COPPEAD/UFRJ

RELATÓRIO COPPEAD Nº 24

"THE EFFICIENCY OF INFLATION EXPECTATIONS IN TREASURY BILL MARKETS: THE BRAZILIAN EVIDENCE"

Ney O. Brito*

Março de 1978
(revised December 1978)

* COPPEAD - Federal University of Rio de Janeiro. The data basic of this paper was organized by André Zambludowski and the data was kindly furnished by the Brazilian Association of Open Market Dealers (ANDIMA). The research assistance of Nelson Baptista and André Zabludowski, the comments of an unknown reviewer and the financial support of FINEP, an agency of the Brazilian government, are also gratefully acknowledged.

I. INTRODUCTION

This paper will examine the efficiency of inflation expectations in Brazilian Treasury Bill markets in the 1972-1976 period. In the recent past inflation in Brazil has varied within a very wide range, from 0.4% to 5.12% on a monthly basis or from 4.91% to 82.06% on an yearly adjusted basis. Moreover, Brazilian inflation appears to follow a very volatile process dominated by a strong first-order autoregressive component. The characteristics of our inflation differ from the characteristics of inflationary processes in other countries and, in particular, from the U.S. process. The study of the Brazilian inflationary process may thus offer a contribution to the general understanding of inflation.

The paper reviews the methodology advanced by Fama [3] for tests of the single-period anticipatory efficiency of Bill markets. The methodology is then generalized for the multiple-period case. Moreover, it is argued that, in general, it is relevant to consider two components of the total real rate demanded by investors in an economic environment subject to uncertain inflation. It is the sum of the basic real rate demanded by investors to carry a perfectly indexed bond and a risk premium demanded by investors to bear pure inflation risk. Assuming that the sum of these components (and the total real rate) is constant the paper proceeds to examine the efficiency of nominal interest rates as a predictor of inflation in the Brazilian Bills market. It is shown that the joint hypothesis that total real rates are constant and that inflation expectations are efficient is rejected by the data.

Assuming that the basic real rate component is constant and also assuming that inflation expectations are efficient, the results suggest that the risk premiums demanded by investors to bear inflation risk have not been constant or, more generally, do not follow a martingale process. The paper them examines possible characteristics of the process followed by risk premiums that are consistent with the results.

II. THE BASIC METHODOLOGY

In an economic environment exposed to certain inflation the relationship between nominal and real forward interest rates at any point in time is approximately given by the well known fisherian expression, i. e., the nominal forward rate demanded is equal to the real forward rate demanded plus the certain inflation in the relevant period. Under uncertain inflation things are not as straightforward. For any single investment period t define:

$\tilde{\Delta}_t$ = inflation observed during period t, a random variable before the end of the period, as indicated by the tilde;

ϕ_{t-n} = the set of all the information available to investors at the end of period t-n;

$E(\tilde{\Delta}_t|\phi_{t-n})$ = expected value of $\tilde{\Delta}_t$ as of the end of period t-n;

$\tilde{\varepsilon}_{t-n} = \tilde{\Delta}_t - E(\tilde{\Delta}_t \mid \phi_{t-n})$ = n-th period inflation forecasting error;

${}_{t-n}r_t$ = real forward interest rate demanded by investors as of t-n to hold a perfectly indexed riskless bond[1] during period t[2];

${}_{t-n}p_t$ = real forward risk premium (over ${}_{t-n}r_t$) demanded by investors as of t-n to hold pure inflation risk (but no default risk) during period t;

${}_{t-n}r^*_t = {}_{t-n}r_t + {}_{t-n}p_t$ = total real forward rate of interest demanded by investors as of t-n to hold a nominally riskless bond during period t;

${}_{t-n}R_t$ = nominal forward rate of interest demanded by investors as of t-n to hold a nominally riskless bond during period t.

Under uncertain inflation[3] ${}_{t-n}R_t = {}_{t-n}r_t + {}_{t-n}p_t + E(\tilde{\Delta}_t|\phi_{t-n})$ which implies that $\Delta_t = -{}_{t-n}r^*_t + {}_{t-n}R_t + \varepsilon_{t-n}$. In an efficient market forecasting errors should be serially uncorrelated with zero mean. The inflation generating process can thus be described as

$$\tilde{\Delta}_t = -{}_{t-n}\tilde{p}_t - {}_{t-n}\tilde{r}_t + {}_{t-n}\tilde{R}_t + \tilde{\varepsilon}_n, \quad \text{for all n,} \qquad (1)$$

where $\tilde{\varepsilon}_n$ is a forecasting error meeting the usual "white noise" assumptions (for all n) and tildes indicate random variables. General tests of the efficiency of inflation expectations should thus be based upon relation (1). Unfortunately these tests would require the availability of data on real interest rates and risk premiums and such data is not available in most countries[4].

To proceed to empirical tests of the efficiency of inflation expectations using only nominal forward interest rates one has to impose further structure upon the problem. If one assumes that ${}_{t-n}r_t + {}_{t-n}p_t = {}_{t-n}r^*_t = -a_{0n}$ is a constant (for all t) defined as $-a_{0n}$ then, net of a forecasting error, any changes in nominal forward rates should reflect changes in inflation expectations. A version of relation (2) that can be tested using only nominal forward interest rates is then obtained[5]:

$$\Delta_t = a_{0n} + a_{1n}\ {}_{t-n}R_t + \tilde{\varepsilon}_n \qquad (2)$$

Empirical tests of this relation should support the hypotheses that $a_{1n} = 1$ and that $\tilde{\varepsilon}_n$ is serially uncorrelated, for all n, if inflation expectations are efficient and total real forward rates are constant. Moreover, $-a_{0n}$ would be the estimate of the constant real forward rate.

