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HUMAN CAPITAL UNDER CONDITIONS OF UNCERTAINTY: THEORY AND EMPIRICAL EVIDENCE

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I. INTRODUCTION

This paper will examine equilibrium in labor markets under conditions of uncertainty. The economic setting is assumed to allow individuals to make marketable and non-marketable human capital investments. An individual may hold a portfolio of marketable capital assets and his human capital asset which is generally referred to as his "occupation". Several occupations are assumed to exist in the economy and throughout this work an occupation is viewed as a non-marketable (occupational) asset which pays an uncertain amount at the end of the relevant investment period. It is assumed that, at the beginning of the investment period, an investor can acquire marketable assets in unrestricted amounts but he must select a non-negative amount of only one non-marketable occupational asset. The investors in the economy are assumed

(i) to maximize expected utility of terminal wealth in the context of a one-period model,

(ii) to have utility functions that are concave and monotonically increasing in wealth,

(iii) to have homogeneous expectations of returns on marketable and occupational assets which can be represented by a multivariate normal distribution with a positive-definite variance-covariance matrix of returns on risky assets (marketable and occupational),

(iv) to have utility functions that induce convergent and well defined expected utility integrals under the homogeneous multivariate normal distribution of returns,

(v) to be able to freely borrow and lend at the riskless rate offered by a marketable and riskless asset,

(vi) to be able to short-sell any marketable asset and

(vii) to face perfect and competitive capital and labor markets.

Under these assumptions the decisions of any investor can be examined in the mean-variance of terminal value space with a strictly convex indifference map. A theory of human capital under uncertainty that explicitly considers the existence of marketable and risky assets in the economy will be derived in the mean-variance space, under the above assumptions. The theory allows occupational assets to be non-homogeneous differing in their risk and return characteristics. The equilibrium conditions implied by the theory are subsequently tested empirically and it is shown that one cannot reject the hypothesis that human capital markets are in equilibrium under conditions of risk.

II. HUMAN CAPITAL THEORY: FROM CONDITIONS OF CERTAINTY TO CONDITIONS OF RISK

In a world of certainty with perfect capital markets any individual considering an investment in human capital should undertake the investment if:

(a) he is certain that his future earnings will be identical to the average earnings - as of the decision time - of older people who have invested in human capital in the past as contemplated;

(b) the present value of the incremental earnings net of the present value of costs is greater than zero - both present values being obtained at the riskfree rate r_F.[1]

Sometimes equivalently, it could be argued that the investment should be undertaken if the internal rate of return of the stream of incremental earnings less costs is greater than r_F.[2] In such a world of certainty with free entry, the equilibrium stock values (i.e., the present value of the stream of earnings less costs) of occupations should be identical. Human capital economists generally find that stock values of occupations differ[3]. They proceed in the direction of rejecting the hypothesis of equilibrium across occupational assets in the context of perfect markets with free entry, arguing for possible market imperfections that would explain the findings[4].

However, an alternative interpretation is possible. The findings may suggest that the fundamentals of the theory are inadequate. Returns to investments in human capital are uncertain at the time investment decisions are made. In general this uncertainty must be explicitly considered when examining equilibrium across occupational assets. Under certain assumptions, the acceptance criterion and the results derived under conditions of certainty will remain valid in a world of uncertainty. These conditions are:

(i) capital markets are perfect and individuals can freely borrow and lend at the rate r_F,

(ii) the best unbiased estimate of the distribution of increments in future earnings associated with the contemplated investment in human capital is obtained from the age-earnings profiles as of the decision time t of individuals who have invested in human capital (as contemplated) and

(iii) investors are risk-neutral and, accordingly, are indifferent to the risk of deviations of actual future earnings from expected future earnings.

The validity of the findings obtained under the assumption of certainty in a world of uncertainty should be evaluated through the adequacy of these assumptions. Unfortunately assumptions (ii) and (iii) do not appear to be appropriate. Assumption (ii) implies that past earnings profiles do not contain any information which could be relevant for human capital investment decisions. This certainly does not appear to be a reasonable assumption since the variability of average profiles over time should convey information about occupational risk. Moreover, assumption (iii) does not appear to be appropriate. There is extensive evidence in economics and in other behavioral sciences that investors are risk-averse and the mere existence of insurance companies is evidence of risk aversion. The inadequacy of the expected present value criterion under conditions of risk should be obvious and occupational risk must be explicitly considered when examining equilibrium across occupational assets in an uncertain world.

The extension of human capital theory to conditions of uncertainty would appear to be relevant. As it will be shown later, once uncertainty is introduced, it can explain why

- different occupations may offer different distributions of rates of return,
- distributions of occupational rates of return may be in equilibrium after adjustment for risk,

- wealth should be a relevant factor for occupation selection,
- personal preferences represented by utility functions should be relevant for occupation selection.

Under the assumptions of this paper[5] every investor will examine his marketable and human capital investment decisions in the mean-variance of terminal wealth space[6]. Any asset will offer a random value at the end of the relevant investment period and its variance is defined as the risk of the asset. Accordingly, the occupation of the investor can be viewed as an asset, his occupational asset, offering a random terminal value (earnings), its variance being the risk of the occupational asset. Without any loss of generality the random terminal earnings of an occupation can be decomposed into two components:

$$\tilde{O} = \tilde{A}_o + \tilde{I}_o \text{ with } E(\tilde{I}_o) = 0$$

where

$\tilde{O}$ = random terminal earnings of occupation O,

$\tilde{A}_o$ = random average terminal earnings of all individuals with the occupation O,

$\tilde{I}_o$ = random deviation of the terminal earnings of an individual with occupation O from the average terminal earnings of all individuals with occupation O.

At the beginning of the period an individual with occupation O is uncertain about the performance of the economy and of his occupation within the economy during the period, i.e., he faces average risk that can be defined as Var $(\tilde{A}_o)$. Moreover, he is uncertain about his relative intra-occupation position, i.e., he faces individual risk which can be defined as Var $(\tilde{I}_o)$. Since the relative performance of the individual within his occupation class should be independent of the average performance of the occupation, it follows that

$$\text{Var}(\tilde{O}) = \text{Var}(\tilde{A}_o) + \text{Var}(\tilde{I}_o)$$

The risk of an occupational asset is the sum of its average risk and its individual risk.

In a world of risk-averse individuals, expected random terminal earnings of the occupation 0 will contain a premium associated with occupation risk. However, society will be willing to pay a risk premium to the holders of occupational asset 0 only if some of its risk cannot be diversified away at a social level. Individuals should be rewarded only for carrying non-diversifiable occupational risk and the diversifiable component of occupational risk should not command a premium[7]. By combining two risky assets adequately, risk may be diversified. It follows that combinations of non-marketable and risky occupational assets with marketable and risky assets (stocks) may diversify occupational risk. The diversification potential of capital markets may thus be relevant for examining equilibrium in labor markets.

The relevance of risk and capital markets for equilibrium in labor markets has been observed before. Weiss [21], Hause [9] and Levhari and Weiss [12] have considered the effect of risk on human capital investments. However, they consider only individual risk and they fall short of deriving equilibrium results and of producing empirical evidence of equilibrium across occupational assets. Mayers [15] observed the relevance of the diversification potential of capital markets and derived equilibrium conditions across wages of different occupations. He assumes that individuals can hold portfolios of different occupational assets and allows individuals to be net short sellers of occupational assets (and of time). The first assumption poses obvious moral hazard problems and the reasonableness of the second assumption can be questioned. Unfortunately he also falls short of examining equilibrium empirically.

