COPPEAD/UFRJ

RELATÓRIO COPPEAD Nº 219

UNDERSTANDING THE VARIABILITY OF SIMULATION ESTIMATES

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Maio de 1989

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ABSTRACT

This paper describes an empirical study of the variability of simulation estimates, which produced some new interesting results. Simulation estimates are determined by the input sample. Any input sample can be decomposed into two basic features: the set of input values and their sequence. Based on this idea, the individual contribution of each feature to the estimates variance can be empirically studied. This is done following a two-way factorial experiment. Using the standard random sampling approach, the set of values was found to affect most simulation estimates in a common way, and to play a relevant role in their variability. The sequence effect, however, was found to be problem dependent. Apart from providing a better understanding of the estimates variability, this study contributed to the proposal of descriptive sampling.

Key words: Monte Carlo, simulation, variance reduction tecniques

Acknowledgements

I would like to thank Dr. Ray Paul from The London School of Economics for his help and co-operation in completing this paper. Thanks are also due to The British Council and IBM Brasil for the financial support given to my visiting program at the L.S.E, during which this paper was written.

INTRODUCTION

In a Monte Carlo simulation, processing time is directly related to the variability of the estimates. Processing time may not be the most important issue in a simulation project, but it is still relevant in many applications. According to Paul and Chew (1), the coding of a simulation program tends to be highly time-consuming, and therefore more critical than processing time. But now, there are many new computer simulation systems (2) which can reduce considerably the time spent in developing a program. Certainly, the most appealing of these products are the program generators like CAPS, written by Clementson (3), DRAFT by Mathewson (4) and, more recently, the VS6 system (5). Processing time, of course, has also been systematically reduced with computer evolution, but this improvement, unlike the software systems, is not derived from any simulation development.

There are cases, however, where processing time may be highly relevant. This happens when the simulated system is too complex, or when there are many replications to be done or also when response time is critical like, for example, in a military application. When the processing time problem arises, it may be worth reducing the variability of the estimates. One possibility of doing this is to use variance reduction techniques (6, 7).

The problem with variance reduction techniques is that it is not usually clear how they work and, as a consequence, how to use them efficiently. They require a better understanding of how simulation estimates vary, which is the purpose of the present work.

In this paper, following an approach similar to Ehrenfeld and Ben-Tuvia (8), the variance of simulation estimates was analysed in terms of two features - set and sequence - which jointly define an input sample. This empirical investigation was based on a two-way factorial model, conducting one experiment for each simulation problem considered. It was found that a substantial amount of the estimates' variance is only due to the variability associated with the set of input values, and that the sequence influence is problem dependent and rather unpredictable.

The identification of the set influence represents an important contribution towards understanding the estimates variability. It allows us to make a more efficient use of variance reduction techniques. But, more important, it was the cornerstone for us to question the use of simple random sampling in Monte Carlo simulation and to propose descriptive sampling as the most appropriate method (9).

THE PROBLEM

As an O.R. technique, it is assumed that the simulation purpose is to estimate parameters related to a response variable distribution. For example, in a port simulation, El Sheikh et al. (10) studied the mean waiting time of a ship; in a risk analysis, however, higher order moments are also studied (11). Unlike the parameters which are unknown constants, simulation estimates vary depending on the particular input sample generated. The higher this variability, the greater the processing time required to achieve a desired level of precision.

Problem formulation

As already explained in Saliby (9), a simulation run, by definition, produces one estimate for each parameter under study. Thus, assuming, for simplicity, that only one input random variable drives the simulation and that only one response variable is present, a simulation run can be seen as a set of function evaluations

$$Y_j = F_j(X_1, \ldots, X_n) \quad , \qquad (1)$$

where

Y_j, $j = 1,\ldots,L$ are the estimators for parameters θ_j, $j = 1,\ldots,L$,

F_j , $j = 1,\ldots,L$, is a set of functions, usually defined by a computer program, that maps the input sample X_i , $i = 1,\ldots,n$, into the corresponding estimators.