Within this framework Fama [3] proceeds to examine the efficiency of inflation expectations in the U.S. Treasury Bill market in the 1953-1971 period. Unfortunately he tests only the one-period anticipatory efficiency of the market[6] but for this case he finds that empirical tests of relation (2) support the joint hypothesis that the real one-period interest rate is constant and that one-period expectations of inflation are efficient. Fama also proceeds to test the one-period efficiency using another relation that can also be generalized for the multiple-period case. Define

$${}_{t-n}\hat{\Delta}_t = f\ (\phi_{t-n}) = \text{any estimate of } \Delta_t \text{ as of the end of period t-n, a function of the information set } \phi_{t-n}$$

and recall that in an efficient market inflation expectations should be fully reflected upon ${}_{t-n}R_t$. It follows that empirical tests of the relation

$$\Delta_t = a_{0n} + a_{1n}\ {}_{t-n}R_t + a_{2n}\ {}_{t-n}\hat{\Delta}_t + \mu_n \qquad (3)$$

should support the hypothesis that $a_{2n} = 0$ if inflation expectations are efficient. Fama [3] finds that the empirical tests of relation (3) supports this hypothesis in the U.S. Treasury Bill market, for the one-period case and defining ${}_{t-n}\hat{\Delta}_t = \Delta_{t-1}$. It would be of interest to test whether these results can be generalized for the multiple-period case in the U.S. market.

The results of Fama [3] were questioned by Carlson [2], Joines [7] and Nelson and Schwert [10]. In the single period context they argue that

(i) the total real one-period interest rate is not constant[7] and that

(ii) if the inflationary process is not autoregressive of first order then the most efficient estimate of inflation in the next period ($_{t-1}\hat{\Delta}_t$) is not a linear function of inflation in the last period and the procedure of Fama ($_{t-1}\hat{\Delta}_t = \Delta_{t-1}$) is biased towards accepting the hypothesis that $a_{2n} = 0$ in tests of relation (3)[8].

However, as pointed out by Fama [4] in his reply, there seems to exist an agreement that "the (nominal) interest rate remains the best single predictor of the inflation rate; and nobody has uncovered variables that make substantial contributions to the prediction of inflation beyond that provided by the interest rate alone. Moreover, the proposition that the largest part of the variation in nominal interest rates reflects variation in expected inflation rates, seems intact." This paper will show that may be such an agreement should be restricted to the U.S. market and should not be generalized to all other markets.

III. THE CHARACTERISTICS OF THE BRAZILIAN INFLATIONARY PROCESS

In the 1968-1976 period monthly inflation in Brazil varied from 0.40% to 5.12%. On an yearly adjusted basis this represented a 4.91% - 82.06% range. With inflation varying whithin this range, and given the redistribution potential of uncertain inflation, the relevance of efficient inflation expectations would seem obvious. The Brazilian experience could thus offer a contribution to the understanding of the impact of inflation on financial markets. Several price indices are reported monthly in Brazil. Two of them are the most comprehensive and representative: the General Price Index (GPI) and the Wholesale Price Index (WPI). For each price index two inflation indices can be obtained: the rate of change in the purchasing power of money and the relative change in the price index. The second inflation index is the one commonly used and this paper will follow this pattern[9].

The autocorrelation structure of inflation in Brazil up to the 10-th lag is shown in Table 1 for the two indices and periods of up to 90-days[10]. For the GPI and monthly inflation rates the correlation coefficients are significant at the 1% level for lags of up to the 4-th order and in general significant at the 5% level for lags of up to the 9-th order. For the WPI and monthly inflation rates the correlation coefficients are in general smaller and significant at the 1% level only for lags up to the 2nd order. For both indices the results for the initial lags support the hypothesis that inflation is a first-order process. The correlation coefficients for lags up to the 4-th order decrease in an almost perfect geometric progression. For lags of up to the 9-th order the correlation coefficients seem to hover around .20 for both indices, being significant at the 5% level for the GPI and non-significant for the WPI. For lags of 10-th and higher order the correlation coefficients drop and are non-significant. The same results apply to 60-days and 90-days periods. During a 6-month period inflation[11] is in general significantly autocorrelated for both indices. The autocorrelation coefficients decline very fast in the initial periods, begin to hover around .20 for a while and then drop to lower levels.

These results suggest that inflation expectations in Brazil are dominated by a short-term and first-order autoregressive component. To be fair there appears to exist a mild long-term component that is not autoregressive of first-order, it would account for the autocorrelation coefficients for lags beyond a 6-month period that are generally positive and non-significant[12], varying around .20. It is possible to examine the hypothesis that the process is predominantly of first order.

