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THE PUBLIC INVESTMENT DECISION UNDER UNCERTAINTY: A MEAN VARIANCE SYNTHESIS

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I. INTRODUCTION

There are basically two types of arguments advanced in the economics literature to justify the participation of government as as investor in a particular economic activity. The first one is based on the premise that from the viewpoint of society, the benefits or costs associated with a certain project may be different form those that arise of the project were undertaken by private agents in the economy. Examples of this situation arise with natural monopolies (e.g. municipal services), projects with positive externalities (e.g roads) and public goods (e.g. defense). The second type of argument concerns opportunity costs of time and risk which, for society as a whole, may be different form those for private investiment. In both cases, the rationale for government investment arises from the fact if the private sector is left to its own devices, social resource allocation may be monoptimal: the value of the marginal private net product of the resources employed diverges form the value of the marginal social net product. Such a divergence leads to an over-investment in some activities and to underinvestment in others, relative to the allocation of resources that maximizes social welfare.

This study focuses on the opportunity cost or risk of public investments. In addition, it examines the view that the risk factor, inherent in certain ventures, may itself justify the role of government as an investing agent participating directly in the economic activity, without any of the other considerations playing a role in the decision. This is not to deny the importance of the other factors. Rather, it is to focus on one aspect regarding which there has been considerable discussion in the literature on both financial and public economics.

In a world where individuals display risk-averse behavior, the choice between investment alternatives cannot be made without regard to the uncertainty associated with the future returns of the venture under consideration. The crux of the argument for government investment based solely on risk considerations is that the ability of the individual to privately allocate his risk is not optimal, if private capital markets are imperfect in any sense. In this case, government may be better able to redistribute risks among individuals than the private market. As a consequence, ventures desirable from a social viewpoint and that have not been assumed privately may be undertaken through the government, with a beneficial effect on welfare.

When a private agent undertakes a new risky venture, his welfare is affected in two opposite directions: a positive effect from the marginal increase in the expected value of his future wealth prospects and a negative welfare effect from the marginal increase in the uncertainty attached to these prospects. The risk of the investment represents, therefore, a costa related to the investment to be borne by the individual. The negative welfare implications ot the incremental risk depend on the arrangement by which the risk undertaken by the agent can then be distributed among other members of society. In the extreme case where no risk-redistributing arrangements exist, each individual is an autarkic entity with respect to his investment decisions, facing a limited set of investment opportunities. Also, the amount of resources each individual allocates to new risky ventures is constrained by his risk bearing ability and, if exchange opportunities are also unavailable, to his own endownment. If a risk-redistributing arrangement is created, such autarkic limitations are diminished. This arrangement would allow the risk face by one individual to be reduced by pooling them with those of others. In other words, an individual is not limited to his exclusive investment opportunities and faces an enlarged investment ooportunity set due to the additional vehicles for diversification thar are created. The welfare of each individual is then enhanced owing to the ability to construct portfolios of investments that are better diversified and the consequent benefits from the reduction is risk.

The risk reduction achieved through the redistributing arrangement represents onlya a portion of the beneficial effect of such a mechanism. The more importante effect is related to the fact that some investment opportunities which are hitherto unacceptable in an autarkic world may become desirable when the redistribution of risks is feasible, since the improvement in the opportunities for diversification would, in general, reduce the marginal risk of these ventures. As a result, it is likely that the amount of resources allocated to risky ventures would increase, contributing to an increase in social welfare.

There are several institutional forms that facilitate the allocation of risk bearing. Competitive capital markets providing for trading in stocks, contingent claims and insurance contracts are prime examples of institutions that allow risk shifting between individuals. In fact, when capital and insurance markets are perfect and complete, it can be demonstrated that the allocation of risk bearing among members of society would be Pareto optimal. (see Arrow, 1964). In this case, no other risk redistribution arrangement is necessary, and consequently, no argument can be made for government investment based upon the risk factor exclusively. Any imposed risk sharing attained through the government's redistributing mechanism is, in general, inferior to the voluntary risk sharing attained through market exchange, unless it happens to coincide with the allocation achieved by the market.

It is generally the case, however, that only limited forms of risk shifting contracts are traded in existing capital and insurance markets. In order to overcome this limitation, many non--market institutions have como into existence to provide additional beneficial risk-redistributing arrangements. Bank ruptcy and limited liability laws and vertically integrated business organizations are examples of such non-market arrangements. Similarly, government may be viewed as another non-market arrangement that facilitates the allocation of risk bearing. Assuming an investing role, government becomes a fiduciary agent for all member of society, as corporate management is for stock-holders of privately held firms. In other words, society forms, through government, a coalition for the purpose of collectively undertaking new risky ventures.

In general, government has many instruments available to redistribute risks among members of society. The income taxation mechanism is one such instrument. The working of this mechanism can be easily explained. Suppose that in a given year the government were to have a balanced budget (or a planned deficit or surplus) and that taxes could be reduced or increased depending on the returns on the investment undertaken through government being positive or negative. In this case, when government undertakes an investment, every taxpayer owns a small share of that investment, with returns being paid through changes in the level of taxation. In other words, governement has added, some what arbitrarily, to each individual's portfolio a randon return which is some fraction of the total net returns on the public venture.

It can be concluded from the above that it may be desirable for government to act as an investing agent for all members of society, even in the absence of externalities and/or public goods considerations, if it is able to redistribute the risk associated with new ventures among all individuals in a manner that would affect their future prospects in a more desirable fashion than could have been achieved through imperfect capital and insurance markets. If this is the case, individulas would bear a lower cost of risk when the new venture is undertaken through public investment.

In less developed economies, where capital markets do not function effectively, one might be tempted to argue, based on the government's potentially superior ability to redistribute risk, that all risky ventures should be undertaken only by government. However, this prescription would be inappropriate and inconsistent with be observed reality that even in such economies, a large proportion of risky ventures are, in fact, undertaken privately. The crucial element missing in this argument is that it makes no reference to the issue of operational efficiency. Is is implicitly assumed that the net benefits realized form investment are invariant with respect to the institutional form of owership claims on the returns of the enterprise. When the ownership structure of the venture plays a role in the venture's prospects, it may be desirable for the owner-manager to bear some of the risk. And. in the extreme, private ownership could be beneficial in cases where the impact of the ownership-management factor on the quality of returns prospects is significant.

The primary purpose of this study is to provide a synthesis of the above arguments within a common framwork. While some of these points have been made earlier in the literatur, it is often unclear as to what assumptions are made in each case and how they relate to the rest of the literature. Although almost all arguments made previoulsy will be found here, no attempt is made here to provide a comprehensive survey of all published papers in the area. This study is developed as follows: Section II reviews the current literature relating to the cost of risk for public ventures from the perspective of the available risk redistributing arrangements, assuming away the effect of ownership on operational efficiency. Section III presents a mean-variance model which puts into perspective the specifics of the discussion presented in Section II. Section IV introduces the ownership issue and relates this aspect to the general problem of agency. Section V expands on the model developed in Section III in order to account for the effect of the ownership aspect on the cost of risk of public ventures. A conclusion follows in Section VI.

II. PUBLIC INVESTMENT AND THE EFFICIENT ALLOCATION OF RISK BEARING

The proper normative rule for public investment decisions remains inconclusive despite considerable academic discussion on the subject. The existing literature on public investment assumes away the effect of ownership on operational efficiency and focuses exclusively on the problem of allocative efficiency. In this context, much of the discussion centers on the question of the cost of capital for public ventures. The cost of capital or required rate of return for a public venture, is used as a cutoff rate in public investment and affects the allocation of resources in the economy. For instance, if the cost of capital for public ventures is incorrectly set low, resources would be transferred from private ventures with good retunr prospects to public ventures with poor return prospects and vice versa.

It is generally accepted that for an efficient allocation of resources between the private and public sectors, the cost of capital, should measure the social opportunity cost of the resources channeled to these ventures. In addition, htere seems to be general agreement on the components that should be considered in measuring the cost of capital. As Baumol (1968, p. 788) points out, these components are:

> "the welfare forgone by not having the benefits available for immediate private consumption and ... a premium corresponding the risk incurred in undertaking government projects".

The proper measure of the first component, which is a function of the individual's time-preferences, has been largely addressed elsewhere and is not our concern here.[1] The second component, which is a function of the individual's risk preference, is of interest to us in this paper. Bailey and Jensen (1972, p. 272) put into perspective the concept that should underlie the measure of the cost of risk of public ventures. In their words:

> "To appraise the risks, we have to look at the distribution of

the cost risks and benefit risks among households and their effects on the portfolios ot these households".

The risk premium required by each individual depends on the marginal increase in the risk of his future prospects, due to the share of the public venture's return allocated to him. From a social viewpoint, the risk premium required for public ventures is a weighted average of the individual's required risk premia. The weights are a function of the percentagens of the net public venture benefits allocated to each individual. Consequently, the required risk premium for public ventures depends on the existing risk distribution mechanisms. In this respect, the ability of individuals to redistribute risk among themselves in the private sector is a critical factor.

The disagreement in the existing literature with regard to the cost of risk for public ventures stem mostley form the assumptions made, implicitly or explicitlye, regarding the risk redistribution arrangements available. The relevant characteristics of these arrangements concern the functioning of private capital (and insurance) markets; specifically,whether they (1) are perfect, (2) are imperfect, or (3) function at all. If perfect and complete private capital markets are assumed to exist, an ideal opportunity for diversification is available in the private sector. In this instance, the distribution of risks is accomplished in the most efficient manner and therefore, society's risk-bearing ability is optimized. The ability of perfect and complete capital markets to promote optimal risk sharing is demonstrated by Debre (1959) and Arrow (1964). Even if the government does not use the private capital markets to finance the public project, any risk misallocation produced by an arbitrary allocation of shares of the public venture through government's instruments can be undone by the individual in the capital markets, since they are assumed to be complete. On these grounds, Hirshleifer (1965 and 1966) and Baumol (1968) argue that no difference sould exist in the cost of risk between private and public ventures belonging to the same risk class. Implicit in this argument is the assumption that the net benefits of all public ventures are marketable (i.e., transferable), so that no problems of public goods or externalities exist.