In principle, one has that $E(Y_j) \simeq \theta_j$, $j = 1,\ldots,L$. It is the variance $Var(Y_j)$, $j = 1,\ldots,L$, which we want to reduce.

Example

Illustrating this formulation, consider the PERT problem formerly studied by Sculli (12). As shown in figure 1, this network comprises eight activities, all them with durations that are independent and identically distributed. Activity durations follow a discrete distibution with equally likely values, so that

$$f(d) = 0.2 \quad \text{for} \quad d = 1,\ldots,5 \quad .$$

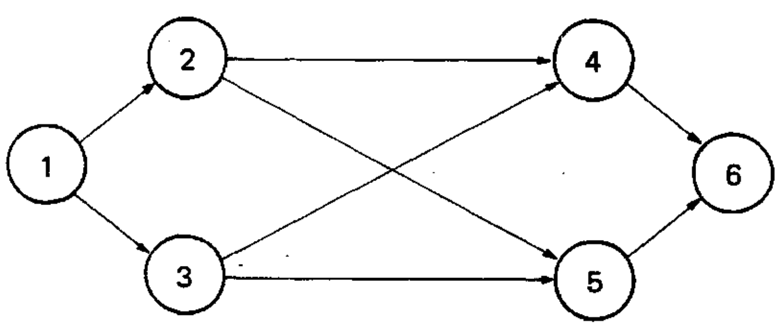

Figure 1. PERT network used as example. Activities are represented by arcs; their durations are independent and identically distributed.

The response variable under study is the total project duration (DT), for which the mean E(DT) and standard deviation SD(DT) are to be estimated. Although there are other methods to deal with this problem, it can be studied using simulation. In this case, a run is defined by a sample of N observations for DT,

$$DT_j \; , \quad j = 1,\dots,N \; .$$

The mean estimator is

$$\overline{DT} = \sum_{j=1}^{N} DT_j \,/\, N \quad ,$$

while the standard deviation estimator is

$$S_{DT} = [\sum_{j=1}^{N} (DT_j - \overline{DT})^2 \,/\, (N-1)]^{1/2} \quad .$$

Defining a run by, say, N=50 response values, the input sample size will be n = 50 x 8 = 400 activity durations. Then, based on (1), each run can be seen as the numerical evaluation of two functions:

$$\overline{DT} = F_1(D_1,\dots,D_{400}) \; ,$$

and

$$S_{DT} = F_2(D_1,\dots,D_{400}) \; .$$

Statistical properties required are that

$$E(\overline{DT}) \simeq E(DT) \; ,$$

$$E(S_{DT}) \simeq SD(DT) \; ,$$

and that $Var(\overline{DT})$ and $Var(S_{DT})$ are as low as possible.

The variability of the estimates

Expression (1) shows that estimates are determined by the input samples. In a simulation experiment, defined by M independent runs, a different sample is drawn each time, resulting in a different estimate too. As a corollary, the estimates variability follows from the input sample variability. Our problem is to find out if there exists a pattern for this variability, thus providing some clues on how to reduce it. Notice, however, that simulation estimates are aggregate measures. Therefore, in principle, the pattern which we are looking for should not be a relationship between estimates and individual input values, but between them and aggregate sample properties too.

Based on this idea, the influence of two global features of the input sample - set and sequence - are initially considered. As will be seen, it was found that one of those features - the set - has a specific and usually important influence on the variability of simulation estimates. This study is now described.

SET AND SEQUENCE: TWO GLOBAL SAMPLE FEATURES

In principle, any input sample can be seen as composed of two global features: the set of input values and their sequence. The set of values is defined by all sample values, without taking their sequence into account. As such, it displays the input variable's distribution pattern. The second feature - the sequence - is defined by the particular order in which the sample values occur. This sequence can be seen as a permutation of the set values and, as such, it is expected to display a "pattern of randomness" or, to be more precise, a lack of order of those values. Therefore, both features describe the two complementary properties that characterize the idea of probability: a pattern of frequencies and randomness.