Index	GPI			WPI		
Lag \ days	30	60	90	30	60	90
1	.688[A] (9.66)	.558[A] (4.75)	.523[A] (3.48)	.657[A] (8.07)	.459[A] (3.31)	.438[B] (2.49)
2	.492[A] (5.74)	.297[B] (2.17)	.322 (1.89)	.385[A] (3.84)	.219 (1.43)	.336 (1.78)
3	.345[A] (3.72)	.212 (1.50)	.224 (1.26)	.217[B] (2.04)	.190 (1.20)	.207 (1.04)
4	.276[A] (2.88)	.276 (1.96)	.288 (1.62)	.184 (1.70)	.257 (1.64)	.221 (1.09)
5	.212[B] (2.18)	.190 (1.30)	.182 (.98)	.235[B] (2.19)	.187 (1.16)	.152 (.72)
6	.167 (1.69)	.259 (1.80)	.141 (.74)	.134 (1.22)	.095 (.57)	.164 (.77)
7	.210[B] (2.12)	.118 (.78)	.224 (1.17)	.192 (1.74)	.148 (.88)	.200 (.91)
8	.237[B] (2.41)	.152 (1.01)	.332 (1.76)	.192 (1.74)	.084 (.48)	.187 (.83)
9	.202[B] (2.02)	.084 (.54)	.349 (1.83)	.197 (1.79)	.118 (.69)	.435 (2.05)
10	.138 (1.36)	.173 (1.12)	.071 (.33)	.170 (1.51)	.187 (1.07)	.141 (.59)
No. of Obs.	107	53	35	89	44	29

TABLE 1

AUTO CORRELATION STRUCTURE OF INFLATION

t-values of regression coefficients appear in parenthesis below the correlation coefficients. A, B indicate significance at the 1% and 5% levels, respectively.

If this is the case the autocorrelation in the series of first differences in inflation ($\Delta_t - \Delta_{t-1}$) should be equal to the autocorrelation in the inflation series, for all lags[13]. The level of autocorrelation in the series of first differences in inflation is remarkably close to the level of autocorrelation in the inflation series, for all lags and all periods. For 60-day periods the autocorrelation of first differences for the first five lags and the GPI are .56, .30, .23, .31 and .22; these results are very close to the results shown in Table 1 for the autocorrelation of the inflation series. Additional evidence that the Brazilian inflationary process is of first order can be obtained by examining the specification of the first-order autoregressive model $\tilde{\Delta}_t = \gamma_0 + \gamma_1 \tilde{\Delta}_{t-1} + \tilde{\varepsilon}$. The model does not show significant specification problems for all periods, the residuals do not appear to be autocorrelated and the Durbin-Watson statistic is 1.87, 1.80 and 2.08 for 30-day, 60-day and 90-day periods, respectively. The U.S. inflationary process appears to have different characteristics. The results of Fama [3] suggest that the U.S. process is dominated by a long term component and is not autoregressive of first-order[14]. Given the characteristics of the U.S. process one should expect to find that, in the U.S., the predictive efficiency of the general ARIMA model is greater than the predictive efficiency of the first-order autoregressive model, as implicitly observed by Nelson and Schwert [10]. Moreover, given the above results, one should not expect a significantly greater predictive efficiency of the general ARIMA model in Brasil. In sum, inflation in Brazil not only varies within a fairly wide range but it varies in a very volatile way[15].

Any empirical work on inflation faces the technical and conceptual problems associated with price indices. Among the technical problems that may introduce spurious autocorrelation in price indices one could mention inadequate sampling, non-clearing prices[16] and lags in reporting the indices. These problems are extensively discussed by Fama [3] and [4]. Among the conceptual problems one could mention the static and dynamic problems of determining a "market" basket of goods as well as the index grouping problem. In a world with multiple goods and several individuals with different tastes it is unclear how one could exactly determine the "market" basket of goods for a given set of prices without extensive information from every individual. Moreover, as the vector of prices change the relevant basket for determining the level of inflation may differ from the new "market" basket of goods. Finally, depending upon the preferences of individuals the structure of the

adequate price index may be multiplicative and not additive as in the usual weighted averages[17]. These conceptual problems appear to be as relevant as the technical problems discussed by Fama. This work is no exception and faces those problems. However, to minimize any problems of spurious autocorrelation in the price index, this paper will proceed to empirical tests using the Brazilian WPI. As shown in Table 1 it shows smaller levels of serial autocorrelation than the GPI[18].

IV. THE DATA AND EMPIRICAL RESULTS

This section will examine the efficiency of inflation expectations using relations (2) and (3) for maturities of 30, 60 and 90 days. For these maturities rates of return on Brazilian Treasury Bills at the last trading day of every month were obtained from the National Association of Open Market Investors (ANDIMA), they form the data basis of this study[19]. Trading in Treasury Bills in Brazil began in 1971 and the market rapidly grew, today the total amount of bills issued is larger than the aggregate of all other government bonds[20]. As in any newly established market, trading in Treasury Bills in late 71 and early 72 may have been atypical. To avoid eventual problems of market maturity this work examined the market only after August 1972 and up to November 1976. A few results on inflation and Bills in this period appear in Table 2. The first four rows show the means (μ) and standard deviations (σ) of inflation and rates of return on Bills. The average inflation is greater than the average nominal return on Bills for every maturity. This suggests that the expected real return on Bills is negative, a result apparently inconsistent with a market of rational investors without "money illusion". Brazilian commercial banks could deposit up to 55% of their reserve requirements in Treasury Bills at their face value during the period covered by this work. This allowed banks to earn interest on resources with a null opportunity cost, i.e., a large fraction of reserve requirements and thus a negative expected real rate of return on Bills is not inconsistent with a market of rational investors. The standard deviation of the inflation series is also much larger than the standard deviation of the series of nominal returns on Bills.

