

ANALYTICAL MODEL FOR FLEXIBLE PIPE ARMOR WIRE LATERAL INSTABILITY

Xiaotian Li

Tese de Doutorado apresentada ao Programa de Pós-graduação em Engenharia Oceânica, COPPE, da Universidade Federal do Rio de Janeiro, como parte dos requisitos necessários à obtenção do título de Doutor em Engenharia Oceânica.

Orientador: Murilo Augusto Vaz

Rio de Janeiro Junho de 2019

ANALYTICAL MODEL FOR FLEXIBLE PIPE ARMOR WIRE LATERAL INSTABILITY

Xiaotian Li

TESE SUBMETIDA AO CORPO DOCENTE DO INSTITUTO ALBERTO LUIZ COIMBRA DE PÓS-GRADUAÇÃO E PESQUISA DE ENGENHARIA (COPPE) DA UNIVERSIDADE FEDERAL DO RIO DE JANEIRO COMO PARTE DOS REQUISITOS NECESSÁRIOS PARA A OBTENÇÃO DO GRAU DE DOUTOR EM CIÊNCIAS EM ENGENHARIA OCEÂNICA.

Examinada por:

Prof. Murilo Augusto Vaz, Ph.D.

Prof. Marcelo Caire, D.Sc.

Prof. José Renato Mendes de Sousa, D.Sc.

Prof. Lavinia Maria Sanabio Alves Borges, D.Sc.

Prof. Celso Pupo Pesce, D.Sc.

RIO DE JANEIRO, RJ – BRASIL JUNHO DE 2019

Li, Xiaotian

Analytical model for flexible pipe armor wire lateral instability/Xiaotian Li. – Rio de Janeiro: UFRJ/COPPE, 2019.

XVI, 69 p.: il.; 29, 7cm.

Orientador: Murilo Augusto Vaz

Tese (doutorado) – UFRJ/COPPE/Programa de Engenharia Oceânica, 2019.

Referências Bibliográficas: p. 64 – 69.

 flexible pipes.
 tensile armors.
 lateral instability.
 Vaz, Murilo Augusto.
 Universidade
 Federal do Rio de Janeiro, COPPE, Programa de Engenharia Oceânica.
 III. Título.

"Você tem duas vidas. A segunda começa quando você percebe que só tem uma." -Confúcio

Acknowledgements

I would like to express my deep and sincere gratitude to my supervisor, Prof. Murilo Augusto Vaz, for providing professional guidance throughout this research. He has shown me, by his example, what a good scientist (and person) should be.

I must thank Dr. Anderson from Petrobras, Sueli, David and all the other staff from the lab for their help in setting up the experiments.

I would like to thank the rest of my thesis committee: Prof. Marcelo, Prof. Lavinia, Prof. Celso, and Prof. José, for their careful reading of the manuscript.

I gratefully acknowledge the funding I received from Petrobras, the National Council of Scientic and Technological Development (CNPq) (No 302380/2013-2) and the Brazilian Coordination for the Improvement of Higher Education Personnel (CAPES) (No 1987/2014) to undertake this research.

I would like to thank all my fellow labmates from NEO, Luis, Victor, Miguel, Nain, Aynor, David, Marcelo, Andressa, Lívia, Nicolau, Nathalia, Rodrigo, Jens, Arvind, Junpeng Liu, Yang Zhou, Yangye He, Hailong Lu, Rongzhi Wei for supporting me and providing a stimulating and fun working environment. I am particular grateful to Eliene for the great Brazilian coffee she made for us everyday.

I am indebted to all my friends scattered around Brazil for the wonderful times we shared. Felipe, Douglas, Flaviano, Pedro, Daniel, Afonso, Luis, Zio, Diogo, Glauter...the list is endless...Without their company there won't be so much fun during my stay in Brazil.

Finally, I must express my very profound gratitude to my parents: Xueqin Li and Jianyu Wang, for loving me and supporting me throughout my life. Resumo da Tese apresentada à COPPE/UFRJ como parte dos requisitos necessários para a obtenção do grau de Doutor em Ciências (D.Sc.)

MODELO ANALÍTICO PARA INSTABILIDADE LATERAL DE ARAMES DAS ARMADURAS DE TRAÇÃO DE DUTOS FLEXÍVEIS

Xiaotian Li

Junho/2019

Orientador: Murilo Augusto Vaz Programa: Engenharia Oceânica

A indústria offshore de petróleo e gás é confrontada com a questão da instabilidade lateral dos arames das armaduras de tração de dutos flexíveis por mais de duas décadas. A presente tese tem por objetivo investigar teórica e experimentalmente o mecanismo subjacente da instabilidade lateral dos arames das armaduras de tração. Um modelo analítico é desenvolvido para descrever a deflexão dos arames das armaduras de tração de um duto flexível submetido a uma carga compressiva axial constante associada à flexão cíclica. Este modelo é capaz de avaliar a trajetória de equilíbrio do arame das armaduras de tração após múltiplos ciclos de flexão em estados estáveis, bem como estimar a carga compressiva axial crítica que pode causar a instabilidade lateral das armaduras de tração. Dois dutos flexíveis são ensaiados numa câmara hiperbárica aplicando alta pressão hidrostática associada a múltiplos ciclos de flexão. Os dados obtidos dos testes, bem como os disponíveis na literatura, são comparados com o limite de estabilidade lateral das armaduras de tração estimado pelo presente modelo analítico, mostrando excelente concordância. Além disso, algumas sugestões úteis para melhorar o projeto do tubo flexível contra a instabilidade lateral das armaduras de tração são apresentadas.

Abstract of Thesis presented to COPPE/UFRJ as a partial fulfillment of the requirements for the degree of Doctor of Science (D.Sc.)

ANALYTICAL MODEL FOR FLEXIBLE PIPE ARMOR WIRE LATERAL INSTABILITY

Xiaotian Li

June/2019

Advisor: Murilo Augusto Vaz Department: Ocean Engineering

The offshore oil and gas industry has been confronted with the issue of armor wire lateral instability in unbonded flexible pipes for over two decades. The objective of the present thesis is by theoretical as well as experimental means to investigate the underlying mechanism of the armor wire lateral instability. Analytical model is developed to describe the armor wire deflection in a flexible pipe subjected to a constant axial compressive load combined to cyclic bending. This model is capable of evaluating the equilibrium path of the armor wire after numerous bending cycles in stable states, as well as estimating the critical axial compressive load that may cause the armor wire lateral instability failure. Two flexible pipes are tested in a hyperbaric chamber applying high hydrostatic pressure combined to numerous bending cycles. The obtained test data, as well as that available in the literature, is compared with the armor wire lateral stability limit estimated by the present analytical model, showing excellent agreement. Also, some useful suggestions for improving the flexible pipe design against the armor wire lateral instability are presented.

Contents

| List of Figures | | | |
|-----------------|-------|--|--|
| Li | st of | Tables xii | |
| \mathbf{Li} | st of | Symbols xiii | |
| 1 | Intr | oduction 1 | |
| | 1.1 | Unbonded flexible pipe | |
| | 1.2 | Armor wire instability in flexible pipe | |
| | 1.3 | Research objectives | |
| | 1.4 | Thesis organization | |
| 2 | Lite | arature review 9 | |
| 3 Theory | | ory 14 | |
| | 3.1 | Field equations for a single armor wire | |
| | | 3.1.1 Geometrical relations | |
| | | 3.1.2 Wire curvature components | |
| | | 3.1.3 Equilibrium equations | |
| | | 3.1.4 Constitutive equations $\ldots \ldots 18$ | |
| | | 3.1.5 Boundary conditions | |
| | 3.2 | Linearization through a perturbation technique | |
| | 3.3 | Axial rotation discussion | |
| | 3.4 | Sliding and frictional forces in dynamic bending | |
| | 3.5 | Equilibrium state | |
| | 3.6 | Lateral instability | |
| 4 Discussions | | cussions 36 | |
| | 4.1 | Equilibrium path | |
| | 4.2 | Frictional forces | |
| | 4.3 | Longitudinal shortening 41 | |
| | 4.4 | Curvatures and moments | |

| | 4.5 | Stress analysis | 43 |
|----------|--------------------------|--|----|
| | 4.6 | Lateral contact | 44 |
| | _ | | |
| 5 | \mathbf{Exp} | Experimental reconstruction of the armor wire lateral instability \sim | |
| | 5.1 | Test of the 6" flexible riser sample | 50 |
| | 5.2 | Test of the 6" flexible flowline sample | 52 |
| | 5.3 | Discussions | 54 |
| | | 5.3.1 Deflection shape \ldots | 54 |
| | | 5.3.2 Lateral contact \ldots | 54 |
| | | 5.3.3 Lateral stability limit | 55 |
| C | | | ٣٥ |
| 0 | Design recommendations 5 | | 58 |
| 7 | Con | cluding remarks | 61 |
| | 7.1 | Recommendations for future research | 63 |
| Bi | Bibliography 64 | | |

List of Figures

| 1.1 | Flexible pipe flowline and riser system examples (4Subsea) | 2 |
|-----|---|----|
| 1.2 | Typical unbonded flexible pipe | 3 |
| 1.3 | Schematic of flexible pipe installation in the free-hanging configuration. | 4 |
| 1.4 | Birdcaging failure mode in a 6" flexible pipe obtained in the labora- | |
| | tory environment | 5 |
| 1.5 | Lateral instability in the inner armor wire layer of a 6" flexible pipe | |
| | obtained in the laboratory environment | 6 |
| 3.1 | Armor wire geometry and coordinate system | 14 |
| 3.2 | Schematic of the armor wire axial rotation: contacts both the upper | |
| | and lower borders (left) and only contacts the upper border (right). $% \left({{\left[{{\left[{{\left[{\left[{\left[{\left[{\left[{\left[{\left[$ | 16 |
| 3.3 | Armor wire cross-section: contacts both the upper and lower borders | |
| | (left) and only contacts the upper border (right). \ldots \ldots \ldots | 16 |
| 3.4 | Initial and deformed paths of half armor wire pitch length. \ldots . | 20 |
| 3.5 | Sliding directions and friction components for a point sliding on the | |
| | toroid tangent space from A to A' . | 25 |
| 3.6 | Schematic of the armor wire tangential sliding direction in cyclic | |
| | bending | 26 |
| 3.7 | Longitudinal load evaluation assuming that each armor wire deflects | |
| | identically | 27 |
| 4.1 | Equilibrium paths of the inner armor wire in different axial compres- | |
| | sive loads. | 37 |
| 4.2 | Equilibrium paths of the inner armor wire when $\bar{P}_t =$ | |
| | $-1500 N, -2182 N, -2500 N. \dots \dots$ | 37 |
| 4.3 | The predicted configuration in the stable state when the compres- | |
| | sive load is close to the stability limit (left) and a typical S-shape | |
| | instability configuration (right) (CALEYRON <i>et al.</i> [1]) | 37 |
| 4.4 | Δu^e at the neutral plane with different \bar{P}_t and bending curvatures | 38 |
| 4.5 | Frictional forces on the inner armor wire in the wet annulus condition | |
| | in the equilibrium state when $\bar{P}_t = -2500 N. \dots \dots \dots \dots$ | 39 |