Given that simulation estimates are determined by the input samples and that an input sample can be decomposed into a (set,sequence) combination, we are going to investigate the influence of both features on the variability of simulation estimates. Following the standard simulation practice, this study will assume the use of simple random sampling (S.R.S.) to generate the input sample.

The set and sequence variability when using S.R.S.

Using S.R.S., both sample features - set and sequence - are allowed to vary. This is illustrated in the example below.

Consider, for simplicity, a simulation run based on n=5 input values, sampled from a standard uniform distribution. Using S.R.S., we initially draw the sample

$$U_1 = (.21, .15, .95, .83, .07) .$$

Adopting the values' ascending order as reference, this sample is jointly defined by a set,

$$SET_1 = (.07, .15, .21, .83, .95) \quad ,$$

and a sequence,

$$SEQ_1 = (3, 2, 5, 4, 1) .$$

The sequence is defined by the permutation in which the set values appear in the sample. The first component of U_1 is the third element in the set, the second component of U_1 is also the second element in the set, and so on...

Replicating the run, a second sample is drawn, say

$$U_2 = (.13, .04, .82, .17, .61) \quad .$$

As in the previous sample, this sample is also defined by a set,

$$SET_2 = (.04, .13, .17, .61, .82) \quad ,$$

and a sequence

$$SEQ_2 = (2, 1, 5, 3, 4) .$$

As seen, the input sample variability is the joint result of the set and sequence variability. Notice also that the set varies at random when using S.R.S., although following the sampled distribution. On the other hand, the sequence is simply a random permutation of the set values. Once both features can be isolated, their influence on the estimates variability can also be isolated. This can be done following an alternative method for random sampling generation, now presented.

An alternative method for random sampling generation

In the same way that we are able to decompose a simple random sample into a random set of input values and their sequence, we can follow the reverse process to generate a simple random sample. Thus, we can define a simple random sample once a random set of values and a random permutation are given. For instance, given the two random sets and the two random permutations from the previous example, we can regenerate both U_1 and U_2.

This follows, because

$$U_1(1) = SET_1 [\ SEQ_1(1)\ ] = SET_1 [3] = .21\ ,$$

$$U_1(2) = SET_1 [\ SEQ_1(2)\ ] = SET_1 [2] = .15\ ,$$

and so on... As a rule, the elements of U_k, $k = 1, \ldots, L$, are defined by

$$U_k(j) = SET_k [\ SEQ_k(j)\ ]\ , \quad j = 1, \ldots, n\ .$$

Notice that even in cases where repeated set values are allowed, like when sampling from a discrete distribution, this idea of composing a random sample is still valid, although more than one random permutations may lead to the same input sample.

We have to generate both sample features separately. The sequence, defined by a random permutation, can be generated following a simple procedure described in Saliby(9). On the other hand, to generate a random set of values with the required variability, it is easier to continue using simple random sampling. Now, S.R.S. is used only to deliver a random set. The values are stored into ascending order, so that the original sequence is neglected.

Given a set of values and a sequence, we can build an input sample. This method increases the computer effort, but we are now able to investigate the contribution of each feature to the variability of simulation estimates. This is done following an experimental approach now described.

EXPERIMENTAL APPROACH TO STUDY THE ESTIMATES VARIABILITY

Since an input sample is defined by a set of values and their sequence, and since both features vary at random when using S.R.S., they can be seen as the two sources of variability of simulation estimates. Knowing how to isolate both features during the sampling process, we are now able to conduct a two-way factorial experiment to investigate their individual influence on the estimates variability. The two factors under consideration are, thus, the set and the sequence. The experiment is defined by NSET different sets of input values, all of them randomly generated, and NSEQ different sequences, also randomly permutated. For each combination of a set with a sequence - thus generating a simple random sample - a simulation run is carried out.