The last three rows of Table 2 show for the August 72/November 76 period, the coefficient of serial autocorrelation in the inflation series, $\rho(\tilde{\Delta}_t, \tilde{\Delta}_{t-1})$, the correlation between inflation and returns on Bills, $\rho(\tilde{\Delta}_t, {}_{t-1}\tilde{R}_t)$, and the correlation between lagged inflation and returns on Bills, $\rho(\tilde{\Delta}_{t-1}, {}_{t-1}\tilde{R}_t)$. Recall that in the 1968/1976 period the inflationary process appeared to be of first-order and, as shown in Table 1, the first-order correlation for the WPI was .657, .459 and .438 for 30, 60 and 90-days periods, moreover, the first two were significant at the 1% level and the last at the 5% level. In the 72/76 period the inflationary process appears to be even more volatile, for the shorter periods of 30 and 60-days the level of serial correlation increased but for 60-days it is significant only at the 5% level. For the longer period of 90-days the level of serial correlation decreased

Period		30	60	90
$\tilde{\Delta}_t$	μ	2.188	4.419	6.703
	σ	1.162	2.243	3.377
${}_{t-1}\tilde{R}_t$	μ	1.610	3.253	4.823
	σ	.569	1.072	1.529
$\rho(\tilde{\Delta}_t, \tilde{\Delta}_{t-1})$		$.741^A$ (7.78)	$.475^B$ (2.65)	.337 (1.38)
$\rho(\tilde{\Delta}_t, {}_{t-1}\tilde{R}_t)$		$.471^A$ (3.22)	$.439^B$ (2.39)	.466 (2.04)
$\rho(\tilde{\Delta}_{t-1}, {}_{t-1}\tilde{R}_t)$		$.457^A$ (3.35)	$.552^B$ (2.50)	$.601^B$ (2.47)

TABLE 2

A FEW RESULTS ON INFLATION AND BILLS

μ = mean, σ = standard deviation, ρ = correlation

t-values appear in parenthesis below the estimate of the correlation coefficient. A, B indicate significance at the 1% and 5% levels, respectively.

and is not significant any longer. These results suggest that in this period the process "looses memory" even faster. Inflation todays conveys information about inflation next month, it conveys some information about inflation two months ahead and little information about inflation three or more months ahead. The results on the correlation between inflation and Bill returns shown in the last two rows of Table 2 are also of interest. The correlation between returns on Bills and inflation in the past period is greater and more significant than the correlation between returns and the unknown inflation in the base period, an intriguing result that probably should be attributed to lags in reporting the index.

Additional results on the structure of the process that generates returns on Bills appear in Table 3. It shows, for various lags, the serial autocorrelation in the series of nominal returns, real returns and first differences of nominal returns on Bills[21] as well as the t-values of the slope coefficients in simple autogressions. For maturities of 30 and 60 days and nominal returns all coefficients of correlation (up to the 10-th lag) are significant at the 1% level with t-values as high as 40.7 for the first lag and a maturity of 30 days. For the 90-day maturity the first five autocorrelations are significant at the 1% level and only the 9-th and 10-th autocorrelations are not significant at the 5% level. The strong autocorrelative structure of nominal returns may be due to the strong autocorrelative structure of inflation. If this is the case one would expect to find small and non-significant levels of autocorrelation in the series of real returns. As shown in Table 3 they are, in general, non-significant. The exception occurs for 30-day ex-post real rates which show a high first-order autocorrelation that is significant at the 1% level, moreover, for all maturities the first-order autocorrelation is positive. These results are not necessarily inconsistent with an efficient market[22]. Notice that even though nominal returns show a strong autocorrelative structure their first differences do not show any evidence of serial autocorrelation. As shown in Table 3 their autocorrelation coefficients are not significant for all maturities and all lags but there exists a predominance of positive coefficients. The results thus suggest that nominal rates follow a martingale process[23].

It is now time to proceed to empirical tests of the joint hypothesis that total real rates are constant and inflation expectations are efficient, using relations (2) and (3). The results for the case of n=1, i.e., the case of single-period antecipatory efficiency, are shown in Table 4 for all three maturities. The