| 4.6 | Frictional forces on the inner armor wire in the wet annulus condition in the equilibrium state when $\bar{P}_t = -1000 N_1 \dots N_{t-1} \dots N_{t-1}$ | 40 |
|------|--|----|
| 4.7 | Effect of axial compressive loads on the ratio of frictional force com- ponents on the inner armor wire in the wet annulus condition in the equilibrium state when the bending curvature approaches the maxi | |
| | mum value | 40 |
| 4.8 | Longitudinal shortening with different axial compression and bending | 40 |
| 1.0 | curvatures. | 41 |
| 4.9 | Curvature components of the armor wire in the equilibrium path | 42 |
| 4.10 | Cross-sectional moments in the armor wire in the equilibrium path. | 42 |
| 4.11 | Distributed torques on the armor wire in the equilibrium path | 43 |
| 4.12 | Critical yielding positions on an armor wire cross-section | 44 |
| 4.13 | Maximum compressive stresses in the cross-sections of the inner armor | |
| | wire in the equilibrium state in repeated bending | 44 |
| 4.14 | Maximum compressive stresses in the inner armor wire in the equi- | |
| | librium state with different axial compressive loads and bending cur- | |
| | vatures. | 45 |
| 4.15 | Schematic of the adjacent armor wire segments in the initial (left) | |
| | and contact (right) conditions | 45 |
| 4.16 | Armor wire lay angle variations in equilibrium paths with different | |
| | bending curvatures | 46 |
| 4.17 | Armor wire lay angle variation at the extrados with different axial | |
| | compression and bending curvatures | 47 |
| 4.18 | Schematic of the armor wire lateral contact with a small gap (left) | |
| | and a big gap (right) | 47 |
| 5.1 | Schematic drawings of the test setup. | 49 |
| 5.2 | Preparation of the connectors for the 6-inch flexible riser sample | 50 |
| 5.3 | Schematic of sample preparation | 51 |
| 5.4 | Collapse of the carcass | 52 |
| 5.5 | Sample appearance after the test (close to the fixed end) | 53 |
| 5.6 | Configuration of the inner armor layer in the 6" flowline sample after | |
| | the test | 53 |
| 5.7 | Lateral contact of armor wires in section 1 (left) and section 3 (right). | 55 |
| 5.8 | Analytical predictions versus test data | 57 |
| 6.1 | Effect of armor wire lay angle on the lateral stability limit | 59 |
| 6.2 | Effect of armor wire width on the lateral stability limit | 59 |
| | | |

List of Tables

| 5.1 | Armor wire layers characteristics of the flexible pipe samples | 50 |
|-----|--|----|
| 5.2 | Tensile armors characteristics | 56 |
| 5.3 | Tests conditions and results | 57 |

List of Symbols

| A | Armor wire cross-sectional area, p. 19 |
|------------------|--|
| E,G | Young's modulus and the shear modulus of the armor wire, p. 19 |
| I_n, I_b, J | Armor wire normal inertia, transverse inertia and stiffness of torsion, p. 19 |
| M_t, M_n, M_b | Armor wire cross-sectional moments in the tangential, normal and bi-normal directions, p. 18 |
| N | Total number of armor wires in the flexible pipe, p. 29 |
| N_{in} | Number of armor wires in the inner armor wire layer, p. 28 |
| P_{1}, P_{2} | Constants corresponding to the armor wire characteristics, p. 23 |
| P_3 | Constant corresponding to the armor wire characteristics, p. 32 |
| P_t, P_n, P_b | Armor wire cross-sectional forces in the tangential, normal and bi-normal directions, p. 18 |
| P_{LI} | Lateral instability limit, p. 34 |
| P_{end} | Longitudinal compressive load carried by the flexible pipe, p. 29 |
| P_{in} | Longitudinal compressive load carried by the inner armor wire layer, p. 28 |
| R | Major radius of the toroid, p. 14 |
| W, H | Armor wire width and thickness, p. 19 |
| $\Delta \bar{L}$ | Flexible pipe longitudinal compressive strain, p. 21 |

 $\Delta u(\theta)$ Armor wire path variation relative to the new helix, p. 21

 $\Delta \kappa_g, \Delta \kappa_n, \Delta \tau^i$ Armor wire geodesic curvature, normal curvature and torsion variations relative to that in the stress-free state, p. 19

- $\Delta \phi^c$ Armor wire lay angle variation when the armor wires are in contact, p. 45
- $\Delta \phi_1$ Angle difference between the initial and new helixes in the straight pipe condition, p. 21
- $\Delta \phi_2$ Armor wire lay angle variation relative to the new helix angle, p. 21
- $\Delta \phi$ Armor wire lay angle variation relative to the initial helix angle, p. 21
- $\Delta u^{e}(\theta)$ Difference between the equilibrium path and the new helix, p. 31
 - Ω Average filling factor of the inner and outer armor wire layers, p. 35
- $\boldsymbol{n}_0, \boldsymbol{b}_0$ Unit vectors in the amor wire normal and bi-normal directions, p. 16
- t, n, b Unit vectors in the armor wire tangential, toroid normal and the corresponding bi-normal directions, p. 15
 - χ, η Variables, p. 29
- κ_g, κ_n, τ Armor wire geodesic curvature, normal curvature and torsion, p. 17
- $\kappa_g^i, \kappa_n^i, \tau^i$ Armor wire geodesic curvature, normal curvature and torsion in the stress-free state, p. 19
- $\kappa_{min}, \kappa_{max}$ Minimum and maximum bending curvatures of cyclic bending, p. 31
 - ν Poisson's ratio, p. 35
 - ω Amor wire axial rotation, p. 16
 - ϕ Armor wire lay angle, p. 15
 - ϕ^i Armor wire lay angle in the stress-free state, p. 19

| σ_s | Armor wire yield stress, p. 44 |
|--|--|
| heta | Angular coordinate, p. 15 |
| $	heta_j$ | Angular position of the j^{th} inner armor wire at a certain pipe cross-section, p. 28 |
| ε | Armor wire axial strain, p. 19 |
| g | Gap between the adjacent armor wires in the initial straight pipe condition, p. 45 |
| m_t, m_n, m_b | External moments on the armor wire in the tangential, normal and bi-normal directions, p. 18 |
| p_t, p_n, p_b | Distributed forces on the armor wire in the tangential, normal and bi-normal directions, p. 18 |
| p_{up}, p_{low} | Contact forces at the upper and lower interfaces with the neighboring layers, p. 24 |
| r | Minor radius of the toroid, p. 14 |
| s | Amor wire arc length in the deformed state, p. 15 |
| s^i | Armor wire arc length in the stress-free state, p. 19 |
| u | Longitudinal coordinate, p. 15 |
| x,y,z | Cartesian coordinates, p. 15 |
| $\sigma_{\max}\left(heta ight)$ | Maximum compressive stress in each cross-section of the armor wire, p. 43 |
| $u^{e}\left(heta ight)$ | Equilibrium path, p. 31 |
| $u^{i}\left(heta ight) ,u^{h}\left(heta ight)$ | The initial helix path on the undeformed toroid and the new helix path on the deformed toroid, p. 20 |
| $u_{n}\left(heta ight)$ | Armor wire path after the n^{th} bending cycle, p. 31 |
| \bar{P}_t | Armor wire tangential cross-sectional force at the neutral plane, p. 28 |
| $ar{P}_{LI}, \xi, \zeta$ | Non-dimensional parameters, p. 35 |

| $\widehat{p}_t, \widehat{p}_b, \widecheck{p}_t, \widecheck{p}_b$ | Frictional forces on the armor wire in the tangential and bi- normal directions when the bending curvature increases and decreases, p. 38 |
|--|--|
| $\widehat{P}_{t}\left(0 ight), \widecheck{P}_{t}\left(0 ight)$ | Concentrated force in the inner armor wire in the tangential direction at extrados when the bending curvature increases and decreases, p. 27 |
| $d\Delta t/d\kappa, d\Delta b/d\kappa$ | Sliding rates of the points on the armor wire relative to the toroid with respect to the bending curvature in the tangential and bi-normal directions, p. 25 |
| $d\Delta u/d\kappa, d\Delta \theta/d\kappa$ | Instantaneous longitudinal and angular coordinate variation rates of the points on the armor wire with respect to the bending curvature, p. 30 |

Chapter 1

Introduction

1.1 Unbonded flexible pipe

Even though many countries are increasingly developing modern renewable energy, fossil fuels, especially hydrocarbons, are still the major sources of the world energy. Currently, approximately 30% of world oil and gas production is from offshore and it is expected to continue to increase in the future. Since the 1990s, driven by the decrease in the available number of oil and gas reservoirs in shallow waters, the offshore industry was forced to move into deeper waters, which requires a wide range of solutions. Unbonded flexible pipes are critical elements in deepwater oil and gas development. Since the first successful application of flexible pipes in Enchova field offshore Brazil in 1978 as part of a floating production system [2], over 3,300 flexible pipes are in service nowadays in various fields in North America, Latin America, Europe, Middle-East, Asia-Pacific and Africa. They are often used either as risers for transporting fluid between subsea installation and topside facilities or as flowlines or jumpers for connecting subsea equipment as depicted in Fig. 1.1. Unbonded flexible pipes are designed in accordance with API 17J [3] together with the associated Recommended Practice API 17B [4]. The ancillary components for flexible pipe system are presented in API 17L1 [5] and API 17L2 [6].

As the name implies, the main characteristic of a flexible pipe is its low relative bending to longitudinal stiffness. The compliant structure provides flexible pipes with many advantages over rigid pipes. Flexible pipes can be easily stored on limited sized reels which simplifies transportation and installation. Besides, the application of flexible risers allows for permanent connection between subsea installations and floating facilities with large motions. Moreover, the use of flexible flowlines enables routing in crowded subsea layouts.

