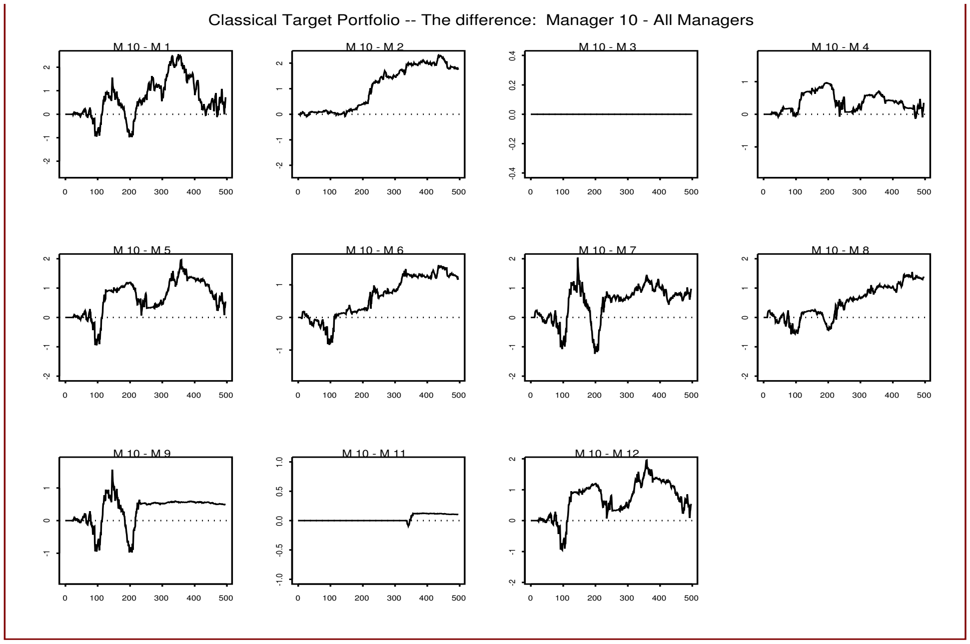

Figure 1: *For the Classical Target portfolio figures show the differences between the accumulated gains under Manager 10 and all other managers.*

for the *Robust Target Portfolio* turned out to be Manager 9, which strategy is based on the VaR value. Even though he/she inspects monthly, only 14 investment updates were done (in contrast, for the Classical Target portfolio, the winner Manager 10 has rebalanced 23 times). Figure 2 shows the differences between the accumulated gain from Manager 9 and all other managers for this portfolio.

On the other hand, when it comes to *Classical Minimum Risk Portfolios*, keeping the allocations unchanged for two years (Manager 1) apparently came out as the best rebalancing strategy. It lead to significant consistently higher accumulated gains when compared to the gains from the remaining strategies. However, this result should be looked at with care since the baseline allocations might not result in an efficient minimum risk anymore. Figure 3 shows the updated efficient frontier after the two-years validation period, and the position of the baseline classical minimum risk portfolio in the risk $\times$ return space.

Thus rebalancing is needed to keep the investment characteristic during its life time. Manager 11 (which monitors the portfolio monthly with a 10% threshold) provided the best maintenance strategy, see Figure 4. The second best manager is number 9 which uses the VaR measure. From Figure 4 it is clear the superiority of these two strategies.