Series	$_{t-1}R_t$			Ex-Post $_{t-1}r_t$			$_{t-1}R_t - {}_{t-2}R_{t-1}$		
Lag \ days	30	60	90	30	60	90	30	60	90
1	.985[A] (40.65)	.980[A] (23.92)	.953[A] (12.21)	.673[A] (6.31)	.279 (1.40)	.055 (.20)	.100 (.68)	.356 (1.83)	.145 (.54)
2	.971[A] (28.08)	.954[A] (15.28)	.905[A] (7.95)	.195 (1.35)	-.148 (-.70)	-.176 (-.65)	.071 (.48)	.283 (1.39)	.362 (1.40)
3	.958[A] (22.84)	.920[A] (11.04)	.830[A] (5.36)	-.118 (-.80)	-.045 (-.22)	.000 (.00)	.110 (.74)	.202 (.95)	.122 (.43)
4	.942[A] (19.14)	.885[A] (8.71)	.736[A] (3.76)	-.187 (-1.28)	-.100 (-.45)	-.283 (-.98)	.214 (1.48)	.341 (1.62)	-.000 (-.02)
5	.921[A] (15.84)	.844[A] (7.06)	.677[A] (3.06)	-.095 (-.63)	-.148 (-.65)	-.089 (-.28)	.100 (.66)	.045 (.18)	.500 (1.83)
6	.896[A] (13.37)	.820[A] (6.26)	.661[B] (2.79)	-.000 (-.01)	-.255 (-1.12)	.032 (.09)	.089 (.60)	.274 (1.21)	-.259 (-.80)
7	.868[A] (11.45)	.811[A] (5.88)	.849[A] (4.81)	-.055 (-.34)	-.000 (-.04)	.141 (.40)	-.071 (-.45)	.597 (3.07)	.567 (1.95)
8	.846[A] (10.28)	.752[A] (4.71)	.696[B] (2.74)	-.089 (-.58)	-.126 (-.51)	.410 (1.19)	-.000 (-.06)	.259 (1.07)	.071 (.18)
9	.830[A] (9.53)	.765[A] (4.75)	.542 (1.71)	-.161 (-1.03)	-.071 (-.26)	.414 (1.11)	.373 (2.54)	-.130 (-.51)	.637 (2.03)
10	.812[A] (8.81)	.832[A] (5.82)	-.100 (-.25)	-.221 (-1.42)	.138 (.53)	-.542 (-1.44)	-.134 (-.84)	.032 (.14)	.224 (.51)
No. of Obs in Series	52	27	18	51	26	17	51	26	17

TABLE 3

SERIAL AUTOCORRELATION OF NOMINAL RETURNS, REAL RETURNS AND FIRST DIFFERENCES OF NOMINAL RETURNS ON TREASURY BILLS

t-values appear in parenthesis below the estimate of the correlation coefficient.

A, B indicate significance at the 1% and 5% levels, respectively.

first three rows show the results of tests of relation (2), i.e., $\Delta_t = a_{01} + a_{11}\ {}_{t-1}R_t + \tilde{\varepsilon}_1$. For all maturities the intercepts were positive but not significantly different from zero at the 5% level. This suggests that expected real rates on Bills are negative, a result consistent with the findings of Table 3. The slope coefficients are not significantly different from 1 at the 5% level and are in general very close to 1. The serial autocorrelation of the regression residuals up to the 4-th order are also shown in the table. For maturities of 60 and 90 days they are not significantly different from zero at the 5% level for all four lags. For the 30-day maturity the first-order autocorrelation of the residuals is great and significant at the 1% level, for higher order lags the autocorrelation is not significant at the 5% level. The significant first-order autocorrelation for the 30-day maturity implies that the joint hypothesis of constant real rates and market efficiency should be rejected, at least for this maturity. In contrast, the results of the tests for maturities of 60 and 90 days seem to support the joint hypothesis.

The results of the empirical tests of relation (3) for n=1, i.e., $\Delta_t = a_{01} + a_{11}\ {}_{t-1}R_t + a_{21}\ \Delta_{t-1} + \mu_1$, are shown in the 4-th to 6-th rows of Table 4. One should recall the first-order autoregressive structure of inflation in Brazil. This suggests that estimates of Δ_t that are a linear function of Δ_{t-1} are more efficient in Brazil than in the U.S. and thus empirical tests of relation (3) using Brazilian data are less subject to the efficiency problems raised by Nelson and Schwert [10] when discussing the work of Fama [3]. Also recall from Table 2 that the coefficient of correlation between Δ_{t-1} and ${}_{t-1}R_t$ is significant at the 1% level for the 30-day maturity and at the 5% level for the other maturities. Empirical tests of relation (3) are thus subject to significant multicollinearity problems. These problems seem to appear in the tests for maturities of 60 and 90-days where none of the coefficients are significant at the 5% level[24]. For these maturities the residuals do not show any significant serial autocorrelation coefficient for lags of up to the 4-th order. Technically these results are consistent with the joint hypothesis being tested but one cannot avoid suspecting that the multicollinearity problems may be hiding the eventual significance of a_{21}. The results for the 30-day maturity are again in contrast with the results for the other maturities, a_{21} is significant at the 1% level and the first-order autocorrelation of the regression residuals is significant at the 5% level.

The results of the tests of relation (2) and (3) for n=1 converge in rejecting the joint hypothesis on efficiency and total real rates for the short 30-day maturity. The significant first-order serial autocorrelation of the residuals of relation (2) is attenuated by the inclusion of Δ_{t-1} in relation (3) but the first-order serial