III. HUMAN CAPITAL THEORY: EQUILIBRIUM UNDER CONDITIONS OF UNCERTAINTY

In the economic setting of this paper there is a set of occupations from which every investor must choose one and only one occupation, of which he must hold a non-negative fraction. The non-negative fractional holdings of occupational assets can be associated with quality levels of occupational training offered in the economy, i.e., within the same occupation investors may ellect to go to universities (schools) of different quality. A high-quality university could be viewed as offering a fraction of the occupation greater than the fraction offered by a low-quality university.

Let $\{0_j;\ j=1,n\}$ denote the set of occupations in the economy. As discussed in Appendix I, the mean-standard deviation efficient frontier of total holdings (occupational plus marketable holdings) of an investor who selects the non-negative fraction α of the j-th occupational asset is the upper segment of a hyperbola with center on the mean axis and with its principal axis parallel to the standard deviation axis. The (slope of the) assymptote of the hyperbola is equal to the maximum reward-to-variability ratio (RV^*) offered by marketable and risky assets[8] and thus is determined exogenously of $\{0_j;\ j=1,n\}$ and is identical for every occupational asset. These results imply that, given the wealth level of the investor, his hyperbolic efficient frontier is completely determined by the feasible total combination ($\alpha\ 0_j$ plus marketable holdings) that offers the minimum variance. Moreover, changes in his wealth will shift all of his efficient frontier vertically by a fixed amount which depends only upon the riskless rate of interest in the economy and the change in wealth[9].

Define

$\tilde{0}_j$ = random terminal earnings (value) of the j-th occupational asset (0_j),

$\tilde{\theta}_j$ = random terminal value of the non-random portfolio θ_j of marketable and risky assets which diversifies away as much of the risk of the j-th occupation as possible in existing capital markets,

P_j^{θ} = the price of θ_j,

c_j = costs which must be incurred to obtain the j-th occupation,

r_F = riskless rate of interest in the economy,

r = $1 + r_F$

$\tilde{MV}_j(\alpha)$ = minimum variance total combination of an investor with null marketable wealth who holds the non-negative fraction α of the j-th occupation,

$\tilde{MV}_j$ = $\tilde{MV}_j\ (1) = \tilde{O}_j + \tilde{\theta}_j - (P_j^{\theta} + c_j)r$,

$\tilde{O}_j^*$ = $\tilde{O}_j + \tilde{\theta}_j$

and recall that, from relation (A.1) of Appendix I,

$$\tilde{MV}_j\ (\alpha) = \alpha\tilde{MV}_j\ . \tag{1}$$

The properties of the total efficient frontier imply that the occupation selection problem of an investor can be reduced to the comparison of the loci of representations of $\{\tilde{MV}_j\ (\alpha)$, non-negative $\alpha\}$, for all j, in the mean-standard deviation plane. It follows from equation (1) that the representation of $\{\tilde{MV}_j\ (\alpha)$, non-negative $\alpha\}$ is linear and goes through the origin in the mean-standard deviation plane, for all j. The linear loci of any two different occupations can only cross at the origin and this implies the following:

<u>Theorem</u>: In equilibrium the linear representations of $\{\tilde{MV}_j\ (\alpha)$, non-negative $\alpha\}$ and of $\{\tilde{MV}_k(\alpha)$, non-negative $\alpha\}$ must coincide, for all j and k.

Proof:

Assume that there exists j and k such that the linear loci do not coincide. Without any loss of generality assume that the locus associated with j is above the locus associated with k in the sense that for all strictly positive α_k there exists α_j such that $\sigma\left[\tilde{MV}_j\ (\alpha_j)\right] =$

$\sigma\left[\tilde{MV}_k(\alpha_k)\right]$ and $E\left[\tilde{MV}_j(\alpha_j)\right] > \left[E\,\tilde{MV}_k(\alpha_k)\right]$. Now assume that in equilibrium an investor decides to acquire a strictly positive fraction α_k of O_k. There will exist a positive α_j satisfying the above conditions and for every point on the efficient frontier associated with $\alpha_k O_k$ there exists a point on the efficient frontier associated with $\alpha_j O_j$ offering the same standard deviation and a higher expected terminal value. This is inconsistent with equilibrium and by contradiction the theorem is established.

QED

Recalling that the linear loci go through the origin, the theorem implies that in equilibrium

$$\frac{E(\tilde{MV}_1)}{\sigma(\tilde{MV}_1)} = \ldots = \frac{E(\tilde{MV}_j)}{\sigma(\tilde{MV}_j)} = \ldots = \frac{E(\tilde{MV}_n)}{\sigma(\tilde{MV}_n)} \qquad (2)$$

Using the principle of corresponding addition these equilibrium conditions can be expressed as

$$E(\tilde{MV}_j) = \gamma_T\,\sigma(\tilde{MV}_j), \text{ for all } j, \qquad (3)$$

where γ_T is the market price of non-diversifiable occupational risk in the terminal value space being equal to the ratio of the average expected terminal value of the $\tilde{MV}_j$'s and the average risk of the $\tilde{MV}_j$'s in the economy. Conditions (2) can be expressed in rate of return space. Since an investor who chooses the j-th occupation will always acquire the portfolio θ_j of marketable and risky assets[10], one could view $O_j^* = O_j + \theta_j$ as the j-th virtual occupational asset in the economy. Rates of return to virtual occupational assets can be defined as $\tilde{r}_j^* = \frac{\tilde{O}_j+\theta_j}{P_j^{\theta}+c_j} - 1$ and, from the definition of $\tilde{MV}_j$, conditions (2) can be reduced to

$$\frac{E(\tilde{r}_j^*) - r_F}{\sigma(\tilde{r}_j^*)} = \frac{E(\tilde{r}_k^*) - r_F}{\sigma(\tilde{r}_k^*)}; \text{ for all } j, k.$$

Using the principle of corresponding addition again, equilibrium conditions in the rate or return space are obtained,

$$E\,(\tilde{r}_j^*) = r_F + \gamma_R^*\,\sigma(\tilde{r}_j^*), \text{ for all } j, \tag{4}$$

where γ_R^* is the market price of non-diversifiable occupational risk in the rate of return space being equal to the ratio of the average expected rate of return to virtual occupational assets in excess of the riskless rate r_F and the average risk of the virtual occupational assets in the economy. In equilibrium, means and standard deviations of virtual occupational assets must plot along an Occupational Market Line (OML) as indicated in figure 1.

Equations (3) and (4) imply that only non-diversifiable occupational risk should be rewarded in labor markets. However, individuals cannot hold portfolios of occupational assets and thus cannot diversify occupational risk across occupational assets. It follows that the existence of an OML does not imply perfect correlation of rates of return to virtual occupational assets and, in general, allocations are not Pareto-efficient. They will be Pareto-efficient only if rates of return to virtual occupational assets are perfectly correlated. Then the social aggregate of virtual occupational assets will offer a rate of return also plotting on the OML because occupational risk cannot be diversified across occupational assets.