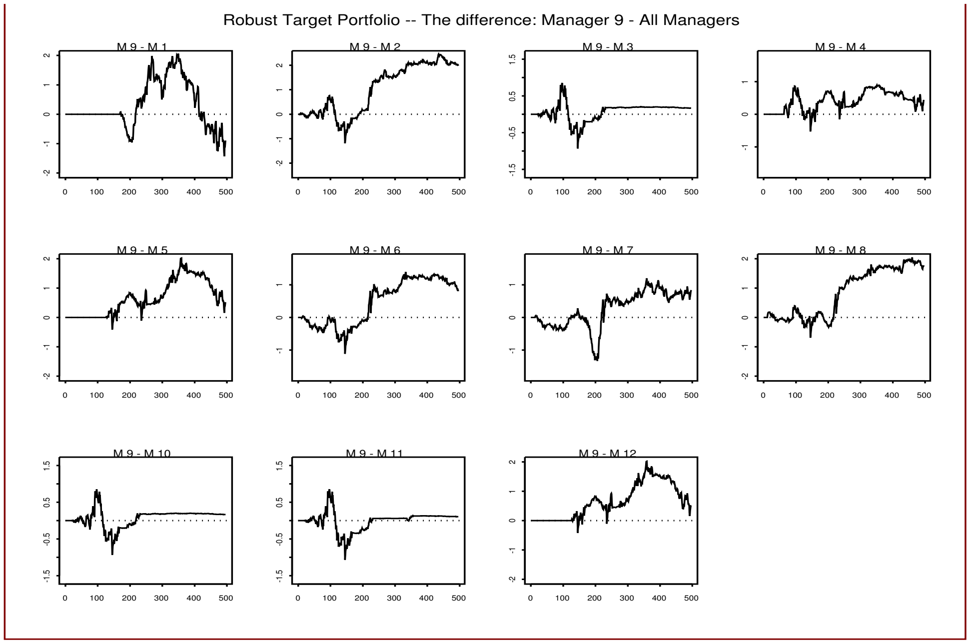

Figure 2: *For the Robust Target portfolio figures show the differences between the accumulated gains under Manager 9 and all other managers.*

Best strategy for rebalancing the Robust Minimum Risk portfolios came from Manager 12, whose strategy is to re-evaluate the allocations each 6 months and rebalance whenever allocations change by 10% (see Figure 5). This is also in line with the long run characteristic of robust procedures. We note that Manager 12 just rebalanced once and outperformed Manager 1 which kept the allocations unchanged for two years. Interestingly, the second best manager is Manager 9, which controls the Value-at-Risk of the portfolio.

Secondly, we compare the performances of the same type of classical and robust investments driven by their best managers. The upper plot in Figure 6 shows the differences between the accumulated gains from the Robust Target & Manager 9 portfolio and the Classical Target & Manager 10 portfolio. We observe that even though the target is the same the robustly estimated pair-copulas based portfolios outperform the classical version. The classical version was updated 23 times whereas the robust one only 14 times. The lower plot in Figure 6 shows the outstanding superiority of the Robust Minimum Risk investment & Manager 12 (rebalanced once) over the classical version & Manager 11 (rebalanced 9 times).

Following a suggestion from a referee, we now investigate the performance of the robustly estimated pair-copulas method combined with the proposed managers for different

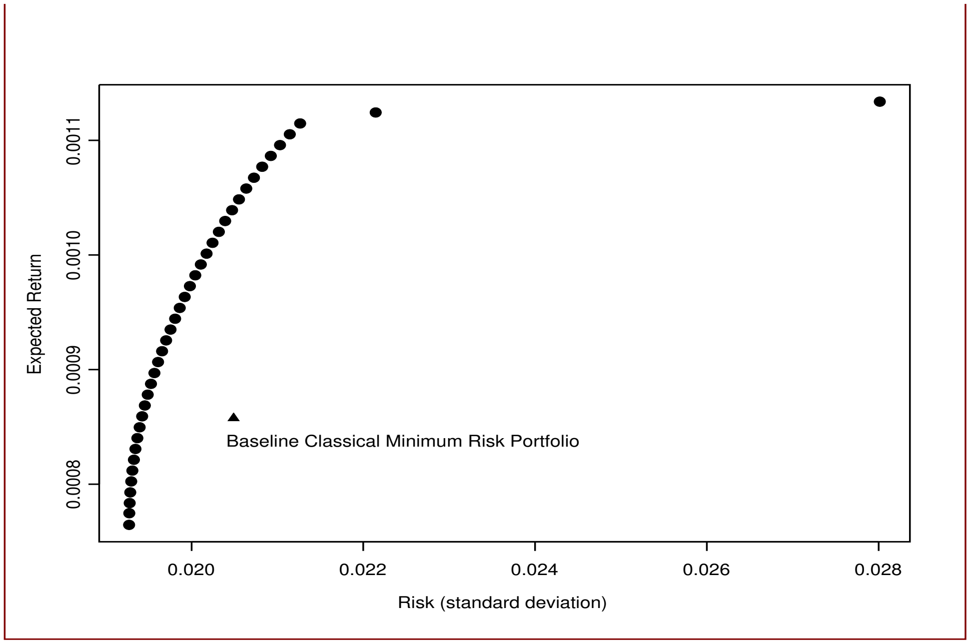

Figure 3: *Updated efficient frontier and baseline Classical Minimum Risk portfolio.*