Example

As a numerical example, table 1 presents a subset of results from this experiment, for the PERT problem. Each run, defined by a pair (set,sequence), produces two estimates: $\overline{DT}$ and S_{DT}. For example, the simulation run based on SET_1 and $SEQUENCE_1$ lead to $\overline{DT}(1,1) = 11.14$ and $S_{DT}(1,1) = 1.906$.

The means of the estimates, for all runs based on the same set of input values, are also presented. The corresponding theoretical values for the parameters under study are also given in table 1. As expected, the estimates varied around those values. The lower part of table 1 also displays the sample frequencies for the activities durations, i.e., the input sets. Finally, the input means are also presented.

Table 1. Sample results of the experiment for the PERT problem. Results based on a subset of NSET=5 different sets and NSEQ=5 different sequences.

ESTIMATE $\overline{DT}$ [E(DT) = 11.2031]

	SET 1	SET 2	SET 3	SET 4	SET 5
SEQUENCE 1	11.14	11.24	10.98	11.16	11.08
SEQUENCE 2	11.36	11.26	11.16	11.48	11.00
SEQUENCE 3	11.46	11.36	10.96	11.32	11.18
SEQUENCE 4	11.52	11.44	11.00	11.42	11.04
SEQUENCE 5	11.32	11.26	10.92	11.32	11.00
MEAN	11.36	11.31	11.00	11.34	11.06

ESTIMATE S_{DT} [SD(DT) = 1.8764]

	SET 1	SET 2	SET 3	SET 4	SET 5
SEQUENCE 1	1.906	1.847	1.790	1.719	1.724
SEQUENCE 2	1.651	1.988	1.670	1.705	2.020
SEQUENCE 3	1.887	1.687	1.538	1.731	2.106
SEQUENCE 4	1.787	1.864	2.138	1.774	1.927
SEQUENCE 5	1.634	2.136	2.284	1.812	1.738
MEAN	1.773	1.904	1.884	1.748	1.903

SAMPLE FREQUENCIES AND MEANS FOR THE INPUT VALUES

VALUE	SET 1	SET 2	SET 3	SET 4	SET 5
1	65	82	89	72	87
2	93	80	79	72	74
3	70	72	77	95	88
4	93	77	82	78	80
5	79	89	73	83	71
TOTAL	400	400	400	400	400
INPUT MEAN	3.070	3.028	2.928	3.070	2.935

Statistical model

Although estimates are fully determined by a pair (set,sequence), we will use a statistical model to describe this relationship. As an approximation, it will be assumed that

$$Y = \theta + \mathcal{E}_{SET} + \mathcal{E}_{SEQ} + \mathcal{E}_{RES} ,$$

where,

$\mathcal{E}_{SET}$ is the influence on estimate Y due only to the input set of values. This component is called the set effect and is randomly generated. It will be assumed that

$$E(\mathcal{E}_{SET}) = 0 \quad \text{and} \quad Var(\mathcal{E}_{SET}) = \sigma^2_{SET} .$$

$\mathcal{E}_{SEQ}$ is the influence on estimate Y due only to the input sample sequence. This component is called the sequence effect and is also randomly generated. It will be assumed that

$$E(\mathcal{E}_{SEQ}) = 0 \quad \text{and} \quad Var(\mathcal{E}_{SEQ}) = \sigma^2_{SEQ} .$$

$\mathcal{E}_{RES}$ is the residual or unexplained influence on estimate Y due to the joint action of both effects. As such, it refers to the interaction component. It will also be assumed that

$$E(\mathcal{E}_{RES}) = 0 \quad \text{and} \quad Var(\mathcal{E}_{RES}) = \sigma^2_{RES} .$$

According to this model, which assumes all effects to be independent, it follows that

$$E(Y) = \theta ,$$

and that

$$Var(Y) = \sigma^2_{SET} + \sigma^2_{SEQ} + \sigma^2_{RES} .$$

The purpose of the sources of variation study is to identify and to estimate the individual variance contribution of both factors. Notice that, without this decomposition, the variance of the estimates would be totally attributed to the residual effect.