Fig. 1.2 shows a typical un-bonded flexible pipe cross-section, which is comprised of a number of layers with specific functions. Essentially, the metallic layers



Figure 1.1: Flexible pipe flowline and riser system examples (4Subsea).

provide structural strength and the polymeric layers offer fluid leak-proof capacity and reduce the friction between metallic layers. The fluid barrier, also known as the inner liner, is an extruded polymeric layer which contains the bore fluid. The carcass locates beneath the fluid barrier, and is an interlocked metallic structure with a high pitch angle that provides the fluid barrier with collapse resistance. The pressure armor, which is also a metallic structure with a high pitch angle, is applied over the fluid barrier to resist the hoop pressure and support the carcass against radial compressive loads. The tensile armor wire layers, conventionally consisting of helically wound steel wires with quasi-rectangular cross-section, bear most of the tensile loads. In most applications of flexible pipes, two tensile armor wire layers are employed with opposite lay angles. The anti-birdcage tapes, commonly made of aramid fibers, are loosely wound over the tensile armor wire layers to reduce the radial expansion of the armor wires in axial compression. Anti-wear tape, which is a nonmetallic extruded thermoplastic sheath or tape wrapping, is generally applied between metallic layers to reduce the friction abrasion. The outer sheath, which is an extruded polymer layer, prevents the ingress of seawater and oxygen to the annulus. The space between the fluid barrier and the outer sheath is referred to as the pipe annulus. Those layers work independently and interact with each other which generates the desired flexible pipe properties. Depending on the application, some of those layers may not exist and additional layers may be employed. For instance, in certain flexible pipe applications, multiple layers of highly insulating polymeric tapes may be needed to improve thermal insulation properties for bore fluid.

To meet the increasing demand for the application of flexible pipe in ultra-deep



Figure 1.2: Typical unbonded flexible pipe.

waters, new techniques have continued to be developed over last years. For the application in ultra-deep waters, the top sections of flexible riser need to withstand a high tensile load imposed by the suspended weight proportional to the water depth, which causes severe fatigue problem in tensile armor wires and increases the installation and operation difficulty. Worse still, the presence of CO_2 and H_2S in some offshore discoveries, such as the pre-salt discoveries in offshore Brazil, requires the use of steel with reduced mechanical properties compared to a sweet service application. Recent works [7-10] have shown the great potential of carbon fiber armor wire in replacing conventional steel armor wire due to its higher tensile strength, excellent corrosion resistance, and lower density. The application of carbon fiber armor wires significantly reduces the overall weight of flexible pipe system, hence making the simpler and less expensive riser configurations, e.g., free-hanging or free-standing hybrid riser, possible in the ultra-deep water. However, it needs to be noted that the carbon fiber composite material is subjected to hydrolysis which leads to degradation of the mechanical performance at elevated temperatures. Other innovative flexible pipe concepts within recent years include new polymer fluid barrier technology for pipe applications subjected to harsh bore environments, new design of carcass to increase the flexible pipe resistance to the external ambient hydrostatic

pressure, new design of pressure armor to increase hydrostatic collapse performance with an optimum weight/strength ratio [11], etc.

1.2 Armor wire instability in flexible pipe

Nowadays, major failures in flexible pipes are often driven by the failure of tensile armor wire layers, which are the principle layers to carry the axial loads. As per Eq. 1.1, the actual axial load carried by the flexible pipe wall N_{real} is a function of the effective axial load N_{ef} and the axial load generated by the end-cap effect $(P_{in}A_{in} - P_{ex}A_{ex})$ [4]:

$$N_{real} = N_{ef} + P_{in}A_{in} - P_{ex}A_{ex} \tag{1.1}$$

where P_{in} and P_{ex} are respectively the internal and external pressures, and A_{in} and A_{ex} denote respectively the cross-sectional areas where the internal and external pressures act on. When the pipe annulus is dry or wet, A_{ex} refers respectively to the cross-sectional areas of the outer sheath and the fluid barrier.



Figure 1.3: Schematic of flexible pipe installation in the free-hanging configuration.

Due to the large effective tensile loads induced by the pipe self-weight and the large internal pressure generated by the bore fluids, significant tensile loads are usually experienced by a flexible pipe in most of its service life. Nevertheless, under certain conditions, large axial compressive loads may also be experienced. For flexible pipe installation in deep water, the pipe is normally free hanging from the pipelay vessel with pipe bore empty due to the limitation of the pipelay vessel tensioning capacity, see Fig. 1.3. Note that such empty bore condition is mandatory for the installation of gas export flexible pipes as there is a high risk of hydrate formation when there is remaining water inside the pipe. Also, there is no guarantee that the flexible pipe bore is filled with fluids during the entire service life. Consequently, a compressive load may be generated by the reversed end-cap effect. Besides, negative effective axial loads may also be generated when the host platform is in near conditions (low top angle) and a large heave of host platform is experienced.

Driven by the axial compressive loads, the tensile armor wires tend to deflect radially. If such radial expansion tendency is not properly restricted, a radial instability mode, also known as birdcaging, may be provoked, as depicted in Fig. 1.4. The birdcaging failure mode has been the subject of research, among others by SOUSA et al. [12], LI et al. [13], RABELO et al. [14], SÆVIK and THORSEN [15], BORGES et al. [16], MALTA and MARTINS [17]. A widely applied technique to prevent this failure mode is wrapping high strength tapes over the tensile armor wires to restrict their radial expansions. Nevertheless, although the radial degree of freedom is restricted by the anti-birdcage tapes, the armor wires still have space to deflect in the lateral direction. In 1997, a pipelay vessel, contracted to run a vertical connection in Marlim South 3 well in offshore Brazil, could not succeed in handling the flexible pipe because its stiffness had unexpectedly diminished. At that time, Marlim South 3, with a water depth of $1709 \ m$, was the deepest offshore production in operation [18]. After retrieval and dissection, an unconventional failure mode, characterized by large lateral deflections in the tensile armor wires, was observed. Such failure mode is nowadays well known as armor wire lateral instability, as depicted in Fig. 1.5. It was then demonstrated that the conventional method to avoid the armor wire instability by reinforcing the anti-birdcage tapes had found a limit.



Figure 1.4: Birdcaging failure mode in a 6" flexible pipe obtained in the laboratory environment.



Figure 1.5: Lateral instability in the inner armor wire layer of a 6" flexible pipe obtained in the laboratory environment.

The armor wire lateral instability usually takes place in the touch-down zone, as depicted in Fig. 1.3, where the largest external hydrostatic pressure is experienced and cyclic bending is generated by the heave of the host platform. Cyclic bending is essential for the formation of armor wire lateral instability since it facilitates the armor wire sliding against the frictional resistance induced by the contact with the neighboring layers. The armor wire lateral instability is prone to be formed when the outer sheath is damaged, i.e., in the wet annulus condition, as the lateral support generated by the frictional forces is substantially reduced. Since the frictional resistance on the inner armor wires is less than that on the outer armor wires, the lateral instability is always triggered by the failure of the inner armor wires. Once the lateral instability is triggered, significant pipe shortening and twist will be generated, which may cause the unlock of the pressure armor layer. Compared with the birdcaging failure mode, the armor wire lateral instability is more difficult to be identified by visual inspection as the outer shield often remains intact and there is no noticeable change in the pipe outer diameter.

1.3 Research objectives

The overall objective of the present work is to gain further insight in the armor wire lateral instability mechanism, in especial to estimate the critical condition that may lead to this failure mode, as well as to provide recommendations for improving the design of flexible pipes against this failure mode. Key aspects of the present work are:

- Analytical treatment of six coupled Love's differential equations for curved beams through the application of a perturbation technique;
- Evaluation of the armor wire sliding direction and velocity in a flexible pipe

subjected to dynamic bending and certain axial compressive loads;

- Evaluation of the armor wire equilibrium path after numerous bending cycles in the stable state as well as the corresponding pipe longitudinal shortening;
- Evaluation of the critical loading condition that may eventually lead to the armor wire lateral instability after numerous bending cycles;
- Discussion of the armor wire axial rotation when a gap is formed between the armor wire and the inner core in axial compression;
- Discussion of the effect of the lateral contact of adjacent armor wires on the lateral instability mechanism;
- Test of two 6-inch flexible pipes in hyperbaric chamber applying hydrostatic pressure combined to cyclic bending concerning the lateral instability failure;
- Recommendations for improving the design of flexible pipes against armor wire lateral instability.

1.4 Thesis organization

The thesis is organized in 7 chapters. Chapter 1 gives an introduction to this topic and the other chapters are briefly summarized as follows.

In Chapter 2, a literature survey of previous work regarding flexible pipe armor wire lateral instability is presented.

In Chapter 3, an analytical model for the armor wire lateral instability is developed. A perturbation technique is proposed to linearize Love's differential equations for curved beams. Whether the armor wire is able to rotate in its own axis when a gap is formed in axial compression is discussed. Under certain circumstance, the sliding direction and velocity of the armor wire in dynamic bending are approximately evaluated. Finally, the armor wire equilibrium path in the stable state, as well as the critical axial compression that may lead to the lateral instability, are obtained.

In Chapter 4, the armor wire mechanical behaviors in cyclic bending in the equilibrium state are discussed including the armor wire equilibrium path, curvatures, moments, frictional forces, stresses, longitudinal shortening as well as the lateral contact of the adjacent armor wires.

In Chapter 5, tests of two 6-inch flexible pipes concerning the armor wire lateral instability are presented and discussed. This is followed by a comparison between the present analytical results and the data of the present tests as well as the existing data in the literature.

In Chapter 6, some recommendations for improving the design of flexible pipes against the armor wire lateral instability are presented.

In Chapter 7, concluding remarks, as well as recommendations for the future work, are presented.

Chapter 2

Literature review

To date, considerable effort has been made by industrial and academic research groups to improve the understanding of flexible pipe armor wire lateral instability mechanism.

Since the first discovery of armor wire lateral instability failure mode, flexible pipes have been tested through DIP (Deep Immersion Performance) tests, which were carried out in offshore fields where the conditions were very similar to what flexible pipes would encounter during their installation and service life. In the DIP tests, the flexible pipe samples, with inner core empty, were usually suspended from the installation vessel in the free-hanging configuration with a small top angle and tested in both dry and flooded annulus conditions for a few hours. DIP tests have been reported by BECTARTE and COUTAREL [19], CUSTÓDIO *et al.* [20], SECHER *et al.* [21]. Although the DIP tests provide reliable results since they simulate directly the installation process, they are very expensive and time-consuming, hence it is necessary to have alternative protocols more practical and economic for armor wire lateral instability tests.

To develop testing alternatives to reduce the high cost of DIP tests, several test rigs were constructed by few laboratories. A compression rig and a bending-compression rig were assembled respectively at the Research and Development Center of Petrobras (CENPES) and Federal University of Santa Catarina (UFSC) in Brazil [18]. Afterward, similar bending-compression test rigs were constructed by NKT-Flexibles in Denmark [22] and SINTEF Ocean in Norway [23]. While the compression rig is only applicable for the test in the straight pipe condition, bending-compression rigs enable the application of axial compression and cyclic bending simultaneously. To avoid the out-of-plane curvature induced by gravitational effects, the bending was applied in the vertical plane in those bending-compression rigs. Nevertheless, it needs to be noted that the hydrostatic end-cap effect was represented in an indirect way that the mechanical loading was applied at the pipe sample ends at atmospheric pressure. Thus, it only partially represents the wet annulus con-

dition. Besides, the mechanical system may generate out of plane deformation which jeopardizes the test representativeness. To overcome those drawbacks brought by mechanical rigs, bending hyperbaric chambers were implemented, through which the axial compressive loads on the pipe sample are generated by the hydrostatic pressure directly. An example of a bending hyperbaric chamber was reported by SECHER *et al.* [21]. Besides, a bending hyperbaric chamber was constructed in the laboratory Núcleo de Estruturas Oceânicas (NEO) at Federal University of Rio de Janeiro.