financial instruments in the same country. Two stocks (Eletrobras and Tim) were replaced by the series IMA-C and IMA-S, respectively a long-term inflation-indexed Brazilian treasury bonds index and a floating rate Brazilian Treasury bill index, both computed by the Brazilian Association of Financial Institutions, ANBIMA (`www.anbima.com.br`). The IMA-S consists of the price changes of Letras Financeiras do Tesouro (LFT), which are zero-coupon shorter term securities whose interest rate is compounded daily using the average treasury repo market rates computed by the Brazilian Central Bank (the SELIC rate) and that is paid according to this accruing in one single payment at the maturity date.

We again split the data in two parts, a in-sample 8-years period for data estimation and the 2-years out-of-sample period for models validation. The Fixed Target portfolio now has a daily target return of 0.17%. The new series present positive in-sample average daily return, respectively, 0.0786 and 0.0617. Their main characteristic is their much smaller sample standard deviations, respectively, 0.204 and 0.054, when compared to those computed for the remaining four stocks, respectively, 2.542, 2.452, 3.185 and 3.033. The same estimation steps were followed and optimal portfolios' weights computed. The difference now between the classical and the copula based efficient frontiers is not so dramatic, but the robust-pc based curve is still located above and at the left of the

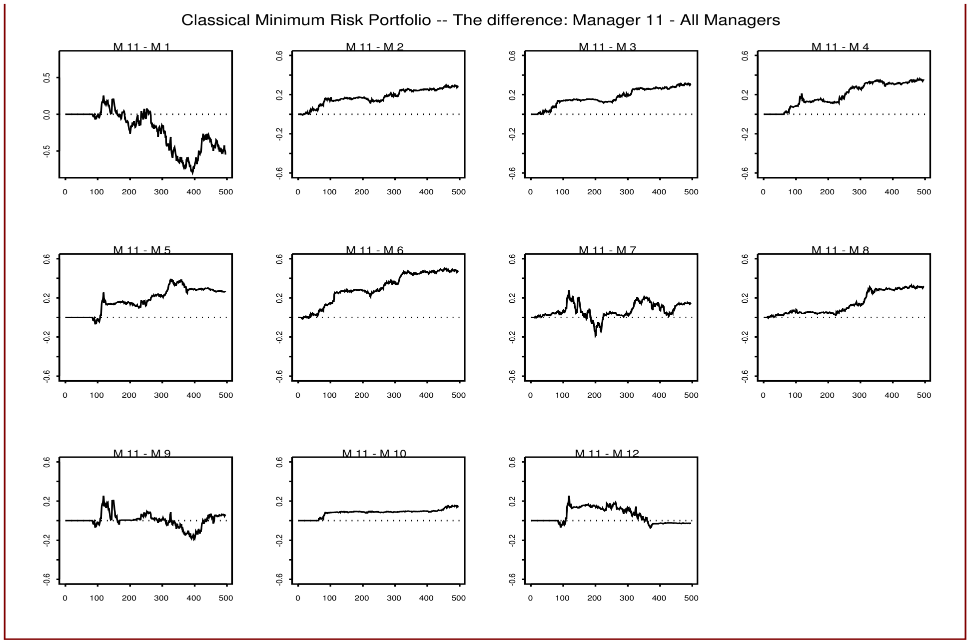

Figure 4: *For the Classical Minimum risk portfolio figures show the differences between the accumulated gains under Manager 11 and all other managers.*

classical version. In particular, the weights defining the Minimum Risk portfolios are very close, both methods allocating around 95% on the IMA-S, due to the obvious reason. The target portfolios had almost all weight allocated to three components, the Vale and Usiminas stocks and the IMA-C. We repeated the same strategies for evaluating and comparing the performance of classical and pc-based portfolios.