The ideal functioning of the private market to redistribute risks can be achieved even with incomplete markets if special additional assumptions are made. For instance, Diamond (1967) demonstrates that the private investment allocation in the economy would still be Pareto optimal with incomplete but perfect capital markets, under the assumption that the consumption pattern across the states of nature is a linerar combination of the production output patterns. The same result is also obtained by JEnsen and Long (1972) and Merton and Subrahmanyam (1974) with the more specific assumption that an individual's ability to distinguish between uncertain payoff schemes is reduced to two parameters (mean-variance) and perfect competition is assumed to exist in all sectors of the economy[2].

As one would expect, Hirshleifer's argument is confirmed for perfect but incomplete markets in the above instances. However, because capital markets are incomplete, it is possible for the returns of public ventures to be non-transferable (i.e., nonmarketable). The relevance of the nonmarketability aspect on the measure of the cost of risk for public ventures is pointed out by Bailey and Jensen (1972). They note that the methods usually employed by government to redistribute the costs and benefits associated with a public project may not be optimal in terms of allocation of risk bearing. If individuals can perfectly compensate for this misallocation by trading in complete and perfect markets, the public investment decision can once agina be reduced to that of a purely private economy. However, if capital markets are imperfect in any respect, investors cannot adjust their positions in the capital markets and the resultant allocation of risk bearing is non-optimal. As a consequence, they suggest that public ventures should command a higher risk premium than similar projects in the private sector.

The effect of the nonmarketability aspect in the investment decision of public enterprise in analyzed by Stapleton and Subrahmanyan

(1978). In computing the cost of capital for public enterprises, they conclude (p. 407) that

> "... the public investment level, should be reduced from tis level in the pure provate case in order to improce aggregate welfare. Thighter investment criteria should be adhered to in the public sector after nationalization. In other words, the cost of capital to he pbulic firm... is higher than it would be if the firm were private".

Combining the above arguments, one can conclude that if capital markets are perfect and complete, they provide an ideal mechanism for edistributing risks. As a consequence, the risk premium required on a given venture should be identical for both private and public sectors. Where the risks from the public ventures are non--transferable due to market incompleteness, the imperfect redistribution of riks increases the cost of risk for such ventures vis-a-vis similar projects in the private sector.

This conclusion is, however, no longer valid when some practical considerations are taken into account. When the redistribution of risk through the market mechanism is also imperfect, market returns can no longer provide a benchmark for the cost of risk of public ventures. For this reason, Bailey and Jensen (1972, p. 289)conclusion is somewhat misleading:

> "If private markets in risk are as 'imperfect' as many have claimed, that merely tends to increase the rate (cost of capital) that should be used, because government is even less albe to distribute risk than are these markets."

The reason is that when capital markets are imperfect, limitations exist in the private sector for individuals to fully diversify their portfolios. Therefore, conditions might exist where society benefits by redistributing risks through the government's fiscal instruments. Conceivably, situations could exist where the cost of risk for investments undertaken through the government would be lower than if they were borne privately.

Samuelson (1964) concludes correctly, but for the wrong reason, that the cost fo risk for ventures undertaken by the public sector might be lower than its cost in the private sector, where capital markets are imperfect. Commenting on a paper by Hirshleifer, Samuelson (1964, p.96) states

> "One can look at much of government as primarily a device for mutual reinsurance. General Motors can borrow at a lower rate than American Motors because it is a pooler of more independent risks. It would absurd for G.M. to apply the same high risk-interest-discount factor to a particular venture that A.M. must apply. The same holds for We, Inc., which is a better pooler of risks than even G.M. ...
>
> Obsiously, stochastic elements make for bigness and 'collectvism'. Collectivesm need not be governmental; it can also mean monopolies and mutual funs. Oftne, government is one of the 'cheapest' ways of providing insurance against important risks. Why are there no mutual funds that enable me to invest in 15 percent risks (and that could enable society to bring down the cost of risky projects)? If such institutions where efficiently possible and existed, that could drive government out of some activities just as civilian job opportunities drive me out of the peacetime army".

Although he argues that the government is an important device for mutual reinsurance, Samuelson places emphasis on the risk pooling aspect and ignores the redistribution of risks[3]. The simple fact that the government undertakes a large number of risky projects does not imply that each individual's risk is unaffected when a new risky venture is undertaken through government. The best that can be expected is that the risk of the project is not evaluated on its own but as a contribution to the risk of the overall portfolio.

Arrow and Lind (1970) explicitly recognize that the risk redistribution ability of government is the relevant consideration in computing the cost of risk for public ventures. However, they come to the extreme conclusion that such cost is negligible. Their position results, as shown later, from their additional assumption that the new public venture is small and its return is stochastically independent of the existing returns. Arrow and Lind (1970, p.244) state that:

> "When the risks associated with a public investment are publicly borne, the total cost of risk-bearing is insignificant and therefore, the government should ingonre uncertainty in evaluating public investments... This result is obtained not because the government is able to pool investments but because the government distributes the risk associated with any investment among a large number of people. It is the risk-spreading aspect of government investment that is essential to this result".

Unfortunately, the results os Arrow and Lid do not apply to mixed economics where public ventures produce a large share of the national income and the returns form an individual project are likely to depend stochastically on other economic activities. Hence, the cost of risk in public investment cannot be disregarded. Further, given that in such economies, capital markets tend to be imperfect, they do not offer an explicit benchmark for the cost of risk that can be applied to the public investment decision - stock market returns do not convey proper information about hte cost of risk for projects undertaken through the government. In these cases, no simple rules can be offered for the measurement of the cost of risk relevant to public ventures.

Sandmo (1972) sets forth the theorical ground rules under which central planners can encourage private enterprise in an optimal manner. First, he assumes that markets are constrained-complete in the sense of Diamond (1967) (linear sharing rules are feasible) and shows that the social rate of discount is the same as the private cost of capital for equivalent risks and hence the investment allocation is Pareto optimal. He then analyzes a setting where security markets do not function (i.e. an "unincorporated economy") and assumes that government has perfect instruments for risk distribution and shows that the same results hold. In Sandmo's framework, an optimal risk distribution can be achieved either by a (constrained) complete market or by a central planner with perfect instruments for allocation of risk bearing.

Mayshar (1977 and 1979) notes that, in reality, government has limited instruments at its disposal for redistribution of risks.

An important mechanism is the income tax system which is, in general, not optimal from the risk redistribution point of view. Hence, he argues, an additional risk subsidy by government for private projects may partially compensate for this imperfection.

It is apparent form the above discussion that the models available in the literature differ mainly in respect of their assumptions regarding the mechanisms available for risk distribution. In order to focus on these differences and their implications, rather that the specific details of modelling, we use a common framework that is flexible enough to incorporate the various assumptions used in the literatur.

III. A MEAN-VARIANCE MODEL OF THE PUBLIC INVESTMENT DECISION

The considerations discussed above may now be formally modelled. The key feature of the following analysis is the relationship between the specific assumptions regarding the working of private capital market and the public investment decision rules that are appropriate in each case. Since the focus of attention here is on the mechanisms available for redistributing risks, we employ the simplest framework that is rich enough to include all the essential features of the problem.[4] Accordingly, the following assumptions are made:

i. The economy is composed of decision makers with a single period horizon;

ii. There are two classes of individuals in the economy, indexed by 1 and 2;

iii. Two risky investment opportunities, a and b, are available in the economy; the former invesment ("a") having already been made, with the latter ("b") proposed to be undertaken in the public sector;

iv. The returns form the investment opportunities are assumed to be joint normally distributed;

v. Individuals have utility functions for end-of-period wealth of the form $u_i = - \exp(-\gamma_i w_i)$. (See Table I for details of the notation). In combination with (iv) above, this assumption implies that investors have utility functions that depend only on the mean and the variance of their end-of-period wealth prospects;

vi. Decisions regarding consumption at the beginning of the period are assumed to have been taken; and

vii. Riskless borrowing and lending is possible at a given riskless rate of interest.

A few comments are in order regarding the assumptions stated above. The single period assumption simplifies the analysis in order to abstract from problems of multiperiod asset pricing which are not germane to the issues addressed here. Assumptions (ii) and (iii) are not essential but are made for notational simplicity. Assumption (iv) is a sufficient condition for mean-variance analysis, the framework used here. The assumption (v) of exponential utility functions implies that the market price of risk is a constant and hence unaffected by the investment decision analysed here. This avoids the cumber-some comparisons of the market prize of risk that would be involved if other mean-variance utility functions are employed which do not add any insight to the basic problem[5]. The last two assumptions allow us to focus on the risk aspect of the investment decision, rather than pure time value of money.

The variables relevant to the utility functions of the individual are defined in Table 1. Given an initial endowment w_{oi}, each individual faces the following budget equation:[6]

$$w_{oi} = m_i + a_i y_a + b_i y_b \tag{1}$$

where m_i represents individual i's net borrowing or leading. The i^{th} individual's end-of-period wealth, w_i, is therefore given by

$$w_1 = m_i r_f + a_i y_a r_a + b_i y_b r_b \tag{2}$$

Substituting m_i fromt he budget equation (1) into (2) yields:

$$w_i = w_{oi} r_f + a_i y_a \ (r_a - r_f) + b_i y_b \ (r_b - r_f) \tag{3}$$

Since r_a and r_b are joint normally distributed, the expected utility function, u_i, is the moment generating function of multi-variate normal distribution and can be written as:[7]

$$u_i = E_i - \frac{\gamma_i}{2} V_i \tag{4}$$

where E_i and V_i are, respectively, the expected value and the variance of individual i's end-of-period wealth.

TABLE 1

Notation

UTILITY FUNCTIONS:

$$u_i = E_i - \frac{\gamma_i}{2} V_i \qquad \text{for } i = 1,2;$$

where:

u_i : utility of end-of-period wealth of individual i.

i : risk aversion parameter of individual i.