Besides the investigation by experimental means, computational tools for the analysis of complex armor wire mechanical behavior are also in great demand. Two configurations have been widely used to describe the armor wire path within the pipe wall: geodesic and loxodromic curves. By employing the geodesic curve, the armor wire is assumed seeking an equilibrium path corresponding to the shortest line on the torus. On the other hand, by employing loxodromic curve, the armor wire transverse deflection is assumed ignored. The geodesic curve has been employed among others by OUT and VON MORGEN [24]. However, due to the frictional resistance, the armor wire is actually not able to reach the path corresponding to the geodesic curve. The study of SÆVIK [25] demonstrated that, due to the frictional resistance, the armor wire sliding is primarily axial when a flexible pipe is subjected to tension and dynamic bending as proposed by WITZ and TAN [26]. Consequently, applying the loxodromic curve, the slip initiation and progression of the armor wire subjected to bending and tension have been investigated by KRAINCANIC and KEBADZE [27]. Also based on the loxodromic curve assumption, finite element formulations were developed by $S \neq VIK$ [28] for the prediction of armor wire sliding and stresses. Nevertheless, the real path of armor wire on a bent cylinder is to be found between the geodesic and loxodromic curves. In other words, the geodesic and loxodromic curves represent respectively the upper and lower bounds for the armor wire deflection. In addition, the pre-defined configurations prevent these models from being applicable for the armor wire lateral instability analysis.

Without pre-assumed configurations, more general models have been continuously developed in an effort to investigate the armor wire lateral instability mechanism. CUSTÓDIO [29] proposed an analytical model to access the stability limit of flexible pipe armor wires under axisymmetric loading through the application of a perturbation method by establishing an eigenvalue problem. BRACK *et al.* [30] presented a finite element model through which the importance of torsional resistance on the armor wire lateral instability was discussed by artificially changing the torsional inertia of the armor wire. The bending effect was treated by altering the initial helical geometry of the armor wire in a straight pipe and the frictional resistance was ignored. TAN *et al.* [31] developed an analytical model for the sim-

ulation of the armor wire instability and post-instability behaviors based on a total strain energy approach. The results were calibrated by a series of DIP and pressure chamber tests, but no details were presented. VAZ and RIZZO [32] developed a finite element model using the commercial software Abaque for the investigation of armor wire instability in a straight pipe subjected to monotonic compressive loads. Only two armor wires were built representing the inner and outer armor wire layers and the anti-birdcage tapes were modeled by spring elements, which makes the analysis computationally light. Their study indicated that the armor wire instability modes highly depend on the friction coefficient between the armor wire and the adjacent layer and the radial expansive restriction from the anti-birdcage tapes and external hydrostatic pressure. Also, a new radial instability mode characterized the wrinkling of armor wires was reported. This model was thereafter improved by LI [33] by considering the plastic behavior of armor wires and introducing the friction between external armor wire and the outer sheath. A similar finite element model was then developed by YANG et al. [34] using the curved beam method and the obtained results were in accordance with the results of VAZ and RIZZO [32].

Moreover, a series of studies were presented by ØSTERGAARD et al. [35, 36, 37] focusing on the armor wire lateral instability on a frictionless toroid, which was principally achieved by establishing and solving a system of six order nonlinear differential equations. Meanwhile, a series of tests were also carried out, which indicates that the estimation of lateral stability limit given by this numerical model lies on the conservative side. Afterward, based on the results obtained by the numerical model developed by \emptyset STERGAARD *et al.* [35] in the straight pipe condition, an empirical model was proposed by PAIVA and VAZ [38] using symbolic regression to evaluated the armor wire lateral stability limit. In addition, by applying the concept of elastic stability theory, a finite element model was developed by GONZALEZ [39] to investigate the armor wire lateral instability in a flexible pipe subjected to monotonic axial compression and constant bending curvature. The frictional resistance between the armor wires and the neighboring layers is modeled through spring elements. Based on the obtained numerical results, an empirical model was proposed by means of symbolic regression. Also, a series of studies were presented by MALTA and MARTINS [17, 40, 41] investigating the armor wire behavior in axial compression using finite element method, where the effects of the sample length and friction coefficient were discussed.

More recently, few analytical models were proposed in an effort to identify the critical axial load that may trigger the armor wire lateral instability. Assuming that the buckling shape is sinusoidal and the full magnitude of frictional force is available in the transverse direction, SÆVIK and THORSEN [42] proposed an analytical formulation for the prediction of armor wire lateral instability. However, since the

frictional resistance in the transverse direction was overestimated and the effect of cyclic bending was not considered, the lateral stability limit given by this model was much higher than the test data. Besides, ignoring the frictional force, another analytical model was proposed by SÆVIK and LI [43] considering varying buckling length. It was demonstrated that the minimum stability limit can be found when the buckling length is a little bit more than half armor wire pitch length. Afterwards, also based on a predefined harmonic buckling shape, an analytical model was presented by SÆVIK and JI [44] and SÆVIK and THORSEN [15] for the prediction of armor wire lateral stability limit on a frictionless cylindrical surface, whose results were in accordance with the numerical results obtained by ØSTERGAARD [22]. Thereafter, a similar analytical model was developed by LI *et al.* [45] using a perturbation technique, however, without predefined deflection shape.

As previously mentioned, frictional resistance and cyclic bending play an important role in the armor wire lateral instability mechanism. Nevertheless, none of the abovementioned analytical and numerical models consider those factors simultaneously. There are few numerical models in the literature for the armor wire lateral instability analysis that consider both friction and cyclic bending. ØSTERGAARD et al. [46] presented a numerical approach to detect the lateral instability of a single armor wire in constant axial compression and cyclic bending with frictional effect included. Besides, a finite element model was developed by SÆVIK and THORSEN [42] and SÆVIK and JI [44] which is applicable for the armor wire lateral instability analysis. To save computational effort, only a few armor wires were modeled representing the entire armor wire layer. The finite element results were compared with the test data from ØSTERGAARD [22] and good correlation was demonstrated. Furthermore, also based on this finite element model, ZHOU et al. [47] investigated the effect of anti-birdcage tape on the armor wire lateral instability mechanism. It was demonstrated that the anti-birdcage tape may resist the pipe rotation in the post-buckling state and hence affect the armor wire transverse deflection process. In addition, considering uniform bending curvature and assuming each armor wire within the same layer deflects equally, numerical models were developed by CA-LEYRON et al. [1, 48], LUKASSEN [49] applying periodicity boundary conditions. By this mean, each armor wire layer can be modeled by only one pitch of a single armor wire, hence reducing significantly the computational effort. Their work indicated that the lateral contact between neighboring armor wires may increase the structural stabilization. It needs to be noted that, in all abovementioned numerical models, a bi-linear Coulomb friction law was employed where the stick displacement was chosen in a manner that the convergence can be obtained. However, since the relative slip during each calculation step may not exceed the chosen stick displacement, the full frictional force may not be applied.

Despite it is demonstrated that the results given by some numerical models are in good accordance with test data, large processing resources are demanded since a significant number of bending cycles needs to be simulated. Thus, to save the computational effort, as well as to improve the understanding of the underlying mechanism, it is worth devoting effort in developing an analytical model for the analysis of armor wire lateral instability with both frictional resistance and cyclic bending considered.

Chapter 3

Theory

3.1 Field equations for a single armor wire

In this section, a single armor wire in a flexible pipe is modeled as a thin curved beam over a toroid through a consistent system of differential equations. The mathematical development is similar to the work presented by ØSTERGAARD *et al.* [35] except that the armor wire rotation around its own axis is considered in the present context.

3.1.1 Geometrical relations



Figure 3.1: Armor wire geometry and coordinate system.

Consider a flexible pipe bent to a toroid with minor and major radii respectively denoted by r and $R = 1/\kappa$, see Fig.3.1. A parameterization of the toroid is established through u and θ , which respectively represent the longitudinal and circumferential positions of the points on the toroid. Besides, a Cartesian coordinate system with principle directions x, y, z is defined by aligning x to the direction from the center of the toroid to the center of the tube as depicted in Fig. 3.1. Consequently, a point on the toroid in the Cartesian coordinate system can be defined by:

$$\boldsymbol{X}(u,\theta) = \begin{bmatrix} (1/\kappa + r\cos\theta)\cos(\kappa u) - 1/\kappa \\ -r\sin\theta \\ (1/\kappa + r\cos\theta)\sin(\kappa u) \end{bmatrix}$$
(3.1)

The path of an armor wire in the flexible pipe can then be represented by a curve on the toroid, which can be determined by specifying a relation between u and θ . Such relation is herein defined by the angle between the curve tangent and its projection in the longitudinal direction, represented by ϕ , as depicted in Fig.3.1. Attach a local curvilinear coordinate triad of unit orthonormal vectors (t, n, b) on the curve representing the tangential, normal and bi-normal directions respectively. The unit tangent vector can be given by:

$$\boldsymbol{t} = \frac{\partial \boldsymbol{X}/\partial u}{\|\partial \boldsymbol{X}/\partial u\|} \cos \phi + \frac{\partial \boldsymbol{X}/\partial \theta}{\|\partial \boldsymbol{X}/\partial \theta\|} \sin \phi$$
(3.2)

Substituting Eq. 3.1 into Eq. 3.2, the unit tangent vector can be obtained as:

$$\boldsymbol{t} = \begin{bmatrix} -\cos\phi\sin(\kappa u) - \sin\phi\sin\theta\cos(\kappa u) \\ -\sin\phi\cos\theta \\ \cos\phi\cos(\kappa u) - \sin\phi\sin\theta\sin(\kappa u) \end{bmatrix}$$
(3.3)

Alternatively, parameterizing the curve by its arc length, denoted by s, the unit tangent vector can also be expressed as:

$$\boldsymbol{t} = \frac{\partial \boldsymbol{X}}{\partial u} \frac{du}{ds} + \frac{\partial \boldsymbol{X}}{\partial \theta} \frac{d\theta}{ds}$$
(3.4)

Comparing Eq.3.2 and Eq.3.4, the following relations can be identified:

$$\frac{du}{ds} = \frac{\cos\phi}{1 + \kappa r \cos\theta} \tag{3.5}$$

$$\frac{d\theta}{ds} = \frac{\sin\phi}{r} \tag{3.6}$$

As previously discussed in the introduction section, a gap may be formed between the armor wires and the inner core in axial compression since the armor wires tend to expand radially and push the anti-birdcage tapes. As a consequence, the armor wire may have space to rotate around its own axis, see Fig. 3.2. Herein, such axial rotation is denoted by ω which represents the angle between the toroid normal vector n_0 and the armor wire normal vector n as depicted in Fig. 3.3.