We found that the best rebalancing strategy for the *Classical Target Portfolio* came from Manager 6 which updates the optimal weights whenever three consecutive losses are observed. The proportions of times Manager 6 Classical Target Portfolio accumulated gains is higher than all others is around 84%, highly statistically significant.

For the *Robust Target Portfolio* the best manager turned out to be Manager 7, which strategy is to update whenever there is a sequence of 6 negative returns. Only one update was made, whereas for the Classical Target portfolio, the winner Manager 6 has rebalanced 19 times. For the *Robust Target Portfolio* he second best performance was provided by Manager 4 which inspected at the end of 63 business days, having carried out seven investment updates.

On the other hand, the *Classical Minimum Risk Portfolio* driven by Manager 12 provided higher accumulated gains than all others during time percentages varying between

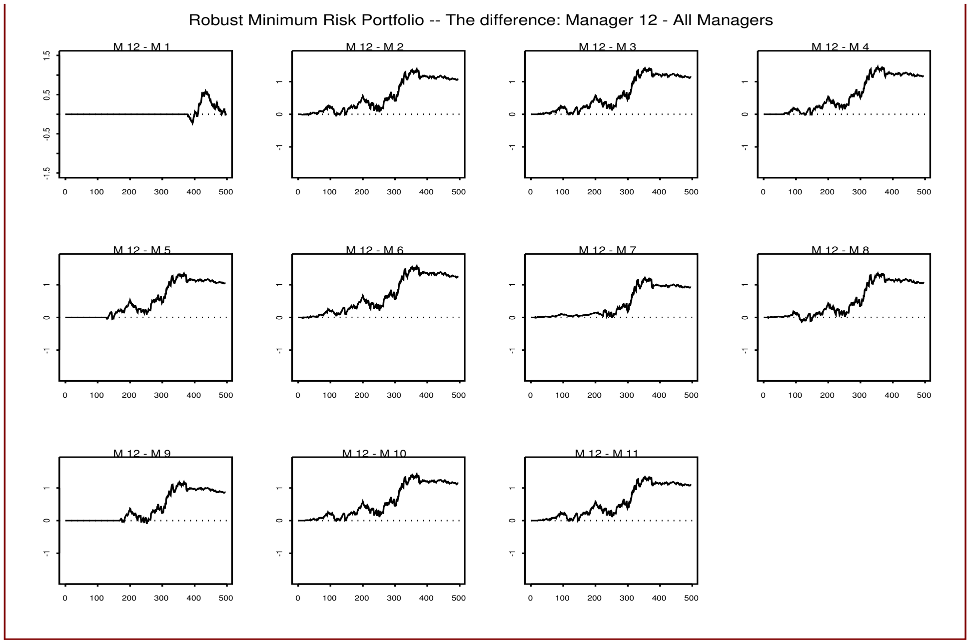

Figure 5: *For the Robust Minimum Risk portfolio figures show the differences between the accumulated gains under Manager 12 and all other managers.*

67% and 97%. This manager has inspected the allocations every semester using a fluctuation margin of 10%, but has performed only one update. For the minimum risk portfolio we observed that the weights distribution along the validation period was very stable, explaining the second place been occupied by three managers which performed zero updates, namely managers 1, 7, and 8. We note that the minimum risk portfolio has allocated approximately 95% weight to the low risk IMA-S.

Best strategy for rebalancing the *Robust Minimum Risk Portfolio* came from Manager 10 , which re-evaluates the allocations monthly and rebalances if the updated allocation changes by 5% or more for any asset. The number of updates carried out was 9.

Finally, we compare the performances of the same type of classical and robust investments driven by their best managers. Likewise Figure 6, Figure 7 shows the differences between the accumulated gains from the Robust Target & Manager 7 portfolio and the Classical Target & Manager 6 portfolio (upper row). In the lower row, Figure 7 shows the differences between the accumulated gains from Robust Minimum Risk investment & Manager 10 and the classical version & Manager 12.