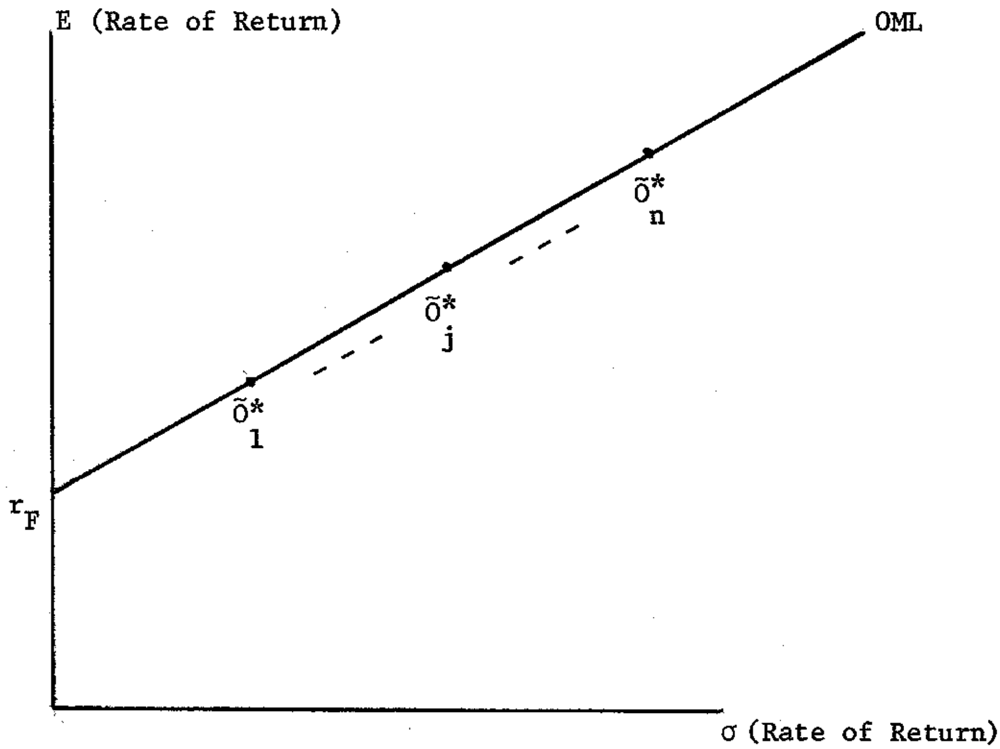

Figure 1

THE OCCUPATIONAL MARKET LINE (OML) IN RATE OF RETURN SPACE the two sets of assets is null. It would thus appear to be relevant to examine the degree and significance of the covariance between marketable and risky assets and risky occupational assets. Moreover, after testing for equilibrium across risky occupational assets using conditions (4) and (5), inferences can be drawn about the extent with which labor markets perceive the risk diversification potential offered by existing capital markets with marketable and risky capital assets.

The assumption that risky occupational assets and a marketable and riskless asset are the only assets in the economy offers the potential of explaining a few problems in human capital theory. In a general economy with marketable and risky assets and with risky occupational assets, the effect of wealth upon risky human capital investments of risk-averse individuals is unclear: it will effect both the level of risky human capital investments and the level of risky marketable investments. However, in an economy in which the only marketable asset is riskless, the effect of wealth upon risky human capital investments can be isolated and examined. If it is assumed that investors' absolute risk aversion[11] is decreasing in wealth, the linearity of conditions (5) implies that the wealthier individuals of a group with identical preferences will invest more in human capital[12]. The conditions also imply that an investor who is everywhere less risk-averse than another investor will invest more in human capital, if both have identical wealth. These results follow directly from Arrow [1] and point out to the relevance of wealth and individual preferences for human capital investments[13].

IV. A SPECIAL CASE: RETURNS TO MARKETABLE AND RISKY ASSETS ARE NOT CORRELATED WITH RETURNS TO RISKY OCCUPATIONAL ASSETS

Corrective portfolios are null whenever returns to marketable and risky assets are not correlated with returns to risky occupational assets. This result follows from the definition of corrective portfolios in Appendix I. In this case $O_j^* = O_j$ and if the rate of return on the j-th occupational asset is defined as $\tilde{r}_j = \tilde{O}_j / c_j - 1$, the equilibrium conditions (4) are reduced to

$$E\,(\tilde{r}_j) = r_F + \gamma_R\, \sigma(\tilde{r}_j), \text{ for all } j, \tag{5}$$

where γ_R is the market price of occupational risk being equal to the ratio of the average expected rate of return to occupational assets in excess of the riskless rate and the average occupational risk.

The previous work in human capital theory has ignored the existence of marketable and risky assets in the economy. Following Friedman and Kuznets [6], and Becker [2], it is generally assumed that only non-marketable occupational assets and a marketable and riskless asset exist in the economy. Investors must choose an occupation but they may borrow or lend at the riskless rate. Under conditions of risk, there would exist risky occupational assets and a marketable and riskless asset in such an economy. Equilibrium conditions in this economic setting have not yet been derived. Their derivation is the problem advanced by Levhari and Weiss [12] for future research. This economic organization is a special case of the more general zero correlation case. If marketable and risky assets are assumed not to exist, corrective portfolios are null and conditions (5) are again the equilibrium conditions. This is the solution of the problem advanced by Levhari and Weiss [12], in the mean-variance space. It follows that it is not inadequate to ignore the existence of marketable and risky assets in the economy when examining equilibrium across occupational assets if the covariance between the returns on

V. EMPIRICAL METHODOLOGY

It follows from Sharpe [17] and Lintner [13] that if occupational assets were treated in the marketplace as perfectly divisible and marketable assets which could be held in unrestricted amounts in portfolios, their returns in equilibrium would be such that

$$E(\tilde{r}_j) = r_F + \lambda \text{Cov}\ (\tilde{r}_j, \tilde{r}_M), \text{ for all } j, \qquad (6)$$

where λ is the market price of risk in the economy and $\tilde{r}_M$ is the rate of return of the aggregate of all risky assets in the economy, the "market" portfolio. A testable version of relation (6) is

$$E\ (\tilde{EX}_j) = a_1 + b_1\ \text{Cov}\ (\tilde{EX}_j, \tilde{r}_M) + \tilde{\varepsilon}_1 \qquad (6')$$

where $\tilde{EX}_j = \tilde{r}_j - r_F$ = excess rate of return offered by the j-th occupation and $\tilde{\varepsilon}_1$ is a random disturbance which is assumed to meet the usual "white noise" conditions. If occupational assets are priced as marketable assets in an economy of risky averse investors, one would expect that $a_1 = 0$ and $b_1 > 0$. It would appear to be relevant to test whether marketable and occupational assets are perceived as different by the market.

If occupational assets are treated as assets that cannot be held in unrestricted amounts in portfolios, then in equilibrium rates of return to occupational assets will satisfy conditions (5), if the risk diversification potential offered by risky and marketable assets does not exist or if it is not perceived by investors. They will satisfy conditions (4) if such a diversification potential exists and is perceived by investors. A testable version of conditions (5) is

$$E\ (\tilde{EX}_j) = a_2 + b_2\ \sigma(\tilde{EX}_j) + \tilde{\varepsilon}_2 \qquad (5')$$

and a testable version of conditions (4) is

$$E\ (\tilde{EX}^*_j) = a_3 + b_3\ \sigma(\tilde{EX}^*_j) + \tilde{\varepsilon}_3 \qquad (4')$$

where $E\tilde{X}_j^* = \tilde{r}_j^* - r_F$ and $\tilde{\varepsilon}_2$, $\tilde{\varepsilon}_3$ are random "white noise" disturbances. One would expect to find $a_2 = a_3 = 0$ and b_2, $b_3 > 0$. Moreover, if the explanatory power of (4') exceeds the explanatory power of (5'), the results will suggest the perception by the market of the occupational risk diversification potential offered by risky capital assets. In the following section the results of the empirical tests of these models are discussed.