Figure 3.2: Schematic of the armor wire axial rotation: contacts both the upper and lower borders (left) and only contacts the upper border (right).



Figure 3.3: Armor wire cross-section: contacts both the upper and lower borders (left) and only contacts the upper border (right).

The unit toroid normal vector can be given by:

$$\boldsymbol{n_0} = \frac{\partial \boldsymbol{X} / \partial \boldsymbol{u} \times \partial \boldsymbol{X} / \partial \boldsymbol{\theta}}{\|\partial \boldsymbol{X} / \partial \boldsymbol{u} \times \partial \boldsymbol{X} / \partial \boldsymbol{\theta}\|}$$
(3.7)

and \boldsymbol{b}_0 is defined by:

$$\boldsymbol{b_0} = \boldsymbol{t} \times \boldsymbol{n_0} \tag{3.8}$$

Applying Eqs.3.1 and 3.3 into Eqs. 3.7-3.8 yields:

$$\boldsymbol{n_0} = \begin{bmatrix} \cos\theta\cos\left(\kappa u\right) \\ -\sin\theta \\ \cos\theta\sin\left(\kappa u\right) \end{bmatrix}$$
(3.9)

$$\boldsymbol{b_0} = \begin{bmatrix} -\sin\phi\sin(\kappa u) + \cos\phi\sin\theta\cos(\kappa u) \\ \cos\phi\cos\theta \\ \cos\phi\sin\theta\sin(\kappa u) + \sin\phi\cos(\kappa u) \end{bmatrix}$$
(3.10)

Consequently, the triad of unit vectors on the armor wire (t, n, b) can be determined by rotating the unit vector frame (t, n_0, b_0) in the axial direction through the following transformation matrix:

$$\begin{bmatrix} \boldsymbol{t} \\ \boldsymbol{n} \\ \boldsymbol{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \omega & \sin \omega \\ 0 & -\sin \omega & \cos \omega \end{bmatrix} \begin{bmatrix} \boldsymbol{t} \\ \boldsymbol{n}_{\boldsymbol{0}} \\ \boldsymbol{b}_{\boldsymbol{0}} \end{bmatrix}$$
(3.11)

Thereafter, substituting Eqs. 3.9-3.10 into Eq. 3.11, the unit normal and binormal vectors on the armor wire can be obtained as:

$$\boldsymbol{n} = \begin{bmatrix} \cos\phi\sin\theta\cos(\kappa u)\sin\omega - \sin\phi\sin(\kappa u)\sin\omega + \cos\theta\cos(\kappa u)\cos\omega\\ \cos\phi\cos\phi\sin\theta\sin\omega - \sin\theta\cos\omega\\ \sin\phi\cos(\kappa u)\sin\omega + \cos\phi\sin\theta\sin(\kappa u)\sin\omega + \cos\theta\sin(\kappa u)\cos\omega \end{bmatrix} (3.12)$$
$$\boldsymbol{b} = \begin{bmatrix} -\cos\theta\cos(\kappa u)\sin\omega - \sin\phi\sin(\kappa u)\cos\omega + \cos\phi\sin\theta\cos(\kappa u)\cos\omega\\ \sin\theta\sin\omega + \cos\phi\cos\theta\cos\omega\\ -\cos\theta\sin(\kappa u)\sin\omega + \cos\phi\sin\theta\sin(\kappa u)\cos\omega + \sin\phi\cos(\kappa u)\cos\omega \end{bmatrix} (3.13)$$

3.1.2 Wire curvature components

In the differential geometry of surfaces, the first order derivatives of the triad vectors with respect to the curve arc length can be defined by the Darboux frame, which is the analog of the Frenet–Serret frame as applied to surface geometry:

$$\frac{d}{ds} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 0 & \kappa_n & -\kappa_g \\ -\kappa_n & 0 & \tau \\ \kappa_g & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix}$$
(3.14)

where κ_g , κ_n , τ are respectively the geodesic curvature, normal curvature and torsion. Thus, the curvature components can be determined by:

$$\kappa_g = -\boldsymbol{b}\frac{d\boldsymbol{t}}{ds} = \boldsymbol{t}\frac{d\boldsymbol{b}}{ds}$$
(3.15a)

$$\kappa_n = \boldsymbol{n} \frac{d\boldsymbol{t}}{ds} = -\boldsymbol{t} \frac{d\boldsymbol{n}}{ds}$$
(3.15b)

$$\tau = -\boldsymbol{n}\frac{d\boldsymbol{b}}{ds} = \boldsymbol{b}\frac{d\boldsymbol{n}}{ds}$$
(3.15c)

Substituting Eqs. 3.3, 3.12-3.13 into Eqs. 3.15a-3.15c, the geodesic curvature,

normal curvature and torsion of the armor wire can be obtained respectively as:

$$\kappa_g = -\frac{\sin^2 \phi}{\kappa r^2 \cos \theta + r} \left[\sin \omega - (1 + \kappa r \cos \theta) \frac{r \cos \omega}{\sin^2 \phi} \frac{d\phi}{ds} - \frac{\kappa r}{\sin^2 \phi} \left(\cos \phi \sin \theta \cos \omega - \cos \theta \sin \omega \right) \right]$$
(3.16a)

$$\kappa_n = -\frac{\sin^2 \phi}{\kappa r^2 \cos \theta + r} \left[\cos \omega + (1 + \kappa r \cos \theta) \frac{r \sin w}{\sin^2 \phi} \frac{d\phi}{ds} + \frac{\kappa r}{\sin^2 \phi} \left(\cos \theta \cos \omega + \cos \phi \sin \theta \sin \omega \right) \right]$$
(3.16b)
$$\tau = \frac{d\omega}{ds} - \frac{1}{2} \frac{\sin 2\phi}{\kappa r^2 \cos \theta + r}$$
(3.16c)

3.1.3 Equilibrium equations

Considering the armor wire as a thin curved beam, the componentwise equilibrium equations for a curved beam derived by LOVE [50] can be applied:

$$\frac{dP_t}{ds} - \kappa_n P_n + \kappa_g P_b + p_t = 0 \tag{3.17a}$$

$$\frac{dP_n}{ds} + \kappa_n P_t - \tau P_b + p_n = 0 \tag{3.17b}$$

$$\frac{dP_b}{ds} - \kappa_g P_t + \tau P_n + p_b = 0 \tag{3.17c}$$

$$\frac{dM_t}{ds} - \kappa_n M_n + \kappa_g M_b + m_t = 0 \tag{3.17d}$$

$$\frac{dM_n}{ds} + \kappa_n M_t - \tau M_b - P_b + m_n = 0 \tag{3.17e}$$

$$\frac{dM_b}{ds} - \kappa_g M_t + \tau M_n + P_n + m_b = 0 \tag{3.17f}$$

where P_t , P_n , P_b , M_t , M_n , M_b , p_t , p_n , p_b , m_t , m_n , m_b are respectively the crosssectional forces and moments, the external forces and moments on the armor wire in the tangential, normal and bi-normal directions. Assuming the neighboring armor wires in the same layer do not contact each other before the lateral instability failure, m_n can be deemed zero. Besides, m_b is also assumed zero.

3.1.4 Constitutive equations

Considering that the armor wire cross-sectional dimensions are small compared to the pipe diameter and the armor wire strains are small, the following linear constitutive relations can be applied:

$$M_t = GJ\Delta\tau = GJ\left(\tau - \tau^i\right) \tag{3.18a}$$

$$M_n = E I_n \Delta \kappa_g = E I_n \left(\kappa_g - \kappa_g^i \right)$$
(3.18b)

$$M_b = EI_b \Delta \kappa_n = EI_b \left(\kappa_n - \kappa_n^i\right) \tag{3.18c}$$

$$P_t = EA\varepsilon = EA\left(\frac{ds}{ds^i} - 1\right) \tag{3.18d}$$

where $E, G, A, I_n, I_b, J, \varepsilon$ are respectively the Young's modulus, the shear modulus, the cross-sectional area, the normal inertia, the transverse inertia, the stiffness of torsion and the axial strain of the armor wire, $\kappa_g^i, \kappa_n^i, \tau^i$ and s^i denote respectively the armor wire curvature components and arc length in the stress-free state, and $\Delta \kappa_g, \Delta \kappa_n, \Delta \tau$ represent the curvature variations relative to the stress-free state. Assume that the armor wire is installed stress-freely during flexible pipe fabrication where the armor wire is helically wound with a constant lay angle ϕ^i in the straight pipe condition and the armor wire normal direction coincides with the toroid normal direction. Thus, the curvature components in the stress-free condition can be obtained by substituting $\kappa = 0, \omega = 0$ and $\phi = \phi^i$ into Eqs. 3.16a-3.16c respectively as:

$$\kappa_g^i = 0 \tag{3.19a}$$

$$\kappa_n^i = -\frac{\sin^2 \phi^i}{r} \tag{3.19b}$$

$$\tau^{i} = -\frac{\sin 2\phi^{i}}{2r} \tag{3.19c}$$

Considering the armor wire cross-section as a wide rectangle and denoting the armor wire width and thickness respectively by W and H, the stiffness of torsion can be approximately evaluated by [51]:

$$J = WH^{3} \left(\frac{1}{3} - \frac{64}{\pi^{5}} \frac{H}{W}\right)$$
(3.20)

3.1.5 Boundary conditions

For a flexible pipe subjected to longitudinal compressive loads combined to cyclic bending, a longitudinal shortening will be generated due to the armor wire axial strain and transverse slip. Meanwhile, since a flexible pipe conventionally consists of an even number of armor wire layers with opposite lay angles, it is reasonable to ignore the axial rotation of a flexible pipe in axial compression before any armor wire lateral instability takes place. Moreover, as the armor wire lateral instability generally occurs in the touchdown zone which is far away from the pipe ends, the boundary effect from the pipe ends can be disregarded and it is thus reasonable to consider the deflection of each armor wire within the same layer identical and periodical. Due to symmetry, the armor wire can be assumed approximately fixed on the underlying toroid in the positions of intrados and extrados. Thus, the following boundary conditions are employed in the present context: (i) the toroid is deformable in the longitudinal and bending directions, however, with the rotational degree of freedom fixed; (ii) the armor wire is pinned on the toroid in the positions of intrados and extrados.