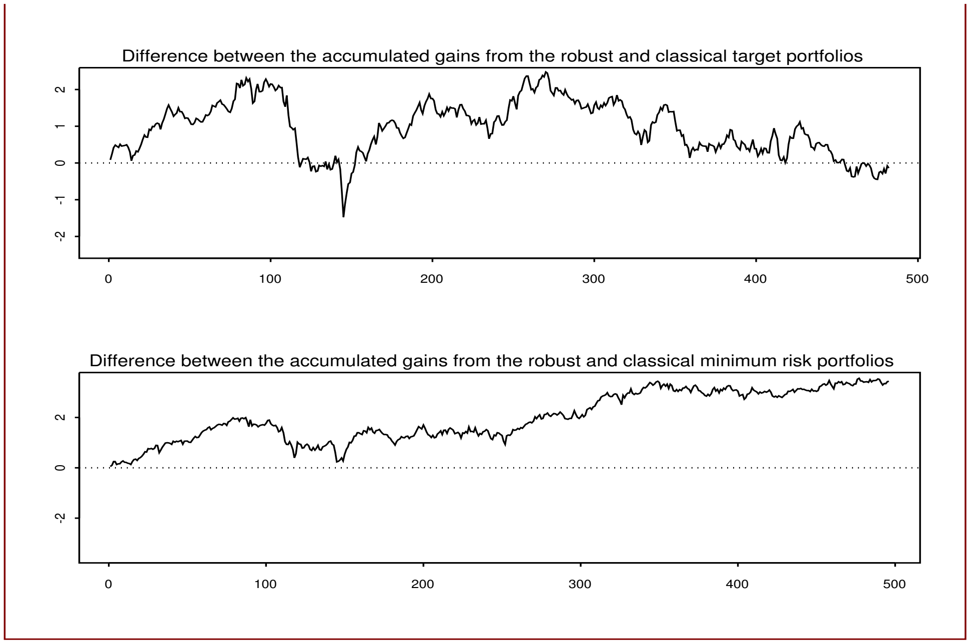

Figure 6: *Difference between the accumulated gains from the best robust and best classical portfolios.*

5 Discussions

As usual, the conclusions drawn in this paper only apply to data possessing similar characteristics. However, the results and discussions may shed some light on the largely discussed topic of portfolio allocation and rebalancing strategies, and may be easily tested and extended to other investments based on different asset characteristics (expected return, volatility and correlations). More importantly, this work has shown that the robustly estimated pair-copulas based mean-variance inputs are more accurate thus increasing the chances of producing a financial instrument which will truthfully yield what was expected from it.

From the analyzes carried on it was clear the superiority of the robust portfolios. But even a robust portfolio must be properly managed. According to the empirical analysis of the first data set composed only by daily stock returns, for the Robust Minimum Risk portfolio the second best manager with respect to higher accumulated gains is Manager 9 (which uses the VaR). However, Manager 9 has done 9 rebalancings, whereas Manager 12 carried on only one after one year and a half. One may be tempted to conclude that the good performance of the winner may be just due to the changes in the economy which