INVESTMENT OPPORTUNITIES

a, b : index of investment opportunity in place and new investment respectively.

r_a, r_b : end-of-period return per dollar invested in investment opportunities "a" and "b" respectively

y_a, y_b : amount invested in the opportunities "a" and "b" respectively.

r_f : riskless return per dollar borrowed or loaned

RISK ALLOCATION VARIABLES

a_i : fraction in investment "a" held by individual i.

b_i : fraction in investment "b" held by individual i.
(By definition, $a_1 + a_2 = 1$ and $b_1 + b_2 = 1$).

Taking the expected value and the variance of individual i's end-of-period wealth given in (3), his utility function (4) becomes:

$$u_i = w_{oi} r_f + a_i y_a [E(r_a) - r_i] + b_i y_a [E\ (r_b) - r_f]$$

$$- \frac{\gamma_i}{2} \left[a_i^2 y_a^2 \text{var}\ (r_a) + b_i^2 y_b^2 \text{var}\ (r_b) + 2 a_i b_i y_a y_b \text{cov}(r_a,\ r_b)\right] \qquad (5)$$

From the perspective of society, the distribution of risks form a new investment can take place either through capital markets or through governmental intervention. As discussed in Section II, with perfect and complete capital markets, it is possible to achieve an optimal allocation of shares in any new venture, whether public or private, since individuals can perfectly "undo" any misallocation of risk bearing. However, if the new venture is in the public sector and its risks, which are allocated amongst the individuals in the society, are even partially non-marketable, the allocation of risk bearing cannot be optimal. Hence, in the absence of perfect and complete markets, it is not possible, in general, to achieve a Pareto-optimal allocation of risk bearing.

In cases where individuals are constrained in their attempt to redistribute the risk of a new venture through portfolio decisions in the rest of their holdings, it is necessary to make welfare comparisons across individuals to verify the desirability of this venture. Thus, Pareto optimal decision rules cannot be derived and it is necessary to make tradoffs between individuals in an expected utility sense. The simplest os such social welfare functions is a weighted average of the expected utilities across individuals. This analysis is in the spirit of Wilson (1968) who is concerned with the conditions under which the decisions of a syndicate are Pareto optimal i.e. in the best interests of all its member[8]. While Pareto-optimality can be achieved with as aggregate criterion under specific sharing rules for other cases, he shows, that for the case of negative exponential utility and normally distributed returns, linear sharing rules are sufficient. Since expected utility is a linear function of the mean and variance of portfolio returns, a social welfare functions based on the simple sum of the expected utilities, i.e.,

$$u_0 = u_1 + u_2$$

can lead to a Pareto optimal allocation, if investors make side payments (i.e. change theri respective $E_{i's}$). Even if such side-payments do not take place, the allocation is optimal in the Kaldor-Hicks sense[9]. We will employ this simples welfare criterion in what follows.

Substituting in u_o for the individual's expected utility functions from (5) yields:

$$u_0 = (w_{01} + w_{02}) r_f + y_a [E(r_a) - r_f] + y_b [E(r_b) - r_f]$$

$$- \frac{\gamma_1}{2} [a_1^2 y_a^2 \text{var } (r_a) + b_1^2 y_b^2 \text{var } (r_b) + 2a_1 b_1 y_a y_b \text{cov } (r_a, r_b)]$$

$$- \frac{\gamma_2}{2} [a_2^2 y_a^2 \text{var } (r_a) + b_2^2 y_b^2 \text{var } (r_b) + 2a_2 b_2 y_a y_b \text{cov } (r_a, r_b)] \qquad (6)$$

The investment "a" assumed as givern, represents the existing risky prospects available to individuals while the collective investment decision regarding the new risky venture "b" is to be made. The essence of the public investment problem is to derive the optimal investing rule for "b", given one of the following three risk redistribution arrangements:

Case I: <u>All risks are marketable through perfect capital markets</u>. Here, the existing risk redistribution mechanism is such that the individuals are free to risks transfer between themselves in an optimal manner[10]. In this case, the optimal investment rule for the public venture is obtained by maximizing the social welfare function with respect to the level of public investment and the risk sharing variables for a given level of y_a. The optimal investment rule is obtained by solving the following program:

Magazine $u_0(y_b,\ a_1,\ a_2,\ b_2 |\ y_a)$

with respect to y_b, a_1, a_2, b_1 and b_2.

Case II: <u>Perfect capital markets but incomplete with respect to new public ventures</u>. In this case, individuals cannot, through the existing market mechanism, trade their shares in the public venture. They can, however, make some partial adjustment by rearranging their current holdings of marketable risks through the market mechanism. Such action reduces but does not eli minate the negative impact on the individual's welfare due to the possible undersirable allocation of the risk of the public venture. In this case, the optimal investing rule is derived by solving:

Magazine $u_0(y_b,\ a_1,\ a_2 |\ y_a,\ b_1,\ b_2)$

with respect to y_b, a_1 and a_2.

Case III: <u>Capital markets do not function</u>. That is, there are no opportunities for individuals to exchange risks. The shares of the new risky venture are allocated through the government risk redistribution mechanism. In this case, the optimal investing rule for the public venture is obtained by solving:

Magazine $u_0(y_b\ |\ y_a,\ a_1,\ a_2,\ b_1,\ b_2)$

with respect to y_b.

The optimal investing rules and the optimal risk sharing between both individuals, for the three risk redistribution arrangements above, are presented in Tables 2, 3 and 4, respectively. These results follow directly from the first order optimality conditions derived form each optimazation program (see Appendix).

TABLE 2

Case I: Optimal investment decision rule for public ventures and risk allocation when the returns of the public venture are transferable through the perfect capital markets

PUBLIC INVESTMENT DECISION RULE

$$y_b^* = \max \left[0, \frac{E(r_b) - r_f - \gamma_0 y_a \operatorname{cov}(r_a, r_b)}{\gamma_0 \operatorname{var}(r_b)} \right]$$

where $\gamma_0^{-1} = \gamma_1^{-1} + \gamma_2^{-1}$

OPTIMAL RISK ALLOCATION

	Investment "a" (existing)	Investment "b" (new)
Individual "1"	$a_1^* = \dfrac{\gamma_0}{\gamma_1}$	$b_1^* = \dfrac{\gamma_0}{1}$
Individual "2"	$a_2^* = \dfrac{\gamma_0}{\gamma_2}$	$b_2^* = \dfrac{\gamma_0}{\gamma_1}$

TABLE 3

Case II: Optimal investment decision rule for public ventures and risk allocation when the returns of the public venture are non-marketabele through the existing capital markets

PUBLIC INVESTMENT DECISION RULE

$$y_b^* = \max \left[0, \frac{E(r_b) - r_f - \gamma_o y_a \operatorname{cov}(r_a, r_b).}{\gamma_0 \operatorname{var}(r_b) + (\gamma_1 b_1^2 + \gamma_2 b_2^2 - \gamma_0)\left[\operatorname{var}(r_b) - \operatorname{cov}(r_a, r_b)^2 \operatorname{var}(r_a)\right]}\right]$$

OPTIMAL RISK ALLOCATION

	Investment "a" (existing)	Investment "b" (new)
Individual "1"	$a_1^* = \frac{\gamma_0}{\gamma_1} - \frac{\gamma_1 b_1 - \gamma_2 b_2}{\gamma_1 + \gamma_2}\left[\frac{y_b \operatorname{cov}(r_a, r_b)}{y_a \operatorname{var}(r_a)}\right]$	b_1
Individual "2"	$a_2^* = \frac{\gamma_0}{\gamma_2} - \frac{\gamma_1 b_1 - \gamma_2 b_2}{\gamma_1 + \gamma_2}\left[\frac{y_b \operatorname{cov}(r_a, r_b)}{y_a \operatorname{var}(r_a)}\right]$	b_2

TABLE 4

Case III: Optimal investment decision rule for public ventures when capital markets do not function

PUBLIC INVESTMENT DECISION RULE

$$y_b^* = \max \left[0, \frac{E(r_b) - r_f - (\gamma_1 a_1 b_1 + \gamma_2 a_2 b_2)\, y_a \operatorname{cov}(r_a, r_b)}{\gamma_0 \operatorname{var}(r_b) + (\gamma_1 b_1^{\,2} + \gamma_2 b_2^{\,2} - \gamma_0) \operatorname{var}(r_b)}\right]$$

OPTIMAL RISK ALLOCATION

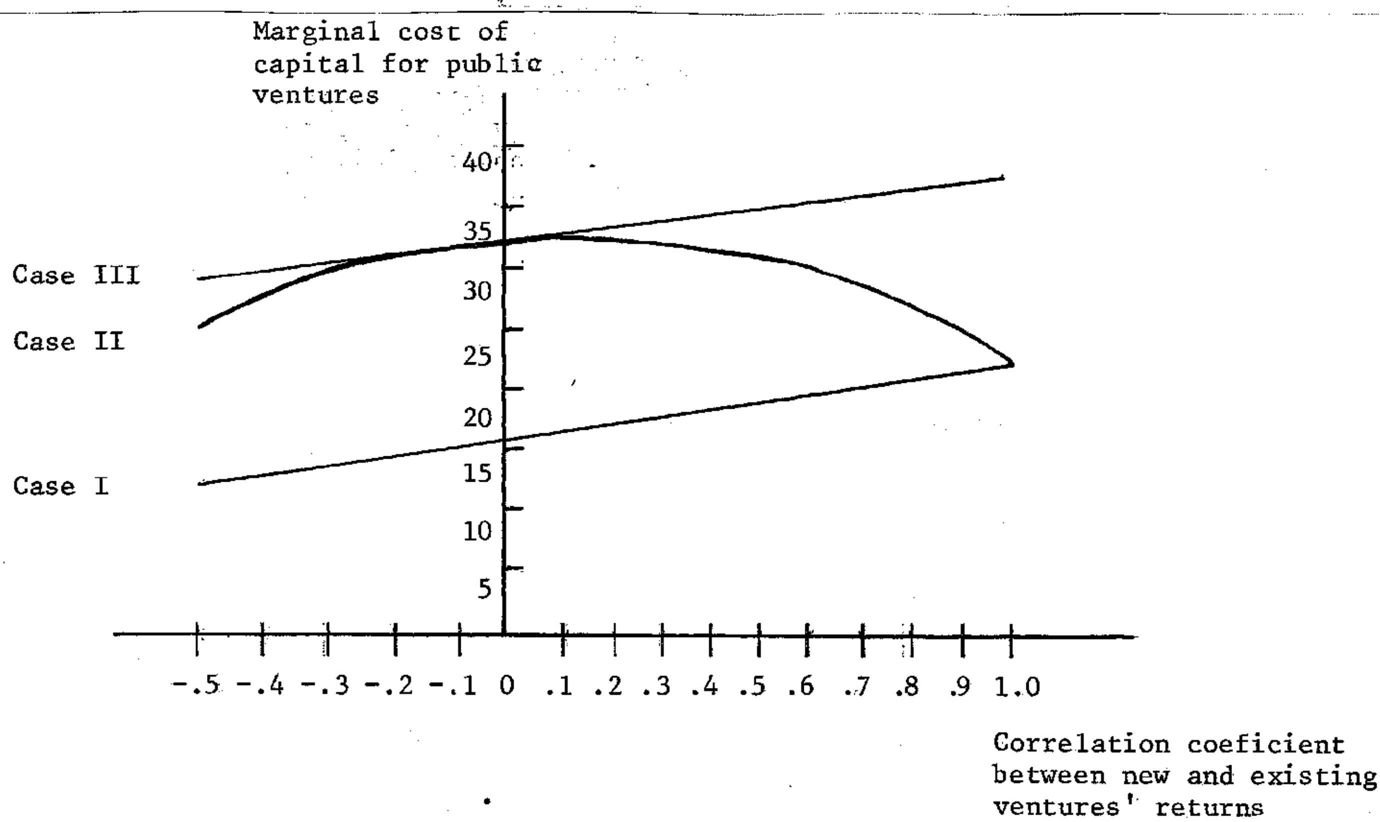
	Investment "a" (existing)	Investment "b" (new)
Individual "1"	a_1	b_1
Individual "2"	a_2	b_1