The variables involved in all the models are ex-ante variables defined by expectations and distributions for the relevant investment period. For empirical tractability these ex-ante variables must be estimated using ex-post data. It is thus assumed that distributions of rates of return to all assets in the economy are stationary over time. Expectations, standard deviations, and covariances between the rates of return to all assets for any investment period can be estimated from observed rates of return in previous periods. The training period associated with the occupation is the relevant investment period. The rate of return on a marketable asset is uncertain at the beginning of the period but is known at the end of the period and equal to the terminal value of the asset (adjusted for dividends and splits) divided by the initial value of the asset, minus one. In the one-period context of this work, an individual incurs certain costs to acquire an uncertain value of the occupational asset at the end of the period when he decides at the beginning of the period to obtain the training associated with an occupation[14]. Similar to the case of marketable assets, the rate of return on an occupational asset is uncertain at the beginning of the period but is known at the end of the period and it can be defined as the terminal value of the occupational asset divided by the cost of occupational training, minus one. Terminal values of occupational assets are not directly observable in existing markets[15] and they will be estimated as the discounted value of the stream of incremental average age-earnings of the occupation at the end of the period. The cost of occupational training is estimated as the discounted value of the stream of direct plus opportunity costs to be incurred during the training period. The estimates of terminal values and costs of training of the various occupations depend upon the discount rates used. The age-earnings profile of any individual who has just completed the training required by an occupation is uncertain and the rate to be used in discounting the average age-earnings profile observed at the completion time should contain a risk premium in excess of the riskless rate observed at the same time, i.e., the end of the training period. Risk premiums ranging from 5% to 9% were used but the results of the empirical tests do not appear to be sensitive to the level of risk premium used. The stream of costs required during the training period is assumed to be certain and perfectly anticipated by individuals. The discount rate to be used in discounting this stream to produce an estimate of the cost of occupational training at the beginning of the period is thus the riskless rate observed at the beginning of the period[16]. For each occupation in the data base a time-series of rates of return was obtained using this procedure. Using the time-series of riskless rates for maturities identical to the training period of the occupation, a time-series of excess rates of return to the occupation was obtained and used to estimate the variables of models (6') and (5'). The variables of model (4') were estimated using equations (A.2) and (A.3) of Appendix I[17] where means and covariances were estimated using the time-series of rates of return to occupational assets and to a limited number of marketable and risky assets.

Rates of return on occupational assets are determined using average age-earnings profiles. The measure of occupational risk used in the empirical tests is thus a measure of average occupational risk associated with the variability of average profiles over time. As discussed in section II, besides facing average risk, investors face individual risk, i.e., they face uncertainty with respect to their relative intra-occupation position. It is true that society can diversify individual risk and, at a social level, it will get the average earnings with certainty[18]. It is also true that portions of individual risk may not be diversified at the individual level in existing markets. The first argument suggests that society should not be willing to pay individuals a premium for carrying the portions of individual risk that cannot be diversified away in existing markets. But the second argument suggests that individuals should demand such a premium. As discussed in Appendix II, this paper will proceed with the assumption that the risk premium added to the riskless rate when estimating terminal values fully adjusts for individual risk. The empirical tests will thus focus on the relevance of average risk.

The data base to be used in the empirical tests consists of time-series of average age-earnings profiles by occupation. In some occupational groups it is likely to exist significant differences in ability. Since for any occupation the population sampled is not held constant over time, significant differences in ability in an occupation are likely to produce estimates of occupational risk that are biased upwards. The estimates would pick-up variability that should be attributed to differences in ability and not to the occupation in itself. To minimize this potential bias this paper should test for equilibrium across occupational assets using occupations which should not be exposed to significant heterogeneity in ability. The set of occupations that require at least a college education should show this property. A college education should serve as a filter, eliminating individuals with lower levels of ability. It could be argued that the heterogeneity problem should be less pronounced the more advanced the educational level. Equilibrium will thus be tested using eight education-occupation groups[19].

1. An average undergraduate education with the average age-earnings profile across individuals with a complete undergraduate education and no graduate education.
2. A teacher with a complete undergraduate education and no graduate education,
3. An engineer with a complete undergraduate and no graduate education,
4. An average graduate education with the average profile across individuals with a complete graduate education (master's or Ph.D.'s degrees),
5. A teacher with a complete graduate education,
6. An engineer with a complete graduate education,
7. A complete graduate law education,
8. A complete medical education (M.D.).

VI. THE DATA BASE AND EMPIRICAL RESULTS

The Survey Research Center of the Institute for Social Research of the University of Michigan conducted yearly surveys every January-February from 1960 to 1970, a total of 11 years. In the surveys heads of family units were interviewed and their earnings and education-occupation groups were among the information collected[20]. With this information it was possible to obtain a time-series of average age-earnings profiles for each of the 8 education-occupation groups described before. For a few education-occupation groups complete profiles were not available in some surveys, these incomplete profiles were deleted when computing rates of return on occupational assets. Time-series of rates of return to the various education-occupation groups were obtained from the profiles using the procedure discussed in the preceding section and in Appendix II. Estimates of rates of return depend upon the risk premium assumed when estimating terminal values of occupational assets but the empirical results do not appear to be sensitive to the risk premium assumption, as shall be seen. Assuming a risk premium of 7%, the values of the summary variables involved in empirical tests of models (4'), (5') and (6') appear in table 1[21].

Before proceeding to empirical tests of such models, it seems adequate to recall that the equilibrium theory developed assumes that occupational assets are non-homogeneous. If this is the case, the hypothesis that rates of return on the various occupational assets come from the same population should be rejected. The non-parametric Mann-Whitney U-test was used in the population tests and the results for the average risk premium of 7% are reported in table 2[22]. At the 10% level, 20 of 28 tests rejected the null hypothesis that the samples came from the same population and at the 5% level, 18 of the 28 tests rejected the null hypothesis. Since almost identical results were obtained for the other risk

(1) Occ	(2) No. of Observ.	(3) $\overline{EX}_j$	(4) $\overline{EX}_j$ (% per year)	(5) $\sigma(\tilde{EX}_j)$	(6) $\overline{EX}_j^*$	(7) $\sigma(\tilde{EX}_j^*)$	(8) $Cov(\tilde{EX}_j,\tilde{r}_M)$
1	8	15.48	3.7	69.52	15.28	38.85	259.94
2	7	-85.16	-38.0	43.14	-78.55	37.21	358.61
3	7	137.81	24.2	80.97	137.08	64.28	269.28
4	7	106.51	19.9	171.86	110.13	164.49	-821.31
5	6	-240.21	--	79.25	-249.76	74.41	-215.12
6	6	12.54	3.0	161.85	40.10	154.16	57.22
7	9	490.15	55.9	286.08	486.79	237.77	-804.43
8	7	162.26	27.3	214.34	115.91	176.27	-2062.56

Table 1

VALUES OF SUMMARY VARIABLES FOR A 7% RISK PREMIUM

(columns (3), (5),(6) and (7) are in % per 4 years, column (4) is the % per year equivalent to column (3) and column (8) is in % per 4 years square).