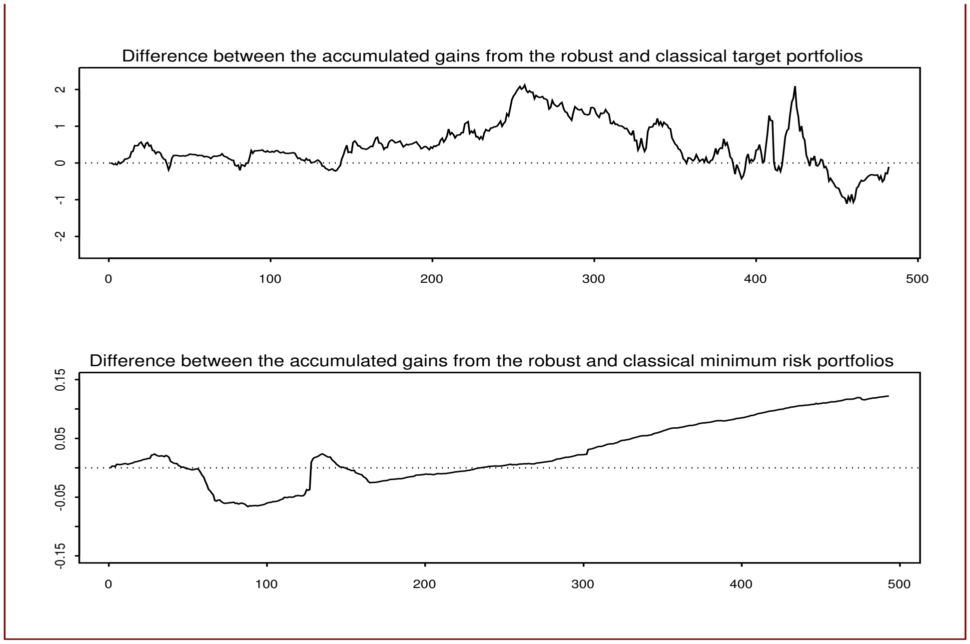

Figure 7: *For the new data set, the difference between the accumulated gains from the best robust and best classical portfolios.*

may have taken the portfolio away from the efficient frontier. It was *not* the case. Figure 8 illustrates and shows the positions of the robust minimum risk portfolios from both managers (based on weights from last rebalancing and using the entire validation period data) along with the updated efficient frontier. We observe that the portfolios are very close and still close to the curve, with the point risk×return from Manager 9 (triangle in pink) even a little bit higher. The success of this procedure may be credited to the timing of Manager 12 combined with the good stability of the robust pair-copulas method of portfolio construction. Actually, for the second data set used based on less volatile assets, Manager 12 was also the best option for the Classical Minimum Risk portfolio.

Manager 9 was also the second best option for the Classical Minimum Risk portfolio, and the best one for the Robust Target. Thus, one may say that controlling the Value-at-Risk is also an efficient strategy. Another issue is whether or not the target portfolio should be restored to the same target. We did not address this problem, but this consideration will certainly not change the result towards the excellent performance of the robust method.

Many other managers' rules could be defined. Another strategy that seems promising is a variation of the rules followed by managers 10, or 11, or 12, where the threshold determining the need for rebalancing would not be fixed for all assets. Instead, it would

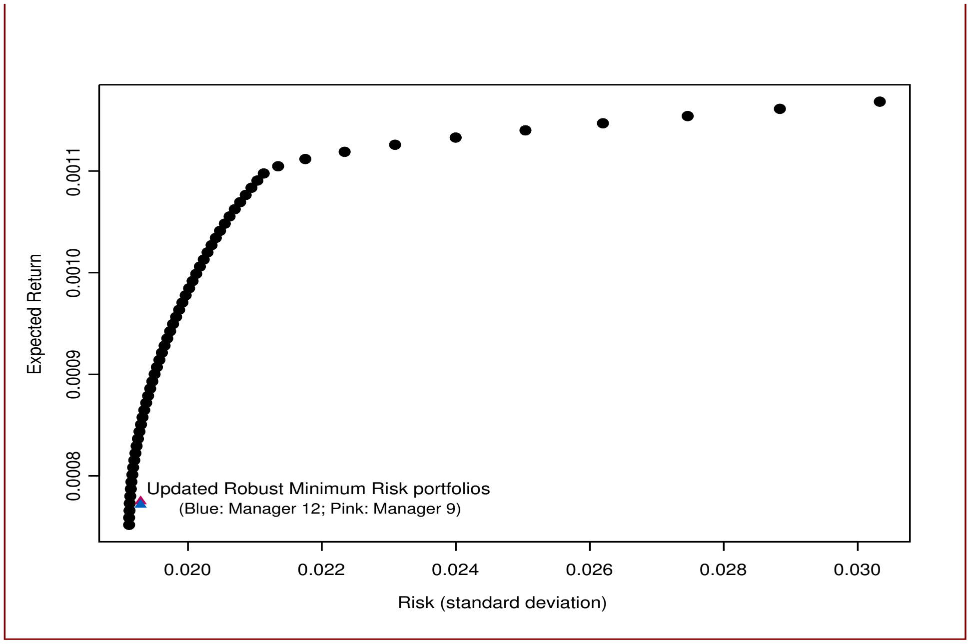

Figure 8: *Updated efficient frontier and updated robust minimum risk portfolios from Manager 12 and Manager 9.*

vary according to each asset weight or importance in the composition. In addition, it deserves further investigation the actual value of the threshold. Other values beyond the assumed 5% and 10% may lead to better performances.

Another important result drawn from this empirical analysis is concerned with costs. We found that despite portfolio type, robust portfolios typically demand a smaller number of updates lowering costs.