Indep. Variables	Period (days)	a_{0n}	a_{1n}	a_{2n}	R^2	$S(\tilde{\mu})$	ρ_1	ρ_2	ρ_3	ρ_4
$_{t-1}R_t$	30	.816 (1.80)	.853^A (3.22)	-	.174	1.066	.675^A (6.34)	.202 (1.42)	-.095 (.66)	-.167 (-1.14)
	60	1.432 (1.09)	.918^B (2.39)	-	.192	2.058	.286 (1.43)	-.138 (-.65)	-.045 (.19)	-.089 (-.41)
	90	1.737 (.68)	1.030 (2.04)	-	.218	3.085	.045 (.19)	-.179 (-.66)	-.000 (-.00)	-.288 (-.99)
$_{t-1}R_t$, Δ_{t-1}	30	.330	.204 (.92)	.699^A (6.41)	.557	.623	.288^B (2.08)	-.197 (-1.38)	-.205 (-1.43)	-.167 (-1.14)
	60	1.226	.531 (1.19)	.334 (1.57)	.271	3.990	.063 (.29)	-.219 (-1.05)	.000 (.02)	-.000 (-.08)
	90	1.739	.913 (1.40)	.087 (.30)	.222	10.131	-.000 (-.05)	-.182 (-.67)	.032 (.09)	-.277 (-.96)
$_{t-2}R_t$	30	.739 (1.47)	.905^A (3.09)	-	.166	1.069	.675^A (6.26)	.202 (1.40)	-.077 (-.53)	-.114 (-.75)
$_{t-3}R_t$	30	.684 (1.30)	.959^A (3.06)	-	.163	1.066	.677^A (6.30)	.230 (1.60)	-.000 (-.15)	-.084 (-.55)

TABLE 4

THE RESULTS FOR $\Delta_t = a_{0n} + a_{1n}\ {}_{t-n}R_t + a_{2n}\ \Delta_{t-n} + \mu_n$

$S(\tilde{\mu})$ = standard deviation of residuals,

ρ_i = i-th order autocorrelation of residuals,

t-values appear in parenthesis below the estimates,

A, B indicate significance at the 1% and 5% levels, respectively.

autocorrelation of the residuals of relation (3) are nevertheless significant. This suggests that the results should not be merely attributed to market inefficiency. In any event whatever is causing the problems of model specification for the 30-day maturity disappears in the longer maturities of 60 and 90 days. For these maturities relations (2) and (3) again converge in accepting the joint hypothesis. It is of interest to test if the model specification problems occur only for the next 30-days period or if it occurs in other subsequent 30-day periods, i.e., within a 90-days horizon the problems occur only in the first 30-days period or do they occur in all three non-overlapping 30-day periods? The last two rows of Table 4 show the results of empirical tests of relation (2) for the 30-day maturity and n = 2,3; they are tests of two and three-period anticipatory efficiency under the assumption that total real rates are constant. The results are remarkably similar to the results in the first row for n = 1 . The R^2, standard deviation of the residuals and coefficients of serial autocorrelation of the residuals are very close for n = 1, 2, 3. The inflation coefficients show a slight increase as n increases but are always significant at the 1% level. The model specification problems shown by the regression residuals appear for all n, the coefficient of first-order serial autocorrelation of the residuals are always significant at the 1% level. In sum, whatever happens and causes the specification problems in the first forward 30-days period is also happening and causing specification problems in the second and third forward 30-day periods. Nevertheless, the problems are diversified away in the aggregate 90-days period. It seems relevant to discuss reasons that may explain these findings but they suggest that one should be careful before generalizing Fama's [4] conclusion that "the (nominal) interest rate remains the best single predictor of the inflation rate" to all markets and kinds of inflationary processes.

V. REVIEWING THE RESULTS

The joint hypothesis of constant total real rates and efficient capital markets does not appear to be supported by the Brazilian data. Of course these results could be attributed to inadequate data. The technical problems with inflation data were extensively discussed by Fama and to these problems one could add the conceptual problems of changing relative prices. Given the level and volatility of Brazilian inflation these problems may be more serious here than with U.S. data. The data on returns to Treasury Bills may also contain measurement erros It was obtained by sampling the major dealers and their "virtual" spreads[25]. Since the dealers know that their quoted spreads will not generate a transaction they may bias the spreads. Even though this bias tends to be diversified across the samples, it could conceptually be strongly autocorrelated across dealers and bias the averages reported by ANDIMA which were used in this work. However, one should recall that the joint hypothesis is rejected only for the 30-days maturity. For the 90-days maturity the joint hypothesis is strongly supported and the level and t-value of the first-order residual autocorrelation is very low. For the 60-days maturity the joint hypothesis is accepted but the level and t-value of the autocorrelaiton of the residuals of relation (2) are not very low. In sum, there appears to exist an association between maturity and adjustment to the joint hypothesis, for 30-days the adjustment is significantly poor, for 60-days the adjustment is not significantly poor but it does not seem to be perfect and for 90-days the adjustment seems to be very good. Even though there may exist measurement errors in the data it does not seem reasonable to attribute these results only to them[26].

It is relevant to examine possible fundamental reasons, beyond errors in the data, which could explain the findings. As discussed in section II the total real forward rate demanded by investors ($_{t-n}r^*_t$) is the sum of two components: the basic real forward rate demanded by investors to hold a perfectly indexed and riskless bond ($_{t-n}r_t$) and the real forward risk premium demanded by investors to hold pure inflation risk ($_{t-n}p_t$). The empirical results for the 30-days maturity could thus be attributed to inefficiency, variable basic real rates, variable real forward risk premiums or a combination of these factors. The Brazilian Treasury Bills market is a very active market with a relatively small number of active dealers[27] handling a great number of large transactions. Over one-half of total outstanding government debt is composed of Bills and this gives an idea of the volume of

transactions in the Bills market. Is is difficult to believe that a systematically inefficient dealer could last very long in such a market. It seems reasonable to assume that the market is efficient. Under this assumption and further assuming that basic real rates ($_{t-n}r_t$) are constant[28] it follows that the empirical results should be attributed to the variability in real forward risk premiums demanded by investors to bear inflation risk.