In Case I, where capital markets are perfect and the shares of public ventures are marketable, the optimal investment rule obtained for public ventures is identical to that in the private sector. This result, as expected, confirms Hirshleifer's argument that no distinction should be made regarding the cost of risk between private and public ventures. The expression for the optimal decision rule for public investment shown in Table 2 yields, upon rearranging, the marginal cost of capital for public ventures when capital markets are perfect:

$$E(r_b) = r_f + \gamma_0 \text{cov}\ (y_a r_a + y_b r_b,\ r_b) \qquad (7)$$

This means that any marginal increase of public invesment is desirable only if its expected return is greater that the riskless return r_f plus a premium for risk $\gamma_0 \text{cov}\ (y_a r_a + y_b r_b, r_b)$. (See Figure 1 for a diagrammatic representation).

FIGURE 1



This Figure was constructed assuming r_f=.15, γ_1=0.01, γ_2=1.0, y_a=2000, y_b=20, var(r_a)=9 x 10^{-4}, var(r_b)=10^{-2}. In addition, for Case II it is assumed that b_1=.1 and b_2=.09; for Case III, it is further assumed a_1=0.99 and a_2=0.01

Marginal cost of capital for public ventures under different risk redistribution arrangements

The risk premium is given by the product of the coeficient γ_0 and the measure of relevant risk, the marginal increase in the overrall risk[11] to be experienced by society if an additional dollar is allocated to the public venture. The coeficient γ_0 can be interpreted as the collective (or society's) risk aversion parameter and is harmonic mean of the risk aversion parameters of the individuals, i.e.,

$$\gamma_0^{-1} = \gamma_1^{-1} + \gamma_2^{-2}$$

Thus, the result presented in Table 2 is identical to the Pareto optimal investment decision rule for a perfectly competitive private economy in a mean-variance setting derived by Jensen and Long (1972) and Merton and Subrahmanyam (1974).

Not surprisingly, the marginal cost of capital for public ventures (7) obtained above is identical to the expected rate of return in equilibrium for private ventures derived by Sharpe (1964), Lintner (1965) and Mossin (1965) under similar conditions. Further, as shown in Table 2, the optimal risk sharing is such that the returns form risky ventures are allocated to each individual in proportion to the inverse of his risk aversion parameter. This is in conformity with Wilson (1969) who shows that, in this case, the sharing rule is Pareto-optimal and uses society's ability to bear risk most efficiently.

Under Case II, (see Table 3) the shares in the public ventures are not marketable in existing capital markets and are arbitrarily allocated among individuals through government's risk redistribution instruments. Hence, the optimal allocation of risk is not proportional to the individual's risk-bearing abilities even though investors attempt to undo part of the misallocation. As a consequence, society's overall ability to bear risk is reduced. For that reason, the optimal level of public investment, in this case, is smaller than the optimal level in the perfect market setting of Case I where all risks are marketable.

The additional second term in the denominator of the public investment decision rule in Table 3 compared to Table 2 reflects the negative impact on the optimal amount to be invested in the public venture in the event of risk-bearing misallocation. In general, this term is positive and would be zero only in two special cases:

(i) if $\gamma_1 b_1^2 + \gamma_2 b_2^2 - \gamma_0 = 0$, i.e., $b_1 = \frac{\gamma_0}{\gamma_1}$ and $b_2 = \frac{\gamma_0}{\gamma_1}$

In other words, if the allocation of the shares of the public venture is proportional to the individual's risk-bearing abilities, i.e., if there is no misallocation of risk bearing. For any other values of b_1 and b_2, this factor is positive.

(ii) if $\text{var}(r_b) - \frac{\text{cov}^2(r_a, r_b)}{\text{var}(r_a)} = 0$, i.e., the return on the public venture is perfectly correlated with the return on the existing risky ventures, positively or negatively. In this case, any misallocation of risk bearing could be perfectly hedged in existing capital markets. If returns are less than perfectly correlated, this factor is positive.

Rearranging the investing rule shown in Table 3 produces the marginal cost of capital for public ventures found by Stapleton and Subrahmanyam (1978):

$$E(r_b) = r_f + \gamma_0 \left[y_a \text{cov}(r_a, r_b) + y_b \text{var}(r_b)\right] + (\gamma_1 b_1^2 + \gamma_2 b_2^2 - \gamma_0)\, y_b \left[\text{var}(r_b) - \frac{\text{cov}^2(r_a, r_b)}{\text{var}(r_a)}\right] \quad (8)$$

Except for the two special cases discusses above, the cost of capital for public ventures presented in (8) is larger than the cost of capital for the private sector where all risks are marketable. (See Figure 1). This result confirms Bailey and Jensen's (1972) arguments that if risk sharing is imperfectly accomplished by the

the government's risks redistribution mechanism, the public ventures should command a higher risk premium than private ventures, provided, of course, that capital markets, although incomplete, are perfect.

For the case of mixed economies where capital markets do not function, it is useful to analyze the public investment deciison rule in Table 4 under the risk redistribution arrangement of Case III. A tighter investment rule is epxected, given the individual's inability to hedge against the risks imposed by government. A comparision of the optimal investing rules shown in Tables 3 and 4 reveals that the level of investment should be lower under Case III since the numerator is smaller $(\gamma_1 a_1 b_1 + \gamma_2 a_2 b_2 > \gamma_0)$, unless the risks are perfectly allocated, and the denominator is larger, $\left[\mathrm{var}(r_b) > \mathrm{var}(r_b) - \frac{\mathrm{cov}^2(r_a, r_b)}{\mathrm{var}(r_a)}\right]$ except for ventures that are perfectly correlated with existing projects.

The marginal cost of capital for public ventures where capital markets do not function is obtained by rearranging the investing rule in Table 4:

$$E(r_b) = r_f + (\gamma_1 a_1 b_1 + \gamma_2 a_2 b_2)\, y_a \mathrm{cov}(r_a, r_b)$$

$$+ (\gamma_1 b_1{}^2 + \gamma_2 b_2{}^2)\, y_b\, \mathrm{var}(r_b) \qquad (9)$$

This expression, illustrated in Figure 1, can be used to reproduced in the mean-variance setting the results presented by Arrow and Lind (1970). Using a somewhat different framework, they conclude that no discount for risk should be made for the special case where the new venture is uncorrelated with the existing wentures $[\mathrm{cov}(r_a, r_b) = 0]$. In this case, (9) becomes:

$$E(r_b) = r_f + (\gamma_1 b_1{}^2 + \gamma_2 b_2{}^2)\, y_b\, \mathrm{var}(r_b) \qquad (10)$$

Assuming now, as Arrow and Lind (1970) do, that shares of public ventures are distributed among a large population, i.e, b_i

tends to be infinitley small, the term $(\gamma_1 b_1{}^2 + \gamma_2 b_2{}^2 + \ldots)$ tends to zero, obtaining Arrow and Lind's results:

$$E(r_b) \doteq r_f \tag{11}$$

Equation (9) provides some qualitative indications of how the marginal cost of risk is affected by the government's risk redistribution mechanism in mixed economies where capital markets do not function. We assume that the new risky venture to be undertaken through government is relatively small and that the risks associated with the costs and benefits of the venture are redistribut ed arbitrarily to a large number of individuals. In this case, the term $(\gamma_1 b_1{}^2 + \gamma_2 b_2{}^2 + \ldots)$ is close enough to zero such that the marginal cost of capital for the public venture can be approximated by:

$$E(r_b) = r_f + (\gamma_1 a_1 b_1 + \gamma_2 a_2 b_2 + \ldots) \text{ cov } (y_a r_a, r_b). \tag{12}$$

The second term in (12), which is the risk premium required for the public venture, depends on:

(i) $\text{cov}(y_a r_a, r_b)$ which is the covariance between the total return on existing investments and the return per dollar from the new investment. (Given that the risks of the new venture are <u>a fortiori</u> applicable only to those individuals who will be assigned to bear the risk, $y_a r_a$ represents only the return available to those individuals. For risks that are borne by the entire population, $y_a r_a$ represents the existing national income prospect). And,

(ii) $(\gamma_1 a_1 b_1 + \gamma_2 a_2 b_2 + \ldots)$ which represents the society's risk aversion parameter in an economy where capital markets do not function. In this case, the parameter depends on the way risks are allocated among the individuals who bear the risks of the new venture. This parameter is lowest when risks are allocated to individuals proportionately to the inverse of their risk aversion parameters. However, it increases when risks are arbitrarily allocated and reaches its maximum when risks are perfectly misallocated, i.e., allocated proportionately to the risks aversion parameters of individuals.