Finally, we found that despite the rebalancing rule, the robust portfolios always outperform the classical versions. Figure 9 shows the differences between the accumulated gains from the robust and the calssical Target portfolios, having fixed the managing rule. Given the same manager and the same portfolio target, the robust method is always superior to the classical method. This is also true for the Minimum Risk portfolios, and the outstanding performance of the Robust Minimum Risk portfolio for all fixed managers can be seen in Figure 10.

[Figure 10 around here]

In summary, observing that the dependence between assets go beyond the linear cor-

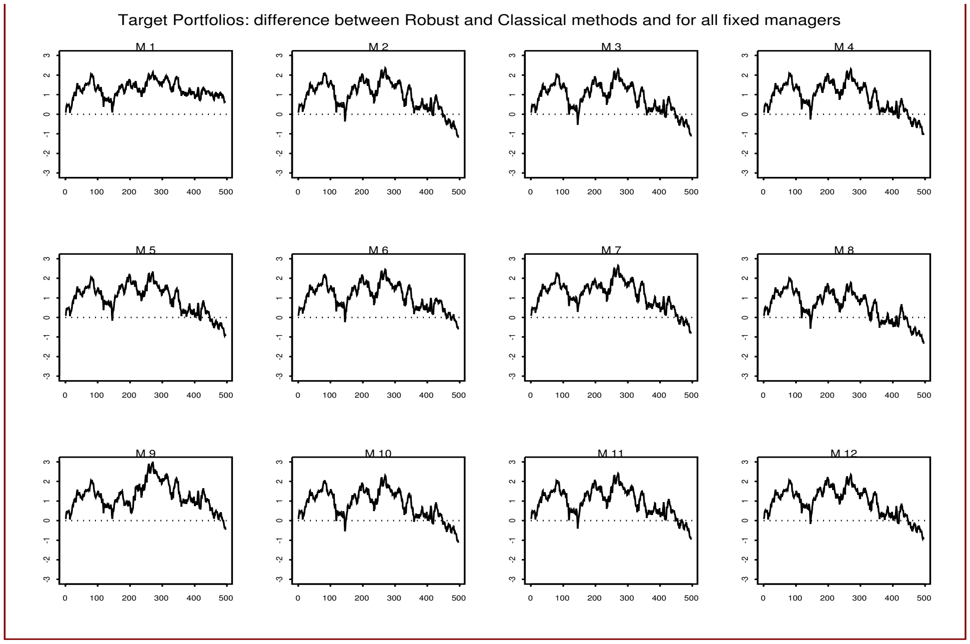

Figure 9: *For the Target portfolio figure shows the differences between the robust and the classical accumulated gains keeping fixed the manager.*

Figure 10: *For the Minimum Risk portfolio figure shows the differences between the robust and the classical accumulated gains keeping fixed the manager.*

relation, in this paper we proposed modeling log-returns data using robustly estimated pair-copula models. The method is appealing simple and able to handle contaminations that may occur when working with financial data. We illustrated the idea in the context of emerging stock markets using two data sets composed by the most liquid Brazilian stocks, a long-term inflation-indexed Brazilian treasury bonds index and a floating rate Brazilian Treasury bill index. The empirical analyzes carried on in this paper indicated that for any type of portfolio we are able to find the best manager strategy. Moreover, they indicated that the robustly estimated pair-copulas based portfolios always outperform the classical versions despite the managing strategy.

Methodology seems promising for any risk profile investor. The interested reader (or investor) may easily tailor these ideas to his/her needs, repeating these exercises using other data sets and considering other investor risk aversion levels, and even defining new rules for managing. We are very confident that it will always be a combination of manager and a robust portfolio outperforming its classical version. The authors will be happy to compute the robust estimates and check the performance of any portfolio for any data set the reader shall have. Future research may include the investigation of different managers' rules for other types of portfolios.

Indeed, we have a final recommendation. If the objective is a minimum risk long run investment the best one can do is to allocate the assets using robustly estimated pair-copulas estimates and inspect the portfolio each 6 months, rebalancing whenever allocations change by 10%. This will guarantee the investment characteristics and provide the cheapest managing strategy.

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