The results of Table 1 suggest that in the 1968-1976 period inflation in Brazil is autoregressive of first-order and in general significantly autocorrelated within six-month periods, i.e., inflation this month conveys little information about inflation six months ahead. The results of Table 2, for the 1972-1976 sub-period, suggest that recently inflation has been even more autoregressive and volatile than in the overall 1968-1976 period. As discussed before, in the sub-period inflation this month appears to convey little information about inflation three months ahead. The data and tests of the Bills market cover the 1972-1976 sub-period and these characteristics of the inflation process may help to explain the findings. If the ex-post inflation series is strongly autoregressive it seems reasonable to assume that the time series of ex-ante inflation risk is also autoregressive of first-order, i.e., it seems reasonable to assume that the autoregressive structure of the two series is identical[29]. Under this assumption and given the characteristics of the Brazilian inflationary process, it follows that inflation risk this month conveys little information about inflation risk three months ahead. The level of inflation risk in non-overlapping 90-days periods would fluctuate randomly, inflation premiums would also fluctuate randomly and one would not expect to find specification problems in tests of relations (2) and (3). On the other hand, for shorter maturities and specially for the 30-day maturity, inflation risk this period conveys significant information about inflation risk next period. Risk premiums for the shorter 30-day maturity should also be serially autocorrelated and one would expect to find specification problems in tests of relations (2) and (3). The results of Table 4 are thus consistent with the joint hypothesis that

(i) the Bills market is efficient,

(ii) basic real rates in Brazil are constant and

(iii) the level of inflation, inflation risk and risk premiums follow the same autoregressive structure.

Also notice that this joint hypothesis is consistent with the results on the autocorrelation of real rates on Bills that are shown in Table 3.

VI. CONCLUSIONS

The results of this paper suggest that the nominal interest rate is not an efficient predictor of inflation in Brazil. The finding is in contrast with Fama's [3] and [4] findings for the U.S. market. The Brazilian inflation process appears to be much more volatile and autoregressive of first-order than the U.S. process. The divergence in the findings in the two markets should probably be attributed to the different characteristics of their inflationary processes. Under conditions of uncertain inflation investors will demand a risk premium to bear pure inflation risk. In Brazil inflation has varied over a very wide range and it seems reasonable to assume that the level of inflation risk and risk premiums has varied widely. It appears to be relevant to consider the process followed by risk premiums whem examining the efficiency of inflation expectations in any Treasury Bill market. Even though the joint hypothesis that total real rates are constant and inflation expectations efficient is rejected by Brazilian data, the results are consistent with the joint hypothesis that, in Brazil,

(i) the Bills market has efficient expectations of inflation,

(ii) basic real rates are constant and

(iii) the level of inflation, inflation risk and risk premiums follow the same autoregressive structure.

FOOTNOTES

(01) There are well known static and dynamic problems when dealing with price indices and indexation. See Michael [9] and Lloyd [8]. Most of this paper will proceed with the usual simplifying assumption of a single-good world.

(02) This real forward rate may contain an eventual liquidity premium.

(03) For a discussion of the structure of interest rates under uncertain inflation see Brealey and Shaefer [1]. Given their assumptions, an additive risk premium will be demanded by investors to bear inflation risk, as shown in the text.

(04) There exists a market on indexed government bonds in Brazil. Unfortunately the bonds are not "perfectly" indexed and trading in the short maturities of Treasury Bills is not very active.

(05) More generally, one could assume that ${}_{t-n}\tilde{r}^*_t = {}_{t-n-1}r^*_{t-1} + \tilde{\mu}$ where $\tilde{\mu}$ is a "white noise" component. If total real forward rates are not constant but follow such a martingale process one can also reduce equation (1) to the testable relation (2). The "white noise" component $\tilde{\varepsilon}_n$ of relation (2) would be viewed as the aggregation of the inflation forecasting error and $\tilde{\mu}$. This is equivalent to assume that total real forward rates are constant and inflation forecasting errors have a larger variance. Such a simpler view of the process was chosen in this paper.

(06) I.e. he examines relation (2) only for the one-period case of n=1 without discussing the multiple-period case of n > 1.

(07) More generally, they argue that total real forward rates do not follow a martingal process. See footnote (5).

(08) The arguments of Carlson are more general than that. He essentially argues that any function of ϕ_{t-1} should have a null coefficient in relations like (3).

(09) This section examines the Brazilian inflationary process from January 1968 to December 1976. This represented 108 observations of the GPI and 89 observations of the more recent WPI. All tests reported here were performed with the two price indices and the two inflation indices. The choice of an inflation index did not effect the results and thus we have chosen to follow common practice.

(10) The autocorrelation coefficients were estimated as the square root of the R-square in simple regressions with lags of multiple order. The t-values reported were the ones of the slope coefficients in these regressions.

(11) I.e., six 30-day periods, three 60-day periods and two 90-day periods.

(12) The exception occurs for the GPI and 30-day periods where the autocorrelation is significant for lags of 7-th up to 9-th order. This is probably caused by spurious

autocorrelation in the GPI.