It is reasonable to assume that individuals with large endowments of wealth are less risk averse. Under this assumption, if risks are allocated through a progressive tax system on wealth, the society's risk aversion parameter tends to be small. However, if the resources for the new venture are captured through inflationay means, which perhaps has a relatively greater impact on individuals with smaller wealth endowments, the risk aversion parameter of society tneds to approach its maximum. Therefore, if the inflationary route is taken for raising resources for the public sector, the cost of risk for public ventures would be correspondingly higher.

In a mixer economy, if capital markets do not function effectively, the difficult task is to provide an administrative criterion for the appraisal of new public investment decisions. This paper does not offer a precise solution to the question of what the precises administrative criteria is. However it has been pointed out that

(i) the risks of new ventures should not be judged in isolation, but by their impact on the overall risk faced by the individuals who bear the risks of the new project. For relatively small projects and/or where risks are redistributed across for large numbers of individuals, this impact is measured by the covariance between their total return prospects (e.g., regional income) and the new project return:

(ii) the society's risk aversion parameter depends on the way cost risks and benefit risks related to the new venture are allocated among individuals. For instance, the risk allocation through the tax system tends to exploit more effectively the individuals' collective ability to undertake risks than allocations through inflationary means, i.e., soceity's risk aversion parameter tends to be smaller in the first case than in the second.

If the elements above are assessed, the cost of capital for public investment can be computed by employing (12). Clearly, an objettive assessment of society's risk aversion parameter is not feasible since it requires knowledge of each individual's risk preferences. The assessment of this parameter is, in the final analysis, dependent on the judgment of public officials. If such a measure is included in the strategy for economic development, careful consideration should be given to possible biases that can cause an undersirable transfer of resources form profitable venture in the private sector to less profitable ones in the public sector.

IV AGENCY PROBLEMS AND THE ALLOCATION OF PUBLIC INVESTMENT

In the analysis of the previous section, the risk premium required on a public project was shown to depend on the manner in which the risk of the venture is borne by members of the society. In other words, the ability to allocate risk among members of the society trhourgh the risk distribution mechanism available is the crucial determinant of the cost of risk associated with new risky ventures. The particular aspects of the institutional form to be created to implement public investments were not taken into account. As a consequence, under the framework of section III, given the mechanism for risk redistribution, the cost of risk associated with a public venture is not dependent on the form of ownership of the institution established to carry out the risky venture. Specifically, under these conditions, it makes no difference whether government undertakes the new venture directly through a public enterprise (public ownership), allows a private enterpreneur to undertake it with public financing or quarantees (private ownership), or enters into partnership with private entrepreneurs forming a mixed enterprise (joint ownership).[12] This assumption needs to be modified so that the ownership aspect is explicitly taken into account and its influence on the cost of risk of a new venture can be analyzed.

In establishing any of the above institutions, government, the principal, delegates to particular individuals, the agents, the operating decisions of the enterprise. The point to be made in this section is that the potential conflict between the interests of the agents and that of government has a definite impact on the manner in which the risks of the new venture ought to be allocated among individuals and, as a consequence, affects the cost of risk of public ventures.[13]

The problem of induceing an "agent" to act as if be maximizes the "principals" welfare is quite general. Recent developments in the theory of agency have focused on the normative question of how to design a contractual relationship between the principal and the agent so as to provide adequate incentives for an agent to make decisions which wiil be most consistent which the interests of the principal. These developments have shown that Pareto optimal incentive schemes do require risk sharing between the principal adn agents in a manner inconsistent with their respective risk bearing capacities.[14]

The operational efficiency of a new venture will, in general, depend on the incentives available for the agent to maximize the returns form the venture and on the opportunities he has for acting in its own interest to the detriment of the interests of the principal. We will focus here only on one such type of such opportunity: the agent's choice of how much effort he should devote to the managerial activity.[15] This include vaious aspects of the agent's activities such as the effort he expedns in the search for new production techniques, sources of inputs, new output markets, the most effective organizational form, and the degree to which he coordinates with middle managment and workers by transmiting goals and conveying information. These are dimensions of the agent's effort which have a definite impact on the venture's return.

In contrasting the operational efficiency of firms managed under an agency contract with those which are not, Leibenstein (1966) concludes that firms that are not managed under an agency contract perform more efficiently than those that are managed under such a contract where at leats one of the following factors is applicable: (1) the firm's activity involves risky technologies where the relation between management's actions and the outcome prospects retunr cannot be perfectly identified, (2) imperfect competition exists in the markets for the factors of production (which includes managerial services) and (3) capital markets are imperfect.[16]

Leibenstein's findings are clearly relevant to the case when public investment is justified by risk considerations in an economy where capital markets do not function (i.e., at least factors (1) and (3) are in effect). As a consequence, for those ventures in which government participation is deemed required, it can be conjectured that the greater the proportion of ownership helb by management, the higher is the level of operational efficiency that can be expected, other things being equal. In the extreme case of public enterprises, where managers to not have an ownership interest, operational efficiency is expected to be clearly lower than in private enterprises which are owner-managed.[17]

There are, of course, several instruments besides the choice of effort on the part of the agent, to use in his own interest and at some cost to the principal but these will not be emphasized here. Many of these alternative explanations for the conflict to interest between the principal and the agent are related to asymmentric access to information between the agent and the principal, for example, regarding the production process used by the firm. In these cases, there is a tendency on the part of agents to appropriate perquisites from the resources of the firm for their own consumption. The impact of the agency conflict in the ownership structure and the attendant costs was analyzed by Jensen and Meckling (1977) for the case of privately owned firms. In their framework, they focus on the conflict of interests between owner-managers and outside stockholders which many be restrained by costly bonding and monitoring schemes. Such schemes can disclose, to some extent, misrepresentation of facts and highlight situations where decisions made by the agent are not conducive to an efficient use of the resources of the enterprise. However, Jensen and Meckling focus almost exclusively on the appropriation of enterprise resources rather than effort choice on the part of the agent. Further, little attention is paid to the risk aspect that is the focus of our attention here.

In one important respect there is a similarity between the research on agency cost and the problem posed here. In both cases, the incentives for the agent to devote a higher degree of effort in the managerial acitivyt are essentially linked with his share (or ownership claim) of the returns of the enterprise. The larger his share, the greater are the potential benefits he can derive form increases in his effort through operational efficiency, i.e., from improvements in the quality of the return prospects of the venture.

From the above, it is clear that maximum operational efficiency would be achieved when the claims on returns of the ventures are fully owned by the agent. However, in this extreme case, the venture may become unattractive due to high cost of risk to be faced by the agent if markets are imperfect in some respect, since the risk cannot be partly sold to other investors. The optimal allocation of risk, on the other hand, would require that the agent's ownership share be consistent with his risk bearing capacity relative to the risk bearing capacity of society as a whole. In this case, his ownership share tends to be small and, consequently, his incentive to devote effort very low. The optimal ownership claim structure for the venture's return is one which provides the best trade-off between the gains in operational efficiency and the reductions in the cost of risk associated within the venture. This is the crux of the problem of designing and optimal ownership structure for the institution through which the new public venture is to be implemented.

V. A MEAN-VARIANCE MODEL OF THE AGENCY PROBLEM AND THE PUBLIC INVESTMENT DECISION

We are now in a position to explicitly model the agency problem discussed above in the mean variance framework of Section III. Since we wish to highlight the effect of the agents' ownership interest in the new venture, it is necessary to modify somewhat the basic structure of the model used earlier. The agency problem can be easily incorporated into the simple "two individual, two investment oportunities" model developed in Section III by making the following changes:

i. individual 1 represents all members of society (other than the agent),i.e.,the government;

ii. individual 2 is the manager or agent to whom the managerial responsability for the new venture is delegated;

iii. investment "a" is the existing aggregate investment in the economy with a rate of return r_a associated with it; and

iv. investment (b) is the new venture under consideration to be undertaken with individual 2 as the manager.

The agency problem is recognized by assuming that the return on the new public venture is a function of both the investment level y_b and the manager's share of ownership, b_2. This dependence is specified by viewing the agent's ownership interest as a factor of production. In the constant returns to scale production function used earlier. the total return is now given by

$$[h(b_2) + r_b]\ y_b$$

where

$h(b_2)$ is the parte of the return per dollar invested which depends on the manager's effort dictated by his ownership proportion, b_2. The positive impact of the manager's ownership proportion on the return from the venture is reflected by specifying that $h' > 0$ and $h'' < 0$, and also that h is sclaed so that $h(0) = 0$; and

r_b is the part of the stochastic return per dollar invested which does not depend on the agent's ownership proportion i.e. the same as in Section III.

The rest of the model structure and notation is the same as in section III above.

Given the above specifications, the expected value and the variance of the return on the public venture are respectively,

$$|h(b_2) + E(r_b)| \; y_b \, , \quad \text{and}$$

$$y_b^2 \text{ var } (r_b); \tag{13}$$

where the agent's ownership proportion, as specified, brings a beneficial impact on the expected value of the venture's return prospects for the same level of variance (risk).