Occ \ Occ	1	2	3	4	5	6	7
2	* 5						
3	* 6	* 0					
4	18	* 2	17				
5	* 0	* 3	* 0	* 0			
6	23	14	** 8	15	* 2		
7	* 4	* 0	* 9	* 8	* 0	* 4	
8	** 12	* 7	22	18	* 1	11	* 11

Table 2

RESULTS OF THE MANN-WHITNEY V-TEST

(* and ** indicate rejection of the null hypothesis at the 5% and 10% levels, respectively) premiums, one cannot escape the conclusion that there are significant differences across distributions of rates of return on occupational assets.

The theory of equilibrium across occupational assets also assumes that the non-marketable nature of these assets is perceived in labor markets. This is an empirically testable hypothesis. Under the null hypothesis that returns to occupational assets are determined as if they were marketable assets, one would expect to find b_1 significantly greater than zero and a_1 not significantly different from zero in empirical tests of model (6'). A commonly used market index in tests of capital asset pricing theory is the Standard and Poor's 500 (SP500)[23]. Rates of return on the SP 500 were used as surrogates for the market portfolio in this work and a time-series of 4-year rates of return on this index was obtained. Moreover, from the time-series of rates of return on each occupational asset and from a time-series of yields on government bonds with 4 years to maturity, a time-series of excess rates of return on each occupational asset was obtained. Expected excess rates of return on occupational assets, $E(\tilde{EX}_j)$'s, and covariances with the SP500, $Cov(\tilde{EX}_j, \tilde{r}_M)$'s, were estimated from the corresponding time-series of excess rates of return. The results of the empirical tests appear in table 3. Regardless of the risk premium used, neither a_1 or b_1 are significantly different from zero at the 10% level. Tests of the model using marketable assets support the hypothesis that the slope coefficient and the market price of non-diversifiable risk is significantly greater than zero[24]. For occupational assets not only the $b_1 > 0$ hypothesis is rejected but the estimate of b_1 is negative. This is inconsistent with the hypothesis that occupational assets are perceived as perfectly divisible assets in a market of risk-averse investors.

It remains an open question whether differences in the rates of return on occupational assets could be explained by differences in occupational risk. If

Coefficient / Premium	a_1 (t_{a_1})	b_1 (t_{b_1})	R^2
5%	73.81 (.76)	- .12 (-1.21)	.20
6%	51.53 (.58)	- .12 (-1.20)	.19
7%	33.95 (.42)	- .11 (-1.17)	.19
8%	19.89 (.27)	- .11 (-1.13)	.18
9%	8.46 (.12)	- .10 (-1.09)	.17

Table 3

REGRESSION RESULTS FOR $E(\tilde{EX}_j) = a_1 + b_1 \text{ Cov } (\tilde{EX}_j, \tilde{r}_M) + \tilde{\varepsilon}_1$

(the critical value of t for two-sided tests at the 10% level is 1.94)

Coefficient / Premium	a_2 (t_{a_2})	b_2 (t_{b_2})	R^2
5%	- .02 (-1.89)	2.14 (3.25)*	.64
6%	- .02 (-2.02)	2.09 (3.32)*	.65
7%	- .02 (-2.16)	2.04 (3.39)*	.66
8%	- .02 (-2.32)	1.99 (3.46)*	.67
9%	- .02 (-2.48)**	1.95 (3.54)*	.68

Table 4

REGRESSION RESULTS FOR $E(\tilde{EX}_j) = a_2 + b_2 \sigma(\tilde{EX}_j) + \tilde{\varepsilon}_2$

(* and ** indicate significance at the 1% and 5% levels, respectively) this is the case and, further assuming that the only risky assets are occupational assets, then empirical tests of model (5') should support the hypotehsis that $a_2=0$ and $b_2>0$. The standard deviations, $\sigma(\tilde{r}_j)$'s, of rates of return on occupational assets were estimated from the time-series of excess rates of return on occupational assets and the results of the empirical tests, for all risk premiums, appear in table 4. At the 1% and 5% levels one cannot reject the hypothesis that $a_2=0$ [25] and $b_2>0$. The results support the hypothesis that differences in rates of return on occupational assets can be explained by differences in occupational risk and the hypothesis that labor markets are in equilibrium in the risk-return space. The estimate of the market price of occupational risk (the slope coefficient) is close to 2% per 4 years, the equivalent of approximately 0.5% per year. As suggested by Friedman and Kuznets [6] lawyers and doctors have large expected excess returns but they also bear a large amount of occupational risk. The large expected excess returns reward lawyers and doctors for such large occupational risk, at the market price of occupational risk. However, there are marketable and risky assets in the economy and they may diversify occupational risk through corrective portfolios. Let us consider four marketable and risky assets in the economy: the portfolios associated with the Standard and Poor's Industrials, Utilities and Railroads indices and the portfolio of bonds associated with the Keystone B-2 fund[26]. Assuming that these were the only marketable and risky assets, corrective portfolios and rates of return to virtual occupational assets were estimated[27] and empirical tests of model (4') were performed. The results appear in table 5 and at the 5% and 1% levels one cannot reject the hypothesis that $a_3=0$ but the $b_3>0$ hypothesis is supported only at the 5% level. The t values and R^2 are consistently smaller than the corresponding results of table 4, indicating that model (5') is better than model (4') by the best of fit criterion. This suggests that the risk diversification potential offered by corrective portfolios does not

Coefficient / Premium	a (t_a)	b (t_b)	R^2
5%	-1.88 (-1.23)	2.16 (2.29)**	.47
6%	-1.79 (-1.40)	2.11 (2.45)**	.50
7%	-1.72 (-1.59)	2.06 (2.61)**	.53
8%	-1.66 (-1.78)	2.01 (2.78)**	.56
9%	-1.61 (-1.99)	1.96 (2.94)**	.59

Table 5

REGRESSION RESULTS FOR $E(\tilde{EX}_j^*) = a_3 + b_3\ \sigma(\tilde{EX}_j^*) + \tilde{\varepsilon}_3$

(* and ** indicate significance at the 1% and 5% levels, respectively) appear to be relevant.

The risk diversification potential offered by marketable and risky assets is relevant if their rates of return are correlated with rates of returns on occupational assets. The significance of the correlation between two-variables can be tested using regression procedures, the correlation is significant if the slope coefficient is significant. Rates of return on occupational assets were regressed upon rates of return on marketable assets. The results for the slope coefficients appear in table 6. None of the coefficients is significant at the 5% level and just one is significant at the 10% level. The results support the hypothesis that returns on occupational assets are not correlated with returns on marketable and risky assets. This implies that the risk diversification potential offered by marketable assets is not relevant. Moreover, this explains the loss in explanatory power of model (4') relative to model (5'). Given the non-significant correlation the estimates of corrective portfolios are nothing but white noise. They introduce measurement errors in the variables associated with virtual occupational assets in model (4'). This results in the observed loss in efficiency.