In summary, the aforementioned differential equation system for a single armor wire contains 18 unknowns including three geometrical parameters u, θ , ϕ , three curvature components κ_g , κ_n , τ , three internal forces P_t , P_n , P_b , three external forces p_t , p_n , p_b , three internal moments M_t , M_n , M_b , one external moment m_t , the armor wire axial strain ε and axial rotation ω . Nevertheless, only 15 independent equations are formulated including Eqs. 3.5-3.6, 3.16a-3.16c, 3.17a-3.17f, 3.18a-3.18d. The imbalance between the numbers of equations and unknowns is due to the lack of formulas regarding the axial rotation ω and the external forces p_t and p_b , which will be discussed respectively in sections 3.3 and 3.4.

3.2 Linearization through a perturbation technique

In practical applications of flexible pipes, the ratio between the pipe radius to the bending radius is generally small, i.e., $|\kappa r| \ll 1$. As a consequence, the armor wire transverse deflection is small before any lateral instability takes place. Thus, the armor wire equilibrium path can be therefore approximately evaluated by adding small corrections to the helix path through a perturbation method.



Figure 3.4: Initial and deformed paths of half armor wire pitch length.

Fig. 3.4 depicts the paths of half armor wire pitch in the initial and deformed states, where $u^{i}(\theta)$ and $u^{h}(\theta)$ represent respectively the initial helix path and the
new helix path on the deformed toroid, $\Delta \phi_1$ represents the angle difference between the initial and new helixes in the straight pipe condition. The armor wire path on the toroid in the deformed state can then be expressed by:

$$u(\theta) = u^{h}(\theta) + \Delta u(\theta)$$
(3.21)

where $\Delta u(\theta)$ denotes the armor wire displacement relative to the new helix due to slips. Similarly, denote the armor wire lay angle in the deformed state by:

$$\phi(\theta) = \phi^{i} + \Delta\phi(\theta) = \phi^{i} + \Delta\phi_{1} + \Delta\phi_{2}(\theta)$$
(3.22)

where $\Delta \phi_2$ is the armor wire lay angle variation relative to the new helix angle and $\Delta \phi$ is the sum of $\Delta \phi_1$ and $\Delta \phi_2$.

Thereafter, combining Eqs. 3.5-3.6, the following relations can be obtained:

$$\frac{du^{h}}{d\theta} = r \cot\left(\phi^{i} + \Delta\phi_{1}\right) \tag{3.23a}$$

$$\frac{du}{d\theta} = \frac{r \cot \phi}{1 + \kappa r \cos \theta} \tag{3.23b}$$

Deriving Eq. 3.21 in terms of the angular coordinate and employing Eqs. 3.23a-3.23b yields:

$$\frac{r\cot\phi}{1+\kappa r\cos\theta} = r\cot\left(\phi^i + \Delta\phi_1\right) + \frac{d\Delta u}{d\theta}$$
(3.24)

Then, expanding Eq. 3.24 and considering κr , $\Delta \phi_1$ and $\Delta \phi_2$ as small terms, the armor wire lay angle variation relative to the new helix angle can be approximately obtained as:

$$\Delta\phi_2\left(\theta\right) = -\kappa r \frac{\sin 2\phi^i}{2} \cos\theta - \frac{\sin^2\phi^i}{r} \frac{d\Delta u}{d\theta}$$
(3.25)

Thereafter, integrating Eq. 3.6 for half armor wire pitch in the initial and deformed states, the following relation can be obtained through Cauchy's definition of strain:

$$\int_{0}^{\pi} \frac{r}{\sin\phi(\theta)} \left[1 - \varepsilon(\theta)\right] d\theta = \frac{\pi r}{\sin\phi^{i}}$$
(3.26)

The lay angle difference between the initial and the new helixes $\Delta \phi_1$ can then be obtained by substituting Eqs. 3.22, 3.25 into Eq. 3.26. With $\Delta \phi_1$ obtained, the flexible pipe longitudinal strain, defined by the longitudinal shortening divided by the original pipe length in the straight pipe condition, can be given by:

$$\Delta \bar{L} = 1 - \frac{\tan \phi^i}{\tan \left(\phi^i + \Delta \phi_1\right)} \tag{3.27}$$

Intuitively, due to the strong constraint generated by the adjacent layers, even

if the armor wire may rotate axially, the axial rotation should be small when no lateral instability takes place, i.e., $|\omega| \ll 1$. Consequently, the previously developed formulas can be approximately linearized by solely keeping the first order small terms of κr , ω and Δu . For the sake of convenience, hereafter all the variables are expressed in terms of the angular coordinate θ instead of the armor wire arc length s by using Eq. 3.6. Substituting Eqs. 3.22, 3.25 into Eqs. 3.16a-3.16c and ignoring the higher order small terms, the armor wire curvature components can be approximately evaluated by:

$$\kappa_g(\theta) = \kappa \left(1 + \sin^2 \phi^i\right) \cos \phi^i \sin \theta - \frac{\sin^3 \phi^i}{r^2} \frac{d^2 \Delta u}{d\theta^2} - \frac{\sin^2 \phi}{r} \omega$$
(3.28a)

$$\kappa_n(\theta) = -\frac{\sin^2 \phi^i}{r} + -\kappa \cos 2\phi^i \cos^2 \phi^i \cos \theta + 2\frac{\sin^3 \phi^i \cos \phi^i}{r^2} \frac{d\Delta u}{d\theta}$$
(3.28b)

$$\tau\left(\theta\right) = -\frac{\sin 2\phi^{i}}{2r} + 2\kappa\sin\phi^{i}\cos^{3}\phi^{i}\cos\theta + \frac{\sin^{2}\phi^{i}\cos 2\phi^{i}}{r^{2}}\frac{d\Delta u}{d\theta} + \frac{\sin\phi^{i}}{r}\frac{d\omega}{d\theta} \quad (3.28c)$$

Afterwards, substituting Eqs. 3.19a-3.19c and 3.28a-3.28c into Eqs. 3.18a-3.18c, the armor wire cross-sectional moments in the tangential, normal and bi-normal directions can be approximately determined respectively by:

$$M_t(\theta) = GJ\left(2\kappa\sin\phi^i\cos^3\phi^i\cos\theta + \frac{\sin^2\phi^i\cos 2\phi^i}{r^2}\frac{d\Delta u}{d\theta} + \frac{\sin\phi^i}{r}\frac{d\omega}{d\theta}\right)$$
(3.29a)

$$M_n(\theta) = EI_n\left[\kappa\left(1 + \sin^2\phi^i\right)\cos\phi^i\sin\theta - \frac{\sin^3\phi^i}{r^2}\frac{d^2\Delta u}{d\theta^2} - \frac{\sin^2\phi^i}{r}\omega\right]$$
(3.29b)

$$M_b(\theta) = EI_b \left[-\kappa \cos 2\phi^i \cos^2 \phi^i \cos \theta + \frac{2\sin^3 \phi^i \cos \phi^i}{r^2} \frac{d\Delta u}{d\theta} \right]$$
(3.29c)

Subsequently, substituting Eqs.3.28a-3.28b, 3.29a-3.29c into Eq.3.17d and ignoring the higher order small terms, the external moment on the armor wire in the axial direction can be obtained as:

$$m_t(\theta) = \left(EI_n \sin^2 \phi^i - GJ \cos 2\phi^i\right) \frac{\sin^3 \phi^i}{r^3} \frac{d^2 \Delta u}{d\theta^2} - \frac{\kappa}{r} \left[EI_n \left(1 + \sin^2 \phi^i\right) - 2GJ \cos^2 \phi^i\right] \sin^2 \phi^i \cos \phi^i \sin \theta \qquad (3.30) - GJ \frac{\sin^2 \phi^i}{r^2} \frac{d^2 \omega}{d\theta^2} + EI_n \frac{\sin^4 \phi^i}{r^2} \omega$$

Thereafter, substituting Eqs. 3.28a-3.28c and 3.29a-3.29c into Eqs. 3.17f, 3.17e and ignoring the higher order small terms, the armor wire cross-sectional forces in

the normal and bi-normal directions can be evaluated respectively by:

$$P_{n}(\theta) = -\left(EI_{n} + 2EI_{b}\right) \frac{\sin^{4}\phi^{i}\cos\phi^{i}}{r^{3}} \frac{d^{2}\Delta u}{d\theta^{2}} + \frac{\kappa}{r} \left[EI_{n}\left(1 + \sin^{2}\phi^{i}\right) - EI_{b}\cos 2\phi^{i}\right] \sin\phi^{i}\cos^{2}\phi^{i}\sin\theta \qquad (3.31) - EI_{n}\frac{\sin^{3}\phi^{i}\cos\phi^{i}}{r^{2}}\omega$$

$$P_{b}(\theta) = -EI_{n}\frac{\sin^{4}\phi^{i}}{r^{3}}\frac{d\Delta u}{d\theta} + \left(EI_{b} - GJ\frac{\cos 2\phi^{i}}{2\cos^{2}\phi^{i}}\right)\frac{2\sin^{4}\phi^{i}\cos^{2}\phi^{i}}{r^{3}}\frac{d\Delta u}{d\theta} + \frac{\kappa}{2r}\left[EI_{n}\left(1 + \sin^{2}\phi^{i}\right) - EI_{b}\cos 2\phi^{i}\cos^{2}\phi^{i} - \frac{1}{2}GJ\sin^{2}2\phi^{i}\right]\sin 2\phi^{i}\cos\theta - \frac{\sin^{3}\phi^{i}}{r^{2}}\left(EI_{n} + GJ\right)\frac{d\omega}{d\theta}$$

$$(3.32)$$

Subsequently, substituting Eqs. 3.28a, 3.28c, 3.31-3.32 into Eq. 3.17b-3.17c and ignoring the higher order small terms, the external forces on the armor wire in the normal and bi-normal directions can be evaluated respectively by:

$$p_n(\theta) = \frac{\sin^2 \phi^i}{r} P_t(\theta)$$
(3.33)

$$p_{b}(\theta) = EI_{n} \frac{\sin^{5} \phi^{i}}{r^{4}} \frac{d^{4} \Delta u}{d\theta^{4}} - [P_{t}(\theta) - P_{1}] \frac{\sin^{3} \phi^{i}}{r^{2}} \frac{d^{2} \Delta u}{d\theta^{2}} + \kappa [P_{t}(\theta) - P_{2}] (1 + \sin^{2} \phi^{i}) \cos \phi^{i} \sin \theta + \frac{\sin^{4} \phi^{i}}{r^{3}} (EI_{n} + GJ) \frac{d^{2} \omega}{d\theta^{2}} - \left(P_{t}(\theta) + EI_{n} \frac{\sin^{2} 2 \phi^{i}}{4r^{2}}\right) \frac{1}{r} \sin^{2} \phi^{i} \omega$$

$$(3.34)$$

In which P_1 and P_2 are constants corresponding to the armor wire characteristics:

$$P_{1} = \frac{\sin^{2} \phi^{i}}{r^{2}} \left(-EI_{n} \cos^{2} \phi^{i} - 4EI_{b} \cos^{2} \phi^{i} + GJ \cos 2\phi^{i} \right)$$
(3.35a)

$$P_{2} = \frac{\sin^{2}\phi^{i}}{r^{2}} \left[-EI_{n} \left(1 + \cos^{2}\phi^{i} \right) + EI_{b} \frac{2\cos^{2}\phi^{i}\cos 2\phi^{i}}{1 + \sin^{2}\phi^{i}} + GJ \frac{\sin^{2}2\phi^{i}}{2 + 2\sin^{2}\phi^{i}} \right] \quad (3.35b)$$