Taking into consideration the changes made above, the equations for end-of-period wealth analogous to equation (2) are

$$w_1 = w_{01}r_f + a_1(r_a - r_f)y_a + b_1|h(b_2) + r_b - r_f| \; y_b, \text{ and} \tag{14}$$

$$w_2 = w_{02}r_f + a_2(r_a - r_f)y_a + b_2|h(b_2) + r_b - r_f| \; y_b \tag{15}$$

Evaluating the expected value and variance of each individual's end-of-period wealth equations |(14) and (15)| , and substituting these values in each individual's utility functions yields:

$$u_1 = w_{or} r_f + a_1 \left| E(r_a) - r_f \right| y_a + b_1 \left| h(b_2) + E(r_b) - r_f \right| y_b \quad (16)$$

$$- \frac{\gamma_1}{2} \left| a_1^2 \, y_a^2 \, var(r_a) + b_1^2 \, y_b^2 \, var\,(r_b) + 2a_1 \, b_1 \, y_a \, y_b \, cov\,(r_a, r_b) \right|$$

$$u_2 = w_{02} r_f = a_2 \left| E(r_a) - r_f \right| y_a + b_2 \left| h(b_2) + E(r_b) - r_f \right| y_b$$

$$- \frac{\gamma_2}{2} \left| a_1^2 \, y_a^2 \, var(r_a) + b_2^2 \, y_b^2 \, var\,(r_b) + 2a_2 \, b_2 y_a y_b \, cov\,(r_a, r_b) \right| (17)$$

As in Section III, the social welfare function $u_0 = u_1 + u_2$ can be computed from (16) and (17) above. Since, $a_1 = 1 - a_2$ and $b_1 = 1 - b_2$, u_0 becomes:

$$u_0\,(a_2 b_2 y_b) = (w_{01} + w_{02})\, r_f + \left| E(r_a) - r_f \right| y_a + \left| h(b_2) + E(r_b) - r_f \right| y_b$$

$$- \frac{\gamma_1}{2} \left| (1-a_2)^2 \, y_a^2 \, var(r_a) + (1-b_2)^2 \, y_b^2 \, var\,(r_b) + 2(1-a_2)(1-b_2) y_a y_b cov(r_a r_b) \right|$$

$$- \frac{\gamma_2}{2} \left| a_2^2 y_a^2 \, var\,(r_a) + b_2^2 y_b^2 \, var\,(r_b) + 2a_2 \, b_2 \, y_a \, y_b \, cov(r_a, r_b) \right| \quad (18)$$

We then proceed to find the investment decision rule for the public venture "b" using the same method as in Section III. Here, however, the analysis is focused only on the two extreme conditions regarding the funtioning of the existing risk redistributing arrangements:

Case I A: <u>All risks are marketable through perfect capital markets</u>. Here, the individuals are free to transfer risk between themselves via market exchanges. Thus, the agent (to whom the operational decisions concerning the new public ventures are delegated) is free to adjust his positions in the existing ventures and in the new public venture. Hence, this case parallels Case I of Section III. As in that case, the optimal investment rule is obtained by maximizing the social welfare function, assuming that the risk--sharing variables can be optimally adjusted. The optimal investment rule is obtained by solving the following program:

$$\text{Mazimeze} \quad u_0 \, (y_0, a_1, a_2, b_1, b_2 \mid y_a)$$

$$\text{with respect to } y_b, a_1, a_2, b_1 \text{ and } b_2$$

Case III A: Capital markets do not function. That is, there are no opportunities to trade risks. Any reallocation of business risk among individuals is arbirarily undertaken through government instruments. Explicitly, it is assumed here that the actual risk allocation of the existing ventures is given and cannot be modified when the new public venture is undertaken. Nevertheless, the risk sharing between the agent and the government dictated by the agency contract is presumed to be freely negotiated. This is an additional condition imposed on Case III of Section III, where the shares of the new risky venture can be optimally allocated. As a consequence, the optimal decision rule for public investments can be obtained by maximizing the social welfare function with respect to the level of public investment, assuming that the risk of the venture can be optimally shared between the agent and government, i.e., through the following optimazation program:

$$\text{Maximize } u_0(y_b, b_1, b_2 \mid y_a, a_1, a_2)$$

$$\text{with respecto to } y_b, b_1 \text{ and } b_2.$$

The optimal investment rules and risk sharing for the two cases above are presented in Tables 5 and 6. These results are derived from the first order optimality conditions of each of the maximization problems previously described.

TABLE 5

Case IA: Optimal investment decision rule for public ventures and risk allocation when the agent's ownership affects the venture's return and this return is transferable through perfect capital markets

PUBLIC INVESTMENT DECISION RULE

$$y_b^* = \max \left| 0, \frac{E(r_b) + h(b_2) - r_f - \gamma_0 y_a \operatorname{cov}(r_a, r_b)}{\gamma_0 \operatorname{var}(r_b) + (\gamma_1 b_1^{*2} + \gamma_2 b_2^{*2} - {}_0) \operatorname{var}(r_b) - \dfrac{\{\operatorname{cov}(r_a, r_b)\}^2}{\operatorname{var}(r_a)}} \right|$$

OPTIMAL RISK ALLOCATION

	Investment "a" (existing)	Investment "b" (new)
Individual "1"	$a_1^* = 1 - a_2^*$	$b_1^* = 1 - b^*_2$
Individual "2"	$a^*_2 = \dfrac{\gamma_0}{\gamma_2} + \left(\dfrac{\gamma_0}{\gamma_2} - b_2^*\right) \dfrac{y_b \operatorname{cov}(r_a, r_b)}{y_a \operatorname{var}(r_a)}$	$b_2^* = \dfrac{\gamma_0}{\gamma_2} + \left(\dfrac{\gamma_0}{2} - a_2^*\right) \dfrac{y_a \operatorname{cov}(r_a, r_b)}{y_b \operatorname{var}(r_b)} + \dfrac{h'}{(\gamma_1 + \gamma_2) y_b \operatorname{var}(r_b)}$

TABLE 6

Case III.A Optimal investment decision rule for public ventures and risk allocation when the agent's ownership affects the venture's return and capital markets do not function

PUBLIC INVESTMENT DECISION RULE

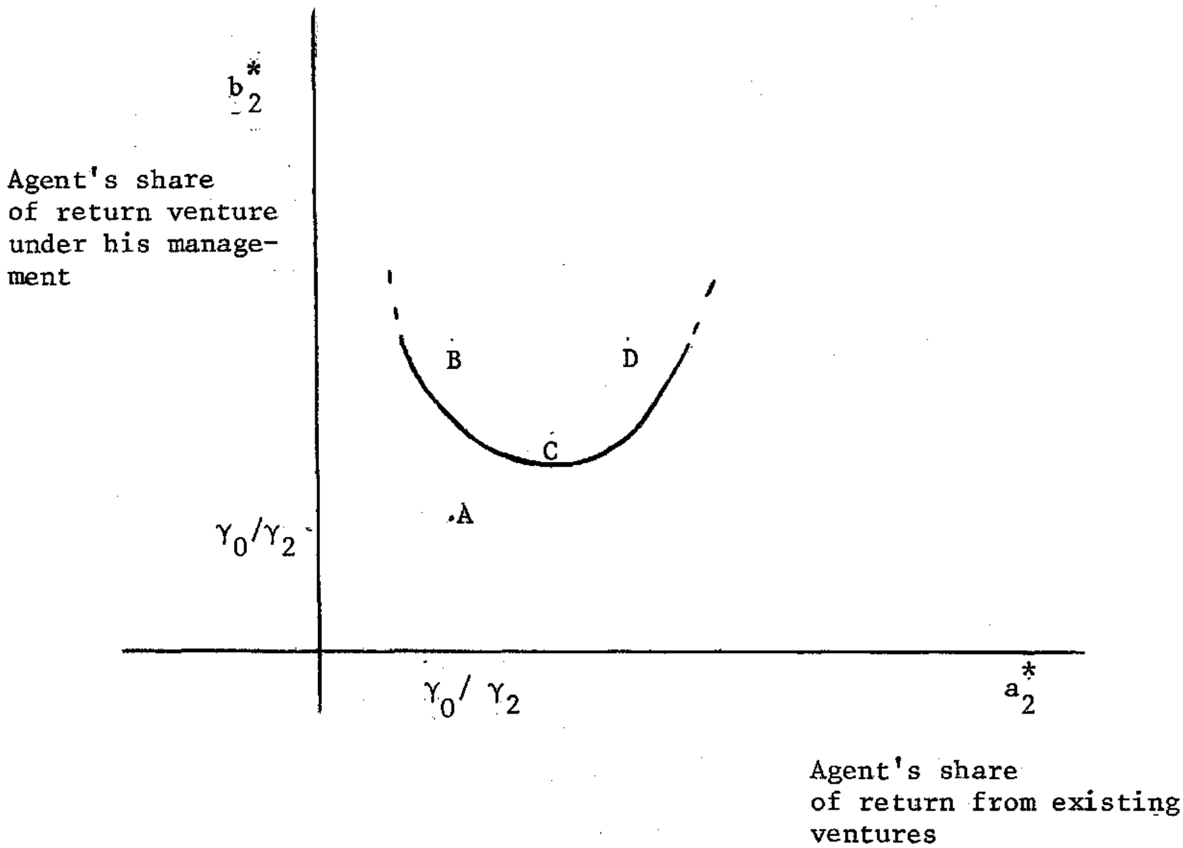
$$y_b^* = \max \left| 0, \frac{h'(b_2^*) + E(r_b) - r_f - (\gamma_1 a_1 b_1^* + {}_2 a_2 b_2^*)\, y_a \operatorname{cov}(r_a, r_b)}{\gamma_0 \operatorname{var}(r_b) + ({}_1 b_1^{*2} + {}_2 b_2^{*2} - \gamma_0) \operatorname{var}(r_b)} \right|$$

OPTIMAL RISK ALLOCATION

	Investment "a" (existing)	Investment "b" (new)
Individual "1"	a_1	$b_1^* = 1 - b_2^*$
Individual "2"	a_2	$b_2^* = \dfrac{\gamma_0}{\gamma_2} + \left(\dfrac{\gamma_0}{\gamma_2} - a_2\right) \dfrac{y_a \operatorname{cov}(r_a, r_b)}{y_b \operatorname{var}(r_b)} + \dfrac{h'}{(\gamma_1 + \gamma_2) y_b \operatorname{var}(r_b)}$

Observing the results of the Case I A, when capital markets are perfect and the shares of public ventures are marketable (Table 5), we notice that the optimal allocation of risk is no longer consistentn with the risk-bearing abilities of the two parties as occurs in Case I. This deviation results form the effect of the agent's ownership interest on the venture's return, i.e., on $h'(b_2)$. Figure 2 shows the behavior of the optimal risk allocation to the

FIGURE 2



Point A: Optimal risk allocation to the agent consistent with his relative risk-bearing capacity.

Point B: Optimal risk allocation to the agent if $cov(r_a, r_b)=0$

Segment CD: Optimal risk allocation to the agent if $cov(r_a, r_b)<0$

Segment BC: Optimal risk allocation if $cov(r_a, r_b)> 0$.