Index / Occupation	SP Industrials	SP Utilities	SP Railroads	Keystone B-2
1	.62 (.38) .02	1.68 (2.37)* .48	- .06 (- .07) .00	1.40 (.63) .06
2	.75 (.75) .10	.92 (1.76) .38	.41 (.78) .11	1.61 (1.21) .23
3	.88 (.45) .04	1.56 (1.52) .38	.08 (.08) .00	1.36 (.49) .05
4	-4.60 (-1.13) .20	-5.22 (-1.42) .29	-1.60 (- .73) .10	-6.18 (-1.01) .17
5	-1.16 (- .56) .07	-1.04 (- .52) .06	- .64 (- .58) .08	- .80 (- .25) .02
6	- .66 (- .15) .01	.55 (.13) .00	.70 (.30) .02	2.54 (.40) .04
7	-2.77 (- .44) .03	-4.97 (-1.56) .26	- .06 (- .02) .00	-3.36 (- .38) .02
8	-5.55 (-1.20) .23	-5.02 (-2.09)* .47	-3.12 (-1.30) .25	-9.70 (-1.58) .33

Table 6

REGRESSION RESULTS FOR $\tilde{EX}_j = a_5 + b_5 \tilde{EX}_{INDEX} + \tilde{\varepsilon}_5$

(The slope coefficients, their t-values in parenthesis and the R-squares are reported, in this order, from above. Significance in two-sided tests at the 10% level is indicated by *. None of the t's are significant at the 5% and 1% levels. A risk premium of 7% was used in estimating $\tilde{EX}_j$.)

VII. CONCLUSIONS

This paper examined occupational choices under conditions of uncertainty in the mean-variance space. Conditions for equilibrium in human capital markets were derived explicitly considering the existence of marketable and risky assets in the economy. The general set of conditions allow returns on marketable and non-marketable occupational assets to covary but equilibrium conditions for the special case of zero-covariance between returns on the two sets of assets were also derived. The effect of wealth and risk preferences upon occupational choices was examined and it was shown that wealthier and less risk-averse individuals will select riskier occupations, under the ceteris-paribus clause.

The equilibrium conditions advanced were tested empirically and it was concluded that one cannot reject the hypothesis that rates of return on occupational assets are in equilibrium under conditions of risk. Doctors and lawyers do make large expected returns but they also bear a large amount of occupational risk. The degree of covariance between returns on marketable and occupational assets was examined empirically. Returns on the two sets of assets do not appear to be correlated. Not surprisingly, the explanatory power of the testable version of the zero-covariance equilibrium conditions is greater than the explanatory power of the testable version of the more general equilibrium conditions. Moreover, the zero-covariance of returns implies that equilibrium across occupational assets can be examined without explicitly considering the existence of marketable and risky assets in the economy.

FOOTNOTES

(1) This acceptance condition goes back to Walsh [20] and Friedman and Kuznets [6]. For a recent application see Haley [7].

(2) Becker [2] and Hansen [8] suggest the internal rate of return criterion. For a good discussion of the limitations of the criterion see Hirshleifer [10].

(3) See Becker [2], Friedman and Kuznets [6] and Walsh [20].

(4) They argue (i) for imperfections in loan markets, (ii) that individuals have monopolistic access to the production of human capital and (iii) that in certain occupations entry is directly restricted by cartelization.

(5) I.e., the relevant investment horizon consists of a single period and the joint distribution of terminal values of marketable assets and of earnings of occupational assets at the end of the period is multivariate normal.

(6) The mean-variance space was advanced by Markowitz [14] and has been extensively used in financial economics. See Sharpe [17] and Lintner [13].

(7) These results will be formally derived in the following section.

(8) The reward-to-variability ratio offered by a portfolio of marketable and risky assets is the ratio of the expected rate of return offered by the portfolio in excess of the riskless rate r_F and the risk (standard deviation) of the rate of return on the portfolio.

(9) These results follow directly from Brito [4] and are reviewed in Appendix I.

(10) This follows from the Three Fund Separation Theorem advanced by Brito [4] and reviewed in Appendix I.

(11) For the definition of absolute risk aversion see Arrow [1]

(12) By investing more in human capital it is meant that wealthier individuals will hold a higher fraction of a reference occupational asset.

(13) Similar results in a different context were simultaneously and independently derived by Weiss [22].

(14) Notice that in an one-period context occupational assets cannot be marketed at the beginning of the period but they can be marketed at the end of the period. They have a terminal value but they do not have an initial value.

(15) I.e., the assumption of an one-period world is obviously a simplification and its validity is largely an empirical issue.

(16) A detailed discussion of the procedure used to estimate rates of return on occupational assets appears in Appendix II.

(17) Recall that $\tilde{r}_j^*$ is the rate of return on the j-th virtual occupational asset and, using the notation of Appendix I, $\tilde{r}_j^* = \tilde{r}_j + \tilde{\theta}_j^r$.

(18) When hiring a large number of engineers an engineering firm knows for sure that it will get services of average quality from the group as a whole. It will thus determine the average salary of engineers by the average quality of services and it will not be willing to add a premium to the average salary to reward engineers for individual uncertainty.

(19) These were all the education-occupation groups with at least a college education that could be composed given the data base available.

(20) To the best of our knowledge this is the only time-series data on age-earnings profiles by education-occupation available on a yearly basis. The education-occupation classification used in the surveys only allowed the composition of the 8 groups described.

(21) Risk premiums from 5% to 9% were used. In the lack of a better criterion the summary variables for the average risk premium (7%) were reported. The number of complete profiles available for each occupation appears in column 2 of Table 1.

(22) It could be argued that the Kolmogorov-Smirnov two-sample test may be more efficient for small samples, for a discussion see Siegel [19]. The test was also performed but the results were identical to the results of the Mann-Whitney U-test.

(23) The SP 500 is a market weighted index, the weighting procedure recommended by capital market theory. It is an index commonly used in capital market studies. See Sharpe [18] for a detailed discussion of the index.

(24) See Black et al [3] and Fama and MacBeth [5]. The results of intercept tests suggest that it is greater than zero and there is some controversy with respect to possible causes of these findings. See Black et al [3] and Kraus and Litzenberger [11].

(25) The only exception is the case of 9% risk premium. There one would reject the hypothesis that $a_2 = 0$ at the 5% level. One must admit that t-values of a_2 are close to significance at the 5% level. Since estimates of a_2 are negative this suggests that borrowing rates for human capital investments may be above the rate of return demanded for investments in government bonds.

(26) The Standard and Poor's Industrial, Utilities and Railroads indices are described in the Standard and Poor's publications. The Keystone B-2 bond fund has maintained a portfolio of medium grade bonds. Its annual report for the fiscal year ended October 31, 1960 states that "the bonds are selected and supervised

with a two-fold long-term goal in view: a) to produce relatively dependable income at as good a rate as may be prudently obtained; and b) to maintain the relative stability of capital". Data on the fund was collected from the Moody's Bank and Finance Manual and from the Wall Street Journal.

(27) A time-series of 4-year rates of return on each of the four marketable assets was obtained. Each series had 33 observations and the last 4-year period ended in 1970. Since the Surveys were conducted in January, the 4-year periods begin and end on a January 31st. The variance-covariance matrix of rates of return on marketable and risky assets (C_r) and the vector of expected rates of return (E_M) were estimated using these time-series. Using the time-series of rates of return on occupational assets, the vectors of covariances between the rate of return on each occupational asset and the rates of return on the marketable assets (C^r_{oj}) were obtained and corrective portfolios were estimated ($\theta^r_j = C_r^{-1} C^r_{oj}$). Expected values and standard deviations of rates of return on virtual occupational assets ($r^*_j = \tilde{r}_j + \tilde{\theta}^r_j$) were then estimated using equations (A.2) and (A.3) of Appendix I. For a 7% risk premium the results appear in Table 1.