Since the lateral instability may only occur when large axial compressive loads are applied, it is reasonable to consider that the armor wire cross-sectional force variation in its tangential direction induced by bending and displacement is small. Thus, substituting Eqs. 3.28a-3.28b, 3.31-3.32 into Eq. 3.17a and ignoring the higher order small terms, the armor wire cross-sectional force in the tangential direction can be approximately evaluated by:

$$P_t(\theta) = P_t(\theta_0) - \frac{r}{\sin \phi^i} \int_{\theta_0}^{\theta} p_t d\theta$$
(3.36)

3.3 Axial rotation discussion

Assume that the armor wire experiences axial rotation and the gap is insufficiently big that the armor wire contacts both the outer layer and the pipe core, as depicted in Fig. 3.3 (left). Denoting the contact forces at the upper and lower interfaces with the neighboring layers respectively by p_{up} and p_{low} , the torque on the armor wire in the axial direction can be evaluated by:

$$m_t = -\left(p_{up} + p_{low}\right) \frac{W}{2} \cos \omega \tag{3.37}$$

Besides, the equilibrium condition of the forces in the toroid normal direction is given by:

$$p_n \cos \omega - p_b \sin \omega = p_{low} - p_{up} \tag{3.38}$$

Making p_{up} in Eq. 3.37 substituted by Eq. 3.38, the torque can be rewritten as:

$$m_t = (p_n \cos \omega - p_b \sin \omega) \frac{W}{2} \cos \omega - p_{low} W \cos \omega$$
(3.39)

Substituting Eqs. 3.33-3.34 into Eq. 3.39 and ignoring the higher order small terms yields:

$$m_t(\theta) = P_t(\theta) \frac{W \sin^2 \phi^i}{2r} - p_{low}(\theta) W$$
(3.40)

If the gap is sufficiently big that the armor wire contacts the upper border but does not contact the pipe core, as depicted in Fig. 3.3 (right), the torque on the armor wire can be given by setting $p_{low} = 0$ in Eq. 3.40.

Noting that Eq. 3.30 only contains small terms and P_t is large in case of lateral instability, it can be seen that the torque given by Eq. 3.40 is much larger than that given by Eq. 3.30 no matter whether the armor wire contacts the pipe core or not. In other words, if the armor wire rotates axially, the generated torque, as given by Eq. 3.40, is much larger than the required torque for equilibrium, as given by Eq. 3.30. This is corresponding to a state that, for a single armor wire in compression, the axial rotation is unlikely to be initiated even the gap is formed since the external constraint on the armor wire axial rotation generated by the contact forces is strong. Thus, to evaluate the armor wire lateral stability limit, it is reasonable to consider the armor wire normal vector coinciding with the toroid normal vector, i.e., $\omega = 0$.

3.4 Sliding and frictional forces in dynamic bending

To investigate the armor wire sliding in cyclic bending, it is important to determine the directions and magnitudes of the frictional forces on the armor wire. Fig. 3.4 shows a diagram for the frictional force components on a point on the armor wire sliding on the toroid tangent space from A to A', where $d\Delta t/d\kappa$ and $d\Delta b/d\kappa$ denote respectively the sliding rates of the point on the armor wire relative to the toroid in the tangential and bi-normal directions with respect to the bending curvature. As frictional force acts in the opposite direction of relative sliding, the following relation should hold:

$$\frac{d\Delta b}{d\kappa}\frac{1}{p_b} = \frac{d\Delta t}{d\kappa}\frac{1}{p_t} < 0 \tag{3.41}$$



Figure 3.5: Sliding directions and friction components for a point sliding on the toroid tangent space from A to A'.

Considering a point on the armor wire at $\theta = \theta_s$, the arc length of the armor wire from a point at $\theta = n\pi$ (*n* is an integer), representing the intrados or extrados, to the point at $\theta = \theta_s$ can be calculated by integrating Eq. 3.6 in terms of the angular coordinate as:

$$s\left(\theta_{s},\kappa\right) = r \int_{n\pi}^{\theta_{s}} \frac{1}{\sin\phi} d\theta \qquad (3.42)$$

Make the armor wire lay angle in Eq. 3.42 substituted by Eqs. 3.22 and 3.25. Then, by deriving Eq. 3.42 in terms of bending curvature and considering that the armor wire axial strain due to bending is ignorable, the instantaneous angular coordinate variation rate of this point on the armor wire with respect to the bending curvature can be approximately evaluated by:

$$\frac{d\theta_s}{d\kappa} = -r\cos^2\phi^i \sin\theta_s - \frac{\sin 2\phi^i}{2r} \int_{n\pi}^{\theta_s} \frac{d}{d\kappa} \left(\frac{d\Delta u}{d\theta}\right) d\theta \tag{3.43}$$

Assuming that the armor wire sliding is primarily axial in cyclic bending, the integral term in Eq. 3.43 can be taken as a small term. Consequently, considering that the armor wire is fixed on the toroid in the positions of intrados and extrados and disregarding the shear deformation of the underlying polyamide anti-friction tape, the sliding rate of this point in the tangential direction with respect to the bending curvature can be approximately evaluated by:

$$\frac{d\Delta t\left(\theta_{s}\right)}{d\kappa} = \frac{d\theta_{s}}{d\kappa} \frac{r}{\sin\phi}$$
(3.44)

Substituting Eq. 3.43 into Eq. 3.44, making the armor wire lay angle substituted by Eqs. 3.22 and 3.25 and ignoring the higher order small terms, the armor wire sliding rate in the tangential direction with respect to the bending curvature can be approximately evaluated by:

$$\frac{d\Delta t\left(\theta\right)}{d\kappa} = -r^2 \frac{\cos^2 \phi^i}{\sin \phi^i} \sin \theta \tag{3.45}$$

According to Eq. 3.45, the armor wire tangential sliding directions in cyclic bending are illustrated in Fig. 3.6.



Figure 3.6: Schematic of the armor wire tangential sliding direction in cyclic bending.

Subsequently, as the armor wire sliding rate in the tangential direction with respect to the bending curvature has been obtained, its sliding rate in the bi-normal direction with respect to the bending curvature can be determined using Eq. 3.41 when the frictional forces in the tangential and bi-normal directions are obtained. Since the inner armor layer is the critical layer for lateral instability due to less frictional resistance especially when the armor annulus is flooded, the next task is thus to evaluate the frictional forces on the inner armor wire in the wet annulus condition.

Assume that the entire armor wire slides in bending. Using Coulomb frictional

model, the frictional forces on the inner armor wire under the given condition should satisfy the following relation:

$$p_t(\theta)^2 + p_b(\theta)^2 = \mu^2 p_n(\theta)^2$$
 (3.46)

where μ is the friction coefficient between the armor wire and the anti-friction tape. Since the armor wire sliding is assumed primarily axial, the frictional force component in the bi-normal direction is thus much smaller than that in the tangential direction. Consequently, Eq. 3.46 can be approximately simplified as:

$$|p_t(\theta)| \approx \mu |p_n(\theta)| \tag{3.47}$$

Combining Eqs. 3.33, 3.36 and substituting into 3.47, the frictional and crosssectional forces on the inner armor wire in the tangential direction for one armor wire pitch $0 \le \theta \le 2\pi$ can be approximately obtained as:

$$p_t(\theta) = \begin{cases} \mp P_t(0) \frac{\mu \sin^2 \phi^i}{r} e^{\pm \mu \sin \phi^i \theta} & 0 < \theta < \pi \\ \pm P_t(0) \frac{\mu \sin^2 \phi^i}{r} e^{\mp \mu \sin \phi^i (\theta - 2\pi)} & \pi < \theta < 2\pi \end{cases}$$
(3.48)

$$P_t(\theta) = \begin{cases} P_t(0) e^{\pm \mu \sin \phi^i \theta} & 0 < \theta < \pi \\ P_t(0) e^{\pm \mu \sin \phi^i (\theta - 2\pi)} & \pi < \theta < 2\pi \end{cases}$$
(3.49)

where the upper and lower signs of \pm and \mp represent respectively the conditions when the bending curvature increases or decreases. $P_t(0)$ is the force in the inner armor wire in the tangential direction at $\theta = 0$. When the bending curvature increases and decreases $P_t(0)$ is denoted respectively by $\hat{P}_t(0)$ and $\check{P}_t(0)$.



Figure 3.7: Longitudinal load evaluation assuming that each armor wire deflects identically.