(13) If the process is autoregressive of the type $\Delta_t = a_n + b_n \Delta_{t-n} + \varepsilon_n$ and if ε_n is independent of the whole inflation series then in empirical tests of $(\Delta_t - \Delta_{t-1}) = \alpha_n + \beta_n (\Delta_{t-n} - \Delta_{t-n-1}) + \mu$ one would expect to find $b_n = \beta_n$, for all n. The result in the text follows for b_n and β_n are the estimates of the n-th lag autocorrelation in the series of inflation and of first differences in inflation, respectively.

(14) The results of Fama's [3] Table 1 show that the levels of serial autocorrelation in the U.S. inflation series are approximately equal for all reported lags, varying around .30. They do not appear to follow a geometric progression as expected in a first-order process. Unfortunately Fama does not report the levels of serial autocorrelation in the $(\Delta_t - \Delta_{t-1})$ series nor does he report any results on the specification of the first-order autoregressive model. However, if real rates are constant (as his results suggest), the level of autocorrelation in the series of first differences in nominal interest rates $(R_t - R_{t-1})$ should be equal to the level of autocorrelation in the $(\Delta_t - \Delta_{t-1})$ series and thus equal to the level of autocorrelation in the inflation series itself, if the U.S. process were of first-order. The levels of autocorrelation in $(R_t - R_{t-1})$ for the U.S. appear in Fama's Table 5 and differ from the levels of autocorrelation in Δ_t reported in his Table 1. This also suggests that the U.S. process is not of first-order.

(15) This probably reflects the instability that appears to caracterize monetary policy in Brazil.

(16) The existence of wage and price controls will destroy the representativeness of the index if it generates shortages of goods and services. During the period examined in this paper there existed wage and price controls in Brazil. However, there is not any evidence of shortages of goods and services in Brazil during the period.

(17) These conceptual problems are discussed in Michael [9], Lloyd [8] and Grauer and Litzenberger [6].

(18) All items of the WPI are sampled monthly and, as opposed to the GPI, services are not included in the index. Given the relevance of public services in Brazil and th close government control of their prices one should expect to find more serious problems of spurious autocorrelation in the GPI, as suggested by Table 1.

(19) All major traders in the Brazilian open market are associates of ANDIMA. At the end of every trading day ANDIMA samples the major traders collecting their bid-ask spreads and reports the average bid price and the average asked price. The rates of return used in this paper were obtained from the average of these reported

average prices. Brazilian Treasury Bills are discount bonds and reported "prices" are in effect reported "discounts". For a given period the rate of return in the Bill is equal to its face value minus the transaction price divided by the transaction price.

(20) Government debt in Brazil is composed of (not perfectly) linked bonds (the ORTNs) and Treasury Bills (the LTNs). Most of the bills are issued with maturities of 91 and 182 days but there is a small amount of bills issued with a one-year maturity. Most of the ORTNs are issued with maturities of 4 and 5 years.

(21) The coefficients of correlation were estimated as the square root of the R^2 in simple autoregressions of multiple order.

(22) Recall that lack of serial autocorrelation is not a necessary condition for efficiency. Examine relation (1), if the level of inflation risk and inflation risk premiums ($_{t-n}\tilde{p}_t$) are non-stationary and serially autocorrelated then the results of empirical tests of relation (2) would be biased towards residual autocorrelation. In this case autocorrelation indicates model specification problems and it does not indicate market inefficiency.

(23) Notice that these results for first differences of nominal returns are also consistent with a first-order autocorrelation coefficient of nominal returns close to 1, as found before and also shown in Table 3. The same results were obtained by Fama [3] for the U.S. market.

(24) The results of Fama [3] also show multicollinearity problems. At a conceptual level he finds that Δ_t is significantly correlated with Δ_{t-1} and $_{t-1}R_t$. This suggests that Δ_{t-1} and $_{t-1}R_t$ are significantly correlated, however, Fama does not report any tests of this correlation coefficient.His empirical results on tables 8 and 9 show some evidence of multicollinearity, the standard deviations of the coefficient of nominal rates increase when Δ_{t-1} is included in the regression. For the longer maturities the increase is proportionally large and the changes in the estimates of the coefficients are large as well.

(25) Virtual in the sense of not being a spread that may generate a transaction, it supposedly represents the operator "feeling" of the market at the end of the day.

(26) I.e., if the joint hypothesis were true and deviations were caused by measurement errors one would not expect to find this maturity effect. To justify this relation one would have to make assumptions about the distribution and nature of measurement errors by maturity, something that may be questionable.

(27) In 1976 the market had approximately 100 dealers with 30 of them handling the great majority of the transactions.

(28) The outstanding government debt is composed of indexed bonds and Bills (see footnote 20). Presumably one could estimate the basic real forward rates in the economy from the real stream associated with indexed bonds. In effect preliminary estimates indicate that basic real forward rates are stable over time. However this evidence should only be considered as suggestive, the bonds are not perfectly linked but are tied to three month averages of the price level, moreover, interest is paid on a six-month average of face values. At this stage it is not clear how one could obtain "clean" real streams associated with the bonds and "clean" estimates of the basic real rates. This paper thus proceeds assuming that basic real rates are constant.

(29) Notice that this assumption is weaker than Gordon and Halpern's [5] assumption of an association between the level of inflation and the level of inflation uncertainty. Their assumption implies the assumption of identical autoregressive structure of this work but the reverse is not true.

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