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APPENDIX I

THE MARKETABLE PORTFOLIO SELECTION PROBLEM OF AN INVESTOR

The marketable portfolio selection problem of the i-th investor who holds the j-th occupational asset was examined by Brito [4]. He shows that the investor's demand for marketable assets will follow a Three Fund Separation Theorem. He demands the unique and well defined portfolio of marketable and risky assets which diversifies away as much of the risk of the j-th occupation as possible in existing capital markets, proceeding to divide the remainder of his wealth between investments in the riskless asset and in the portfolio of marketable and risky assets which offers the highest reward-to-variability ratio (the ratio of the expected returns to the portfolio in excess of the returns offered by the riskless asset and the risk of the portfolio measured by its standard deviation). The unique portfolio which diversifies as much occupation risk as possible is independent of prices, of risk preferences, and of wealth and should be viewed as the "corrective portfolio" of the j-th occupation.

The region of feasible total combinations (marketable plus occupational holdings) available to the i-th investor if he selects the j-th occupational asset can be examined in the mean-variance of terminal value space or in the mean-variance of rate of return space. Define

$\tilde{O}_j$ = random terminal earnings (value) of the j-th occupational asset (O_j),

C = variance-covariance matrix of random terminal values of the marketable and risky assets in the economy,

C_{oj} = vector of covariances between the random terminal earnings of O_j and the random terminal value of the marketable and risky assets in the economy,

c_j = costs which must be incurred to obtain the j-th occupation,

W_i = marketable wealth of the i-th investor,

$\theta_j = -C^{-1} C_{oj}$ = corrective portfolio of the j-th occupational asset in the terminal value space offering the random terminal value $\tilde{\theta}_j$,

P_j^{θ} = the price of θ_j,

$W_j = P_j^{\theta} + c_j$ = cost of acquiring the j-th occupational asset and its corrective portfolio,

$W_{ij} = W_i - W_j$ = remainder wealth of the investor after acquiring the j-th occupation and its corrective portfolio,

r = one plus the riskless rate of interest r_F,

η_{ij} = portfolio of marketable and risky assets which offers the maximum reward-to-variability ratio, has market value W_{ij} and offers the random terminal value $\tilde{\eta}_{ij}$,

$\tilde{MV}_j$ = random terminal value of the total combination associated with the j-th occupation which offers the minimum variance,

RV^* = the maximum reward-to-variability ratio offered by marketable and risky assets in the economy,

$\tilde{r}_j$ = random rate of return offered by the j-th occupational asset in the period,

C_r = variance-covariance matrix of the random rates of return offered by marketable and risky assets,

C^r_{oj} = vector of covariances between $\tilde{r}_j$ and the random rates of return on marketable and risky assets,

$\theta^r_j = -C_r^{-1} C^r_{oj}$ = corrective portfolio of the j-th occupational asset in the rate of return space offering the random rate of return $\tilde{\theta}^r_j$.

It follows from the Three Fund Separation Theorem and from Merton [16] that the locus of efficient total combinations (aggregate of marketable and occupational holdings) of the investor in the terminal value space is the upper segment of the hyperbolic locus of combinations of $\tilde{O}_j + \tilde{\theta}_j + W_{ij}r$ and $\tilde{O}_j + \tilde{\theta}_j + \tilde{\eta}_{ij}$. The hyperbola has its center on the expected return axis, has its principal axis parallel to the standard deviation axis and the slope of its assymptotes is RV^*, the maximum reward-to-variability ratio offered by marketable and risky assets in the economy. Moreover, $\tilde{O}_j + \tilde{\theta}_j + W_{ij}r$ is the combination on the locus that offers the minimum standard deviation (risk) thus lying on the principal axis of the hyperbola. These properties are illustrated in figure A-1.

These results generalize to an economy with occupational assets a simple principle of Capital Market Theory; if risk is added to a position it should be added bringing as much expected return (reward) as possible. They have implications for Human Capital Theory. Recalling that RV^* is determined by the set of marketable and risk assets and exogenously of the set of occupational assets, the assymptote of the hyperbolic locus is known and identical for every occupational asset in the economy. The minimum variance combination $\tilde{O}_j + \tilde{\theta}_j + W_{ij}r$ thus completely determines the hyperbolic locus and contains all the information about the j-th occupation that is

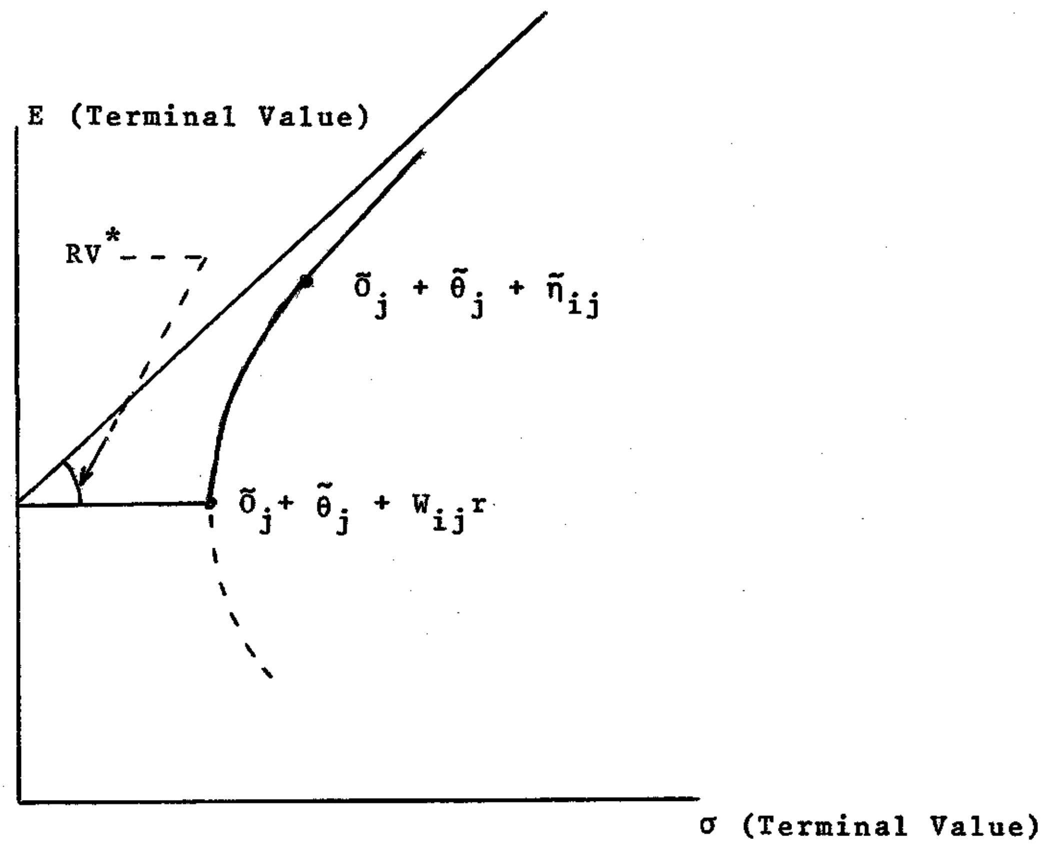

Figure A.1

THE EFFICIENT FRONTIER OF INVESTOR i IN OCCUPATION j relevant to the i-th investor. In his occupation selection problem all the information the i-th investor needs are the means and variances of the minimum variance combinations associated with the various occupations in the economy. Moreover, observe that under the atomistic assumptions of this paper $\frac{\partial W_{ij}}{\partial W_i} = 1$, $\frac{\partial E(\tilde{O}_j + \tilde{\theta}_j + W_{ij}r)}{\partial W_i} = r$ and $\frac{\partial \sigma(\tilde{O}_j + \tilde{\theta}_j + W_{ij}r)}{\partial W_i} = 0$ for all i and j, i.e.. changes in wealth will merely shift the hyperbolic loci of all the occupations vertically by the same amount without changing the relative positions of the loci. It follows that the occupation selection problem of any investor can be based upon the impersonal set of means and variances of $\{\tilde{MV}_j = \tilde{O}_j + \tilde{\theta}_j + W_j r, \text{ all } j\}$. This set contains all the information about returns to occupational assets that is relevant to any investor.