Since the deflection of each armor wire within the same layer is assumed identical, the cross-sectional force of each armor wire within the same layer at the same angular position is deemed the same, as illustrated by Fig. 3.7. Thus, the longitudinal compressive load carried by the inner armor wire layer can be approximately evaluated by:

$$P_{in} = \sum_{j=1}^{N_{in}} P_t(\theta_j) \cos \phi(\theta_j)$$
(3.50)

where N_{in} is the number of armor wires in the inner armor wire layer, θ_j represents the angular position of the j^{th} inner armor wire at a certain pipe cross-section. Considering that the armor wire deflection is small when no lateral instability takes place, the inner armor wires can be considered approximately uniformly distributed on the pipe cross-section, i.e.:

$$\theta_j \approx j \frac{2\pi}{N_{in}} \tag{3.51}$$

Substituting Eq. 3.49 into Eq. 3.50 and applying Eq. 3.51, the longitudinal compressive loads carried by the inner armor wire layer when the bending curvature increases and decreases can be evaluated respectively by:

$$P_{in} \approx \begin{cases} 2\widetilde{P}_t(0)\cos\phi^i \sum_{j=1}^{N_{in}/2} e^{\mu\sin\phi^i \frac{2\pi}{N_{in}}j} & (\kappa \text{ increases}) \\ 2\widetilde{P}_t(0)\cos\phi^i \sum_{j=1}^{N_{in}/2} e^{-\mu\sin\phi^i \frac{2\pi}{N_{in}}j} & (\kappa \text{ decreases}) \end{cases}$$
(3.52)

Due to the symmetry between the inner and outer armor wire layers, it is assumed that the longitudinal load carried by each armor wire layer is approximately identical. As a result, P_{in} should be approximately constant when the bending curvature increases and decreases. Thus, according to Eq. 3.52, the following relation can be obtained:

$$\widehat{P}_t(0) = e^{-\mu \sin \phi^i \pi} \widecheck{P}_t(0) \tag{3.53}$$

Substituting Eq. 3.53 into Eqs. 3.49, it can be seen that the tangential force remains constant at the neutral plane of the pipe ($\theta = 0.5\pi, 1.5\pi$) when the bending curvature increases and decreases, which is herein denoted by \bar{P}_t . Accordingly, Eq. 3.52 can be rewritten in terms of \bar{P}_t as:

$$P_{in} = 2\bar{P}_t \cos\phi^i \sum_{i=1}^{N_{in}/2} e^{\mu \sin\phi^i \left(\frac{2}{N_{in}}i - 0.5\right)\pi}$$
(3.54)

Using the armor wire lay angles and the friction coefficients between the armor wires and anti-friction tapes (0.05-0.15) that may be encountered in a practical flexible pipe structure, the following approximation can be used:

$$e^{\mu \sin \phi^i(\theta - 0.5\pi)} + e^{-\mu \sin \phi^i(\theta - 0.5\pi)} \approx 2 \quad (0 \le \theta \le \pi)$$
 (3.55)

Accordingly, Eq. 3.54 can be simplified as:

$$P_{in} \approx N_{in} \bar{P}_t \cos \phi^i \tag{3.56}$$

Due to the symmetry between the inner and outer armor wire layers, Eq. 3.56 is also applicable to the outer armor wire layer. Considering the tangential crosssectional forces on the inner and outer armor wires at the neutral plane are approximately identical, \bar{P}_t can be approximately evaluated by:

$$\bar{P}_t = \frac{P_{end}}{N\cos\phi^i} \tag{3.57}$$

where N is the total number of armor wires in the flexible pipe and P_{end} is the longitudinal compressive load carried by the flexible pipe. Then, the external loads in the armor wire normal and bi-normal directions and the tangential cross-sectional force given respectively by Eqs. 3.33, 3.48-3.49 can be rewritten in terms of \bar{P}_t as:

$$p_n\left(\theta\right) = \bar{P}_t \frac{\sin^2 \phi^i}{r} \chi\left(\theta\right) \tag{3.58}$$

$$p_t(\theta) = \mu \bar{P}_t \frac{\sin^2 \phi^i}{r} \chi(\theta) \eta(\theta)$$
(3.59)

$$P_t(\theta) = \bar{P}_t \chi(\theta) \tag{3.60}$$

In which

$$\chi(\theta) = \begin{cases} e^{\pm\mu\sin\phi^{i}(\theta-0.5\pi)} & 0 < \theta < \pi\\ e^{\mp\mu\sin\phi^{i}(\theta-1.5\pi)} & \pi < \theta < 2\pi \end{cases}$$
(3.61a)

$$\eta\left(\theta\right) = \begin{cases} \mp 1 & 0 < \theta < \pi \\ \pm 1 & \pi < \theta < 2\pi \end{cases}$$
(3.61b)

where the upper and lower signs of \pm and \mp represent respectively the conditions when the bending curvature increases or decreases.

Subsequently, making the tangential cross-sectional force in Eq. 3.34 substituted by Eq. 3.60 and applying $\omega = 0$ yields:

$$p_{b}(\theta) = EI_{n} \frac{\sin^{5} \phi^{i}}{r^{4}} \frac{d^{4} \Delta u}{d\theta^{4}} - \left[\bar{P}_{t} \chi\left(\theta\right) - P_{1}\right] \frac{\sin^{3} \phi^{i}}{r^{2}} \frac{d^{2} \Delta u}{d\theta^{2}} + \kappa \left[\bar{P}_{t} \chi\left(\theta\right) - P_{2}\right] \left(1 + \sin^{2} \phi^{i}\right) \cos \phi^{i} \sin \theta$$

$$(3.62)$$

Thereafter, substituting the frictional force components given by Eqs. 3.59 and 3.62 and the tangential sliding rate with respect to the bending curvature given by Eq. 3.45 into Eq. 3.41, the armor wire sliding rate in the bi-normal direction with

respect to the bending curvature can be approximately evaluated by:

$$\frac{d\Delta b\left(\theta\right)}{d\kappa} = \frac{r^{2}\sin\theta}{\mu\eta\left(\theta\right)} \left\{ -\frac{EI_{n}}{\bar{P}_{t}\chi\left(\theta\right)} \frac{\sin^{2}2\phi^{i}}{4r^{3}} \frac{d^{4}\Delta u}{d\theta^{4}} + \left[1 - \frac{P_{1}}{\bar{P}_{t}\chi\left(\theta\right)}\right] \frac{\cos^{2}\phi^{i}}{r} \frac{d^{2}\Delta u}{d\theta^{2}} -\kappa r \left[1 - \frac{P_{2}}{\bar{P}_{t}\chi\left(\theta\right)}\right] \left(1 + \sin^{2}\phi^{i}\right) \cot^{3}\phi^{i}\sin\theta \right\}$$
(3.63)

For consistency purpose, the armor wire sliding rate with respect to the bending curvature in its local coordinate system is transformed into that in the toroidal coordinate system. Let $d\Delta u/d\kappa$ and $d\Delta\theta/d\kappa$ denote respectively the instantaneous longitudinal and angular coordinate variation rates with respect to the bending curvature of the points on the armor wire. According to the relation depicted in Fig. 3.5 and applying Eq. 3.1, $d\Delta u/d\kappa$ and $d\Delta\theta/d\kappa$ can be expressed in terms of $d\Delta t/d\kappa$ and $d\Delta b/d\kappa$ respectively as:

$$\frac{d\Delta u\left(\theta\right)}{d\kappa} = \frac{1}{1 + \kappa r \cos\theta} \left[\frac{d\Delta t\left(\theta\right)}{d\kappa} \cos\phi\left(\theta\right) + \frac{d\Delta b\left(\theta\right)}{d\kappa} \sin\phi\left(\theta\right) \right]$$
(3.64a)

$$\frac{d\Delta\theta\left(\theta\right)}{d\kappa} = \frac{1}{r} \left[\frac{d\Delta t\left(\theta\right)}{d\kappa} \sin\phi\left(\theta\right) - \frac{d\Delta b\left(\theta\right)}{d\kappa} \cos\phi\left(\theta\right) \right]$$
(3.64b)

Making the sliding rates in the tangential and bi-normal directions with respect to the bending curvature in Eqs. 3.64a-3.64b substituted by Eqs. 3.45, 3.63 and ignoring the higher order small terms, the instantaneous longitudinal and angular coordinate variation rates with respect to the bending curvature of the points on the inner armor wire can be obtained respectively as:

$$\frac{d\Delta u\left(\theta\right)}{d\kappa} = -r^{2} \frac{\cos^{3} \phi^{i}}{\sin \phi^{i}} \sin \theta \\
+ \frac{r^{2} \sin \phi^{i} \sin \theta}{\mu \eta\left(\theta\right)} \left\{ -\frac{EI_{n}}{\bar{P}_{t}\chi\left(\theta\right)} \frac{\sin^{2} 2\phi^{i}}{4r^{3}} \frac{d^{4}\Delta u}{d\theta^{4}} + \left[1 - \frac{P_{1}}{\bar{P}_{t}\chi\left(\theta\right)}\right] \frac{\cos^{2} \phi^{i}}{r} \frac{d^{2}\Delta u}{d\theta^{2}} \\
-\kappa r \left[1 - \frac{P_{2}}{\bar{P}_{t}\chi\left(\theta\right)}\right] \left(1 + \sin^{2} \phi^{i}\right) \cot^{3} \phi^{i} \sin \theta \right\}$$
(3.65a)

$$\frac{d\Delta\theta\left(\theta\right)}{d\kappa} = -r\cos^{2}\phi^{i}\sin\theta \\
-\frac{r\cos\phi^{i}\sin\theta}{\mu\eta\left(\theta\right)} \left\{ -\frac{EI_{n}}{\bar{P}_{t}\chi\left(\theta\right)} \frac{\sin^{2}2\phi^{i}}{4r^{3}} \frac{d^{4}\Delta u}{d\theta^{4}} + \left[1 - \frac{P_{1}}{\bar{P}_{t}\chi\left(\theta\right)}\right] \frac{\cos^{2}\phi^{i}}{r} \frac{d^{2}\Delta u}{d\theta^{2}} \\
-\kappa r \left[1 - \frac{P_{2}}{\bar{P}_{t}\chi\left(\theta\right)}\right] \left(1 + \sin^{2}\phi^{i}\right) \cot^{3}\phi^{i}\sin\theta \right\}$$
(3.65b)

3.5 Equilibrium state

For a loading condition that does not cause the armor wire lateral instability, each armor wire would eventually reach an equilibrium state after numerous bending cycles in which it slightly oscillates around a path in repeated bending. Such a path is herein named as equilibrium path and denoted by $u^e(\theta)$. The difference between the equilibrium path and the new helix is denoted by $\Delta u^e(\theta)$. Considering a flexible pipe subjected to cyclic bending with the minimum and maximum bending curvatures respectively denoted by κ_{min} and κ_{max} and denoting the armor wire path after the n^{th} bending cycle by $u_n(\theta)$, the equilibrium state of the armor wire can be described by:

$$\lim_{n \to \infty} \left[u_n\left(\theta\right) - u_{n-1}\left(\theta\right) \right] = \int_{\kappa_{\min}}^{\kappa_{\max}} \frac{d\Delta u\left(\theta\right)}{d\kappa} d\kappa + \int_{\kappa_{\max}}^{\kappa_{\min}} \frac{d\Delta u\left(\theta\right)}{d\kappa} d\kappa = 0 \qquad (3.66)$$

which means that the armor wire stops marching furtuer after each bending cycle once the equilibirum path is reached.

For an armor wire in the equilibrium state, considering that its transverse oscillation in cyclic bending is small, the armor wire longitudinal coordinate variation rate with respect to the bending curvature can thus be approximately evaluated through Eq. 3.65a by assuming the armor wire transversely fixed on the equilibrium path. Thus, substituting Eq. 3.65a into Eq. 3.66, using approximation given by Eq. 3.55 and employing $\Delta u(\theta) = \Delta u^e(\theta)$ yields:

$$\frac{EI_n}{\bar{P}_t} \frac{\sin^5 \phi^i}{r} \frac{d^4 \Delta u^e}{d\theta^2} - \left(1 - \frac{P_1}{\bar{P}_t}\right) \sin^3 \phi^i r \frac{d^2 \Delta u^e}{d\theta^2}
+ \frac{1}{2} \left(\kappa_{\min