Optimal risk allocation to the agent under perfect capital markets agent (a_2^*, b_2^*) as a function of the correlation between the returns of the new public venture and thhose of existing ventures. Pure private ownership is optimal when the covariance is high. In the case of high positive correlation, (Point D), the venture seen from the perspective of the existing portfolio of ventures, represents a highg marginal risk for society. Its implementation can still be worthwhile, if the additional return resultant from the private investor's ownership interest is sufficient high. In this case, therefore, the venture can be encouraged by government only through riskless subsidies. Similarly, for high negative correlation, (Point B) private ownership is again justified as long as the agent is able to hedge his position by trading in existing ventures. If such short positions are not feasible, the solution will be similar to the intermediate case of zero correlation, (Point C). In all cases, the agent's share of the new venture is greater than the level consistent with the agent's risk bearing ability.

Rearranging the decision rule presented in Table 5 we obtain the following marginal cost of capital form public ventures in Case IA:

$$h(b_2) + E(r_b) = r_f + \gamma_0 \, |y_a \mathrm{cov}(r_a, r_b) + y_b \, \mathrm{var}(r_b)| \; + \\ (\gamma_1 b_1^* + \gamma_2 b_2^{*2} - \gamma_0) \, y_b \, \left|\mathrm{var}(r_b) - \frac{\{\mathrm{cov}(r_a, r_b)\}}{\mathrm{var}(r_a)}\right| \qquad (19)$$

The expression above indicates that the level of investment in the new public venture should be increased to the point where the marginal expected return from the venture, $|h(b_2) + E(r_b)|$, equals the sum of:

i. the marginal cost of riskless capital r_f;

ii. a marginal risk premium $\gamma|y_a \mathrm{cov}(r_a, r_b) + y_b \mathrm{var}(r_b)|$;
where γ_0 is the risk aversion parameter for a partner ship or syndicate, where risk sharing is undertaken according to the risk-bearing capacities of the partners.

This premium can be negative, when the covariance is negative; and

iii. an additional marginal risk premium, $(\gamma_1 b_1^* + {}_2b_=^{*2} - \gamma_0)y_b \cdot |var(r_b) - cov(r_a, r_b)^2 /var(r_b)|$, which is related to the fact that risk sharing is not consistent with the partner's risk-bearing capacities. This premium is always positive.

The marginal cost of capital for public investment in this case (19) is similar to that obtained in SEction III for Case II, where risk sharing was exogenously imposed in a manner inconsistent, in general, with the risk of bearing of individuals. In Case I A, however, the same inconsistency is caused endogenously because of the assumed beneficial impact of the manager's ownership proportion on the return from the new venture. The expression (19) explicitly shows the trade-off between the gains in expected return resulting form the ownership factor and the increase in the cost of risk resulting form misallocation of risk bearing.

Under Case III A in Table 6, we depict the conditions usually found in mixed economies in a developing stage, i.e., where capital markets do not function effectively and the allocation of risks of public ventures between the agent and government is established by means of negotiated agency contracts. The risk sharing established in such contracts tends to conform to a linear rule, i.e., the contract establishes the fraction of the return to be allocated to each party. In other words, the contract does not include any additional insurance mechanism which could modify such a proportional rule.[18] For example, the non-linear sharing of risks imposed by limited liability rules is not dealt with in our analysis. Thus, more complex risk-sharing arrangements that would improve the allocation of risk bearing are not invoked in this paper.

It is worthwhile to examine again in Case III A the impact of the term $cov(r_a, r_b)$ on the agent's optimal ownership b_2^* . For high positive covariances and a relatively large impact of the agency factor, h', the tendency is towards a large b_2^*, i.e., private ownership. This means that any incentive to be provided by government to stimulate such a venture must be of the nature of a riskless transfer (e.g. a subsidized loan). However, in the opposite case, for low and negative covariances and a small impacto of the agency factor, the tendecy is towards a small b_2^*, i.e., public ownership. The impediment imposed in this case - risk diversification by the agent - gives the government a comparative advantage to directly undertake such ventures, with relatively small attendant agency costs.

A tighter investment criteria is expected in Case III A vis-a-vis Case I A due to the constraint on readjustment of the allocation of risk of existing ventures. Rearranging the investment rule of Table 6 yields the following expression for marginal cost of capital for public ventures when capital markets do not function and the effect of an agency relationship is considered (Case III.A):

$$h(b_2) + E(r_b) = r_f + |(\gamma_1 a_1 b_1^* + \gamma_2 a_2 b_2^*) \quad y_a \operatorname{cov}(r_a, r_b) +$$

$$|(\gamma_1 b_1^{*2} + \gamma_2 b_2^{*2})| \; y_b \operatorname{var}(r_b) \qquad (20)$$

Expression (20) is identical to expression (9) obtained for Case III in Section III. However, b_1^* and b_2^* are negotiated and not exogenously imposed as was assumed in Case III. This reduce the risk premium required in this case relative to Case III.

VI. CONCLUSION

The central concern in this paper has been with the risk aspects of the public investment decision. It was pointed out that the crucial differences between earlier approaches in the literature were with respect to the assumptions made regarding available mechanisms for ditribution of risk across individuals. Further, some of the results in the earlier papers are affected by difference in the modelling assumptions, which are not material to the general issues involved. To avoid this difficulty, in this synthesis the alternative scenarios regarding the working of capital markets are studied within a common mean-variance framework.

There are three major instruments available to government to participate in new projects. The first would be with subsidies for new investment, through the tax system of otherwise. The second alternative would be through insurance arrangements such as loan guarantees to lenders on behalf of the private investor. Finally, the government could invest directly in the equity of the new enterprise. The firts method can ben viewed as a reduction in the required investment in the new project. The other two instruments are similar, in principle, in the context of a complete and perfect capital market. In an incomplete market setting, there are differences in the precise nature of the risks borne by government on behalf of indviduals in the society in the two cases. Apart from this technical distinction, there is a more important issue with respect to insurance arrangements, namely the question of moral hazard. In a sense, this distinction can be interpreted as involving different agency costs for the two types of financing, stemming from differences in their respective bonding and monitoring arrangements. Since this paper is not directly concerned with the details of the agency relationships and their attendant costs, the distinction between insurance and equity arrangements is not relevant in our context. Further, in the mean-variance framework employed in this paper, it is difficult to model insurance arrangements in a meanin full fashion.

An important additional dimension studied here is the role of operational efficiency. This element is missing from much of the literature on the subject and hence, the earlier results derived have shortcomings. The major rationale for government investment form a risk perspective is the improvement in the allocation of risk bearing that is achieved in the absence of perfect and complete capital markets. However, if such an improvement in the distribution of risk can be achieved costlessly, public investment is the obvious prescription whenever caoital markets are less than perfect. This argument suffers from a seriour lacuna namely ignoring the lower operational efficiency of public enterprise due to problem of agency. Thus, in the absence of well functioning capital markets, there is a trade-off between the improvement in allocative efficiency and the reduction in operational efficiency resulting from undertaking a new project in the public sector. This provides a rationale for mixed enterprises, an important institutional arrangement for new investment in many developing economies. Given this trade-off between allocative and operational efficiency for a mixed enterprise, the risk premium and hence the discount rate for public investment have to be modified accordingly. A key variable in the choice of the appropriate discount rate for the jointly owned project turns out to be the statistical correlation between the returns on existing projects and the new venture, which is a measure of the risk borne by the agent in taking an ownership interest in the new project.

Of course, the precise institutional form of the mixed enterprise and, in particular, the agency contract negotiated is a crucial determinant of the attractivenes of the new investment opportunity. An issue not addressed directly here is the optimal design of such an agency contract. Different institutional arrangements elicit different levels of managerial effort and vary in the complexity of the monitoring system necessary to impede moral hazard problems. For this reason, the improvement in social welfare which could be achieved form public investment depends not only on the amount of resources allocated to the venture but also on the institutional arrangement used to conduct the venture's business.

Thus, the government's objectives of maximizing social welfare can best be attained by jointly optimizing the resources allocated to the venture and the institutional arrangement to carry it out.

The characteristics of the institutional arrangement through which public investment is implemented are defined by the form of the contract which is established betwenn the government and the manager of the venture. This contract can be one of direct agency, insurance, or partnership, which corresponds to the institutions of public enterprise, public insurance and mixed enterprise respectively. The essence of the public enterprise (agency) contract is that government provides the capital and management may, through its compensation scheme, share is the enterprise's return. In the public insurance contract, the entrepreneur (manager of the venture) supplies the capital and government underwrites some portion of the risk. The mixed enterprise contract form is an arrangement where both parties supply capital and share in the return. It is a partnership which can be full and complete in the usual sense (quid pro quo), or it can be a partnership of interest where the sharing of returns is disproportionate to the capital contributions.

Based on the arguments presented in this paper, mixed enterprises are likely to be most suitable institutional arrangement for government to adopt when public investment is justified by risk considerations. The enlarged exposure of the entrepreneur to the risk of the venture which is required under a mixed enterprise arrangement, may bring forth a higher level of managerial effort when monitoring costs are high. In other words, an advantageous trade-off occurs between the welfare gains derived from net increases in the quality of the venture's return prospects and the welfare losses associated with increases in the required risk premium of the venture (due to the entrepreneur's enlarged risk exposure).

The entrepreneur's exposure to the venture's risk is enlarged with an increase in his participation in the capital for the venture. Hence, from the entrepreneur's viewpoint, this participation is only permissible to a point dictated by his risk preferences. If it is still desirable from government's viewpoint to improve operational performance by further enlarging the entrepreneur's exposure to the venture's risk, then the mixed enterprise contract may include an insurance policy ("side best") that mitigates the entrepreneur's total risk at the same time that he bears a larger share of the venture's risk. The provision of such an insurance policy requires that a more complex monitoring system be in place which can disclose correctly the contingencies for insurance paye payments (i.e., to impede moral hazard problems related to misrepresentation). Given that the amount of is insurance required by the entrepreneur is directly related to the size of his capital participation and that the cost of monitoring is directly related to the size and complexity of the insurance policy, the sitmuli that government can provide to encourage the entrepreneur to enlarge his capital participation is tempered by the cost of monitoring.