To reduce the burden of the notation the analysis proceeded with the implicit assumption that the investor held exactly O_j. Since non-negative fractional holdings are allowed, these results must be generalized. Define the variables associated with a fraction $\alpha \geqslant 0$ of O_j by a functional α,

$C_{oj}(\alpha)$ = vector of covariances between $\alpha\tilde{O}_j$ and the random terminal value of marketable and risky assets in the economy,

$\theta_j(\alpha) = C^{-1}C_{oj}(\alpha)$ = corrective portfolio of αO_j

and so on. It follows trivially that $\theta_j(\alpha) = \alpha\theta_j$, $P_j^{\theta}(\alpha) = \alpha P_j^{\theta}$, $c_j(\alpha) = \alpha c_j$ and thus

$$\tilde{MV}_j(\alpha) = \alpha\tilde{O}_j + \tilde{\theta}_j(\alpha) - W_j(\alpha) = \alpha\tilde{MV}_j(1) = \alpha\tilde{MV}_j \qquad \text{(A.1)}$$

For any given α the hyperbolic mean-variance efficient frontier of total holdings (marketable plus occupational holdings) of the investor is determined by $\tilde{MV}_j(\alpha)$ (given RV^* and his wealth) and relation (A.1) implies that the set $\{\tilde{MV}_j, \text{ all } j\}$ contains all the relevant information about occupational assets.

Investment decisions can be examined measuring the terminal value of assets. Using such a terminal value measure, a portfolio of marketable and risky assets is a vector and its l-th component is the fraction of the total amount outstanding of the l-th marketable and risky asset held by the investor. Investment decisions can also be examined measuring the interperiod rate of return offered by assets. Since interperiod rates of return are normalized measures, portfolios must also be normalized. If the rate of return measure is used, a portfolio of marketable and risky assets is a vector and its 1-th component is the proportion of the investor's wealth invested in the 1-th marketable and risky asset. Given the well known linear relation between the two measures, the structure of the investment decision problem in the two measures should be in a one-to-one correspondence. Thus far the mean-variance of terminal value space was used in this appendix. Similar results can be derived in the mean-variance of rate of return space. The portfolio that minimizes variance of terminal value of aggregate holdings (marketable plus occupational holdings) is the portfolio that minimizes variance of the rate of return offered by aggregate holdings. In the rate of return space the closed form solution of the corrective portfolio of the j-th occupation is $-C_r^{-1} C_{oj}^r$. It follows that the random rate of return offered by occupational plus corrective holdings is $\tilde{r}_j + \tilde{\theta}_j^r$ and

$$E(\tilde{r}_j + \tilde{\theta}_j^r) = E\,(\tilde{r}_j) + \theta_j^r E_M \tag{A.2}$$

$$\mathrm{Var}(\tilde{r}_j + \tilde{\theta}_j^r) = \mathrm{Var}(\tilde{r}_j) - C_{oj}^r\, C_r^{-1} C_{oj}^r \tag{A.3}$$

where E_M is the vector of expected rates of return to marketable and risky assets. As one would expect, $\mathrm{Var}(\tilde{r}_j + \tilde{\theta}_j^r) < \mathrm{Var}(\tilde{r}_j)$ if there is some covariance between marketable and risky assets and the risky occupational asset. This follows from the positive-definiteness of C_r. Notice that even though $\theta_j \neq \theta_j^r$ they are associated with the same portfolio, the corrective portfolio that diversifies away as much occupational risk as possible.

APPENDIX II

RATES OF RETURN ON OCCUPATIONAL ASSETS

Let us define, in the context of an one period world, the rate of return to an investment in the j-th education-occupation class at time t. Define

d = length of formal training period necessary to acquire the j-th occupational asset,

$r_F^t(i)$ = yield to maturity on government bonds with i years to maturity at time t,

C_{t+i}^j = costs to (opportunity + direct costs) be incurred in the i-th year of the training period of the j-th occupation, if the investment decision was taken at time t,

V_{t+d}^j = incremental market value of the j-th occupational asset at the end of the training period, if the decision is taken at t,

C_t^j = stock value of costs to be incurred to acquire the j-th occupational asset, if the decision is taken at t,

r_{jt} = rate of return to an investment at time t in the j-th education-occupation class $= \left[V_{t+d}^j / C_t^j\right] - 1$

This paper assumes that costs are certain as of the decision time and thus the stock value of costs is

$$C_t^j = \sum_{i=1}^{d} \frac{C_{t+i}^j}{\left[1 + r_F^t(i)\right]}$$

The estimate of V_{t+d}^j requires a more careful analysis. The estimate of V_{t+d}^j is the present value of the stream of incremental earnings associated with the contemplated education-occupation investment. It is unclear which discount rate should be used to obtain this present value but it should contain a risk premium. At time t the investor faces individual risk and average risk that will be resolved only at t+d. Our data base consists of time-series of average age-earnings profiles which generate time-series of rates of return to occupational investments. The variability of this series is the measure of occupation risk used in this work and it is more a measure of average risk than a measure of individual risk. This measure of risk will be used in the empirical tests of equilibrium. However, an attempt to correct for individual risk was made. The discount rate used to estimate V_{t+d}^j was r_F^{t+d} (4) plus a risk premium. Risk premiums ranging from 5% to 9% were used to obtain estimates of V^j_{t+d} and r_{jt}. Since V^j_{t+d} is an incremental value, the rate of return on the j-th occupational asset is also an incremental rate of return. For occupations requiring a complete undergraduate education, incremental values and rates of return were obtained relative to a complete high-school education. For occupations requiring an advanced education (masters or doctoral degrees) incremental values and rates of return were obtained relative to a complete undergraduate education. Finally, the estimates depend upon d, the duration of the training period. Whenever possible the formal training period was used. For example, a 4-year period was used for occupations requiring a complete undergraduate education or an M.D. degree. However, for occupations (and profiles) associated with an advanced degree without specifying the degree, the length of the training period was unclear and a 2 year training period was used. In these cases, for consistency with the 4-year periods, the 2-year rates of return were compounded with the yields on 2-year government bonds observed at the end of period, i.e., (one plus 4-year rate) = $(1 + r_{jt}) \left[1 + r_F^{t+2}(2)\right]$. The same adjustment procedure was used for a law education, (one plus 4-year rate) = $= (1 + r_{7t}) \left[1 + r_F^{t+3}(1)\right]$. The 4-year rates were used in the empirical tests.