Based on this formulation, we will test the hypothesis that the variance components associated with the two factors are meaningful. We will test to see if

$$\sigma^2_{SET} > 0 \quad \text{and if} \quad \sigma^2_{SEQ} > 0 ,$$

estimating their value whenever a significant contribution is detected.

Analysing the results

Results in table 1 already show that, at least for the PERT problem, one of the estimates ($\overline{DT}$) may follow a pattern of variation. In fact, $\overline{DT}$ appears to have a lower variability once the set of input values is fixed. Another important feature is that, as expected, $\overline{DT}$ means are higher for sets with a higher input mean and vice-versa.

A proper analysis of the results is through analysis of variance, using the random effects model (13). The use of this model, also known as the variance components model, is due to the fact that both factors - set and sequence - have their levels randomly selected. In spite of this fact, the calculations are the same as for the standard two-way analysis of variance. But, in addition to the usual calculations, variance components are also estimated.

Variance components are estimated by

$$\hat{\sigma}^2_{SET} = (MS_{SET} - MS_{RES}) / NSEQ$$

and

$$\hat{\sigma}^2_{SEQ} = (MS_{SEQ} - MS_{RES}) / NSET \quad ,$$

where,

MS_{SET} and MS_{SEQ} are the mean squares from the ANOVA table, corresponding to the explained variation by, respectively, the set and the sequence. Such values are only considered when a factor turns out to be significant, otherwise they are pooled together with the residual sum of squares;

$MS_{RES} = \hat{\sigma}^2_{RES}$ is the residual mean square, as given in the ANOVA table.

Based on this method of analysis, the sources of variation of simulation estimates can now be empirically studied.

EMPIRICAL RESULTS

Three simulation problems, the PERT network, an M/M/I queue, and an inventory system, were studied following this experimental approach. Each experiment was based on 10 random sets of input values and on 10 random sequences, thus giving a total of 100 simple random sampling runs. For each problem, two estimates were considered: the mean and the standard deviation of the response variable under study.

The queueing problem

In the queueing problem, the estimates concerned the steady state queue size distribution, including customers in service. For each of 6 different traffic intensity values (ρ), ranging from 0.15 to 0.90, one experiment was conducted. Each run started with the system empty and lasted until 1000 services were completed. In so doing, the sample size for the service times was fixed at NS=1000, whilst the sample size for the interarrival times, NA, varied. But, since the system was empty at the beginning of a run, it follows that NA $\geq$ NS. In reality, only the first 1000 interarrival times were controlled in the experiment; the remaining NA - NS values were generated using the standard simple random sampling procedure. Despite this sample variability, the run length was long enough so that NA - NS was always very small when compared with NS. Therefore, it was possible to achieve a very good control over the set and sequence features. Finally, notice that once there are two input random variables in this problem - interarrival and service times - , each set level in the experiment was defined by a pair of set values, one for each input variable. Similarly, a sequence level was defined by a pair of random permutations.

The inventory system

The inventory system investigated here represents another standard simulation problem, already studied by Naylor and Hunter (14) and Ignall (15). It consists of a single product inventory system, with a daily periodic review. If the inventory level - including pending orders - falls below the reorder point (ROP), a fixed quantity (Q) is ordered. The lead time is assumed constant; products become available in the beginning of the second day after their ordering. Demand not met is holdover. Daily demand follows a gamma distribution composed by 3 stages, each one with mean of 10 units. Demand values are rounded to the nearest integer.

The response variable is the steady state daily cost, composed of three different costs:

C1 = 0.5 , cost of storage per unit per day;

C2 = 175 , cost of placing an order; and

C3 = 5.0 , cost of shortage per unit per day.

The initial inventory is 100 units, and the run length is a period of ND=1000 simulated days. It follows that the input sample size consists of ND=1000 demand values.

For each of the three system configurations below, a complete experiment was carried out:

ROP = 50 and Q = 150 ;

ROP = 75 and Q = 200 ; and

ROP = 40 and Q = 150 .

The results

To illustrate the analysis of the experimental results, we present the steps envolved in the study of estimate DT from the PERT problem. Exactly the same procedure was followed in all other cases.

Each experiment produced NSET x NSEQ = 100 values for each estimate. In our case they are represented by

$$\overline{DT}(i,j) \;, \quad i = 1,\ldots,NSET \;, \quad j = 1,\ldots,NSEQ \;.$$

Based on the values from one experiment, table 2 presents the corresponding two-way ANOVA table.

Table 2. ANOVA Table for estimate $\overline{DT}$

Source of Variation	Sum of Squares	Degrees of freedom	Mean Square	F Value
Between sets	2.994	9	.333	12.95
Between sequences	.387	9	.043	1.67
Unexplained	2.081	81	.026	
Total	5.462	99	.055	

The results in table 2 show that the set variability has a strong contribution to the variance of $\widetilde{DT}$, but not the sequence. The next step is to compute the variance components of $\widetilde{DT}$. They are presented in table 3, with the corresponding relative contribution.

Table 3. Variance components of estimate $\widetilde{DT}$. Results based on an experiment composed by M=100 runs.

Source of variation	Absolute Contribution	Relative Contribution
Set effect	.0306	53 %
Sequence effect	-	-
Residual effect	.0274	47 %
Total	.0580	100 %

From the results in table 3, we can say that 53 % of the total variance of $\widetilde{DT}$ is explained by the set variability. Only 47 % of the original variance remained unexplained by the random effects model.

Following the same procedure, the variance components for the other estimates were similarly computed. Table 4 summarizes those results.

Table 4. Relative contribution of each factor to the variance of the estimates for three simulation problems. Each row totals 100 % .

Problem	Estimate	SET(%)	SEQ(%)	RESIDUAL(%)
PERT network:				
	$\overline{DT}$	53	--	47
	S_{DT}	--	--	100
M/M/1 Queue:				
(ρ =0.15)	$\overline{SIZE}$	96.3	3.4	0.2
(ρ =0.30)	$\overline{SIZE}$	86.1	13.4	0.5
(ρ =0.45)	$\overline{SIZE}$	78.8	20.5	0.7
(ρ =0.60)	$\overline{SIZE}$	69.1	29.5	1.4
(ρ =0.75)	$\overline{SIZE}$	62.9	32.5	4.6
(ρ =0.90)	$\overline{SIZE}$	57.0	30.8	12.2
(ρ =0.15)	S_{SIZE}	76.9	18.5	4.6
(ρ =0.30)	S_{SIZE}	50.0	47.8	2.2
(ρ =0.45)	S_{SIZE}	47.3	51.0	1.7
(ρ =0.60)	S_{SIZE}	41.0	56.6	2.4
(ρ =0.75)	S_{SIZE}	33.3	59.8	6.9
(ρ =0.90)	S_{SIZE}	32.6	59.4	8.0
Inventory:				
(ROP=50,Q=150)	$\overline{COST}$	57	--	43
(ROP=75,Q=200)	$\overline{COST}$	31	--	69
(ROP=40,Q=150)	$\overline{COST}$	54	--	46
(ROP=50,Q=150)	S_{COST}	34	--	66
(ROP=75,Q=200)	S_{COST}	36	--	64
(ROP=40,Q=150)	S_{COST}	36	--	64

Discussion

Observing the results in table 4, we can conclude that:

(a) Set variability appears to be a common feature of most simulation problems. Generally speaking, it usually explains a substantial amount of the estimates' variance. Only in the case of the estimate S_{DT} for the PERT problem, does this not happen.

(b) The sequence variability appears to be problem dependent. It was only observed in the queue problem.

(c) The queue problem deserves special attention. For both estimates, the amount of variance explained by both factors, separately, was quite high. But, the relative contribution of the set variability decreases with the traffic intensity, while the sequence contribution increases. This result can be explained by the fact that in a more congested system, higher values for the servicing times (or lower values for the interarrival times) have a long term effect in the system response, thus creating a sequence dependence. In a less congested system, such extreme values are quickly washed up.

CONCLUSIONS

Three different simulation problems were studied whose results concerning the set contribution are symptomatic of other problems. As a common feature of the variability of simulation estimates, the set effect deserved further investigation. In Saliby (16), it is shown that a regression model, named the Linear Response Model, explains the set effect as the result of deviations between the input sample moments and the corresponding theoretical values. Another important property is that the set relative contribution does not depend on the run length. This means that even by increasing the input sample size, the set still plays the same role, as far as the variance of the estimates are concerned.

Having a better idea of how simulation estimates vary, we are now in a position to make better use of variance reduction techniques. For example, techniques that allow more control over the set variability, like common random numbers, are likely to be more efficient. The same applies to control variates, where the set variability is taken into account. On the other hand, antithetic variates are less efficient techniques once they just provide a partial control over the set variability (17).

However, the main contribution from this study was to suggest that the set variability is a sort of noise which is introduced during the sampling process. Questioning this variability, we derived the proposal of a new sampling approach in simulation: descriptive sampling (9). Descriptive sampling is based on a fully deterministic selection of the set values, thus avoiding the set variability, and their random permutation. Using descriptive sampling, simulation estimates will be more precise. This follows because, in so doing, only the sequence is varying between different runs.

REFERENCES

(1) R. J. PAUL and S. T. CHEW (1987) Simulation modelling using an interactive simulation program generator. J. Opl Res. Soc. 38, 735-752.

(2) J. L. AUTHUR , J. D. FRENDEWEY, P. GHANDFOROUSH and L. P. REES (1986). Comput. & Ops Res. 13, 167-183.

(3) A. T. CLEMENTSON (1982) Extended Control and Simulation Language. Cle. Com. Ltd, Birmingham.

(4) S. C. MATHEWSON (1982) DRAFT II/FORTRAN Manual. Department of Management Science, Imperial College, London.

(5) R. J. PAUL , P. M. KNOX and S. T. CHEW (1988) VS6 Manual. SysPack Ltd, London.

(6) B. A. P. JAMES (1985) Variance reduction techniques. J. Opl Res. Soc. 36, 525-530.

(7) J. P. C. KLEIJNEN (1974) **Statistical Techniques in Simulation**. Part I. Marcel Dekker, New York.

(8) S. EHRENFELD and S. BEN-TUVIA (1962) The efficiency of statistical simulation procedures. Technometrics **4**, 257-275.

(9) E. SALIBY (1989) Descriptive sampling: a more appropriate approach in Monte Carlo simulation. Rio de Janeiro, COPPEAD/UFRJ. (Relatório de Pesquisa, 86)

(10) A. A. R. EL SHEIKH, R. J. PAUL, A. S. HARDING and D. BALMER (1987) A microcomputer-based simulation study of a port. J. Opl Res. Soc. 38, 673-681.

(11) D. HERTZ and H. THOMAS (1983) **Risk Analysis and Its Applications.** Wiley, Chichester.

(12) D. SCULLI (1983) The completion time of PERT networks. J. Opl Res. Soc. 34, 155-158.

(13) P. W. M. JOHN (1971) **Statistical Design and Analysis of Experiments.** Macmillan, New York.

(14) T. H. NAYLOR and J. S. HUNTER (1969) Experimental design. In **The Design of Computer Simulation Experiments** (T. H. Naylor, Ed.). Duke University Press, Durham N.C.

(15) E. J. IGNALL (1972) On experimental designs for computer simulation experiments. Management Sci. 18, 384-388.

(16) E. SALIBY (1989) **Rethinking Simulation: Descriptive Sampling.** Atlas/EDUFRJ, São Paulo. In portuguese.

(17) E. SALIBY (1980) A reappraisal of some simulation fundamentals. Ph.D Thesis, University of Lancaster.