The optimal public investment decision variables are those which exhaust all mutual gains in a bilateral negotiation process between the government and the entrepreneur, i.e. both the optimal levels of investment and contracting terms are Pareto optimal from the entrepreneur's and government's viewpoints. The bilateral aspect of the negotiation does not necessarily imply a monopoly of opportunities for the private entrepreneur: the government may be carrying on bilateral negotiations simultaneously with several entrepreneurs.

In an economy where capital markets do not function, govern ment undertakes the economic role usually ascribed to the capital market, i.e., it provides the means for redistributing risks among individuals. The concept of government as a surrogate institution for the capital market explains the economic meaning of the normative rules derived for government when it participates in risky ventures. From this perspective, the government's certainty equivalent value of return prospects is similar to market valuation, in a pure private economy.

In the public investment decision rule derived, the desirability of increases in the levels of investment is appraised by the government "valuation" of the resulting net marginal increases in the total venture's return. This criterion parallels the Pareto optimal investment rule in an economy with perfect capital markets where the market valuation of the venture's return prospects determines the optimal level of investment. Similarly, this criterion in maintained for determining the optimal contracting terms. The institutional arrangement is chosen according to be government's valuation of its impact on the return prospects of the venture.

In summary, the decision to undertake a new venture, which encompasses the amount of resources to be allocated to the venture and the form of the contract, should be appraised with the government's valuation criterion. As a consequence, optimal resource allocation in a mixed economy cna only be attained if the government's valuation criterion is pursued regardless of the nature of the optimal ownership arrangement of the enterprise that will carry out the venture: public, mixed or private.

FOOTNOTES

(1) Mikesell (1977) provides a summary of the literature regarding this aspect.

(2) If some kind of monopolistic factor is introduced, the private investment allocation will not be Pareto optimal | See Long(1972), Jensen and Long (1972) and Stiglitz (1972) | . A review of an allocation efficiency of security markets is presented by Mossin (1977) and more recently, for the mean-variance setting, by Baron (1979).

(3) The pooling argumento has led Vickret (1964) to conclude erroneously that the cost of capital for public ventures should be represented by the riskless rate of return.

(4) There are well-known problems with the mean-variance approach. However, it has the advantage of leading to a simple valuation model and is the basic paradigm of financial economics, thus providing a framework for comparison.

(5) This is in line with the literature on the private investment decision, e.g. Jensen and Long (1972) and Merton and Subrahmanyam (1974). For a detailed discussion of the comparative statics of the market price of risk, see Stapleton and Subrahmanyam (1979).

(6) Note that b_i can be interpreted either as an allocation of the share of public investment in the market or by government through the tex system or, alternatively, the consequence of a public project where benefits flow to a particular group of individuals in the economy. In most cases, we would observe some combination of the above polar cases.

(7) See for example Stapleton and Subrahmanyam (1978).

(8) Wilson (1968, 1969) proves that, when individual's investment decision is reduced to a mean-variance choice, a collective evaluation function which leads to Pareto optimal investment decisions can be constructed. This requires, however, that sufficient degrees of freedom be provided in order for individuals to establish among themselves optimal sharing procedures regarding the investment returns.

(9) This criterion has been used previously in the literature on public investment. See for example Arrow and Lind (1970) and Stapleton and Subrahmanyam (1978).

(10) This can be achieved through complete capital markets or by a central planner with all the instruments necessary to achieve a Pareto optimal allocation of investment risks.

(11) The marginal increase in the overall risk in given by:

$$\frac{\partial\left|y_a^2\,\mathrm{var}(r_a) + y_b^2\,\mathrm{var}(r_b) + 2y_a y_b \mathrm{cov}(r_a,r_b)\right|}{\partial y_b} = 2\ \mathrm{cov}\ (y_a r_a + y_b r_b, r_b)$$

(12) Another alternative is for government to act as an insurer or underwriter of risk of private business through loan guarantees or other equivalent instruments. This is similar in many respects to the case of public financing of private enterprise on the presence of default risk and raises questions of moral hazard. The insurance type of risk sharing arrangement is no analyzed here as the mean-variance framework employed in this paper is inadequate, in general, to handle such contingent claims.

(13) The effectiveness of agency relations between owners and management of private firms has been the concern of the "behavioral theories of the firm". See for example, Simon (1957, 1959),

Cyert and March (1963), Williamson (1963). These theories reject the assumptions underlying the notion that the single goal of a firm is return maximization. They hypothesize that the firm's behavior is substantially determined by the results of the bargaining process between owners and managers, each party interested in maximizing its own welfare. These theories of the firm indicate that the impact of the agency conflict on the firm's performance can be empirically assessed by observing the difference between the operational performance of firms where an angency relation between owners and managers exists and those where it does not.

(14) See for example Roos (1977), Holmstrom (1979), Shavell (1979) and Harris and Raviv (1979). These studies, conclude that an efficient compensation scheme between the principal and the agent involves a transfer to the agent of a higher share of the risk than warranted in a Pareto optimal sense, i.e, consistent with the agent's risk bearing capacity.

(15) Williamson (1963) observes that the more diffuse the ownership structure, the larger the degree to which a firm's resources are allocated inconsistently with the goal of maximizing the firm's return. In a related analysis, Leibenstein (1966) indicates the conditions under which the relevant variation in operational performance is observed. He notes that certain enterprises perform more efficiencly than others for similar amounts of capital and labor and with similar technology, and suggests that managerial effort is important in creating these disparaties.

(16) Leibenstein (1966) argues that when competitive pressure from markets is limited |factors (2) and (3)| and the relation between input and outpu is not determinate |factor (1)|, managers do not work as hard nor do they search for information regarding new opportunities to improve returns as effectively as they could. Under conditions of intense competition, man-

agerial effort increases, resulting in higler operational performance.

(17) Funkhouser and MacAvoy (1979) compare the performance of privately held enterprises with public enterprises operating under similar market conditions. Using a sample of 150 Indonesian enterprises, they found that public enterprises operate with higher cost and lower productivity than private firms. In explaining their observations they point put the lack of a mechanism in the governmental structure to stimulate managerial interest.

(18) The implicit assumption made here is that the same kind of moral hazard problems which have precluded the existence of market exchanges are in effect for public insurance contracts of business risks.

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APPENDIX

This appendix drevies the necessaru and sufficient conditions for an interior maxiumum of $u_0(a_1,a_2,b_1,b_2, y_a,y_b)$ under the three distinct sets of constrains which characterize the three cases I, II and III. The analysis can be easily modified to take into account the effect of the agency relationship discussed in Section V.

$$a_2 = 1-a_1, \tag{A.1a}$$

$$b_2 = 1-a_2 \text{ and} \tag{A.1.b}$$

$$y_a = y_a \tag{A.1.c}$$

Since y_a or, the investment in existing projects, is given in all the three cases, the optimization problem is reduced to maximizing $u_0(a_2,b_2y_b)$.

Substituting the conditions (A.1) into the expression (5) for u_0 yields

$$u_0 = |w_{01} + w_{02}|\, r_f + |E(r_a) - r_f|\, y_a + |E(r_b) - r_f|\, y_b$$

$$- \frac{\gamma_1}{2} \,|(1-a_2)^2\, y_a^2 \text{ var } (r_a) + (1 - b_2)^2 y_b \text{ var } (r_b) +$$

$$2\,(1 - a_2)\, y_b^2\, (1 - b_2)\, y_a\, y_b \text{ cov } (r_a, r_b)\,| -$$

$$\frac{\gamma_2}{2}\,|a_2^2\, y_a^2 \text{ var } (r_a) + b_2^2 \text{ var } (r_b) + 2a_2\, b_2\, y_a\, y_b \text{ cov } (r_a,r_b)|^2 \tag{A.2}$$

The first order necessary conditions for an interior a maximum, if no other constraint is in effect, is given by

$$\frac{\partial u_0}{\partial a_2} = \gamma_1 \left|(1 - a_2)\, y_a^2 \text{ var } (r_a) + (1 - b_2) y_a y_b \text{ cov } (r_a, r_b)\right|$$

$$- \gamma_2 \left| a_2\, y_a^2 \text{ var } (r_a) + b_2\, y_a y_b \text{ cov } (r_a, r_b)\right| = 0 \qquad \text{(A.3a)}$$

$$\frac{\partial u_0}{\partial b_2} = \gamma_1 \left|(1 - b_2)\, y_b^2 \text{ var}(r_b) + (1 - a_2) y_a y_b \text{ cov } (r_a, r_b)\right|$$

$$- \gamma_2 \left| b_2 y_b^2 \text{ var } (r_b) + a_2 y_a y_b \text{ cov } (r_a, r_b)\right| = 0 \qquad \text{(A.3b)}$$

$$\frac{\partial u_0}{\partial y_b} = E(r_b) - r_f - \gamma_1 \left|(1 - b_2)^2\, y_b \text{ var } (r_b) + (1-b_2) y_a \text{cov}(r_a, r_b)\right|$$

$$- \gamma_2 \left|b_2^2\, y_b \text{ var}(r_b) + a_2 b_2 y_a \text{ cov } (r_a,\ r_b)\right| = 0 \qquad \text{(A.3c)}$$

Conditions (A.3) are also sufficient for a maximum, given that u_0, by definition, is an aggregation of strictly concave utility functions.

In Case I, all risks are marketable, so that a_2, b_2 and y_b can be adjusted optimally and the interior maximum (a_2*, b_2*, y_b*) must simultaneoulsy satisfy all the conditions in (A.3) above. The resultant condition us summarized in Table II. In Case II, however, the risk of the new venture is not marketable so that it is necessary to impose the additional restriction that the value of b_2 is given. Hence, oncly a second-best optimum can be obtained which should simultaneoulsy satisfay only equations (A.3a) and (A.3c). The characteristics of this optimum are reported in Table III. Finally, in Case III, where no risks can be traded, the optimzation is further constrained by imposing a given value on a_2. Again, the solution is second best with the optimum satisfying only (A.3c) and is shown in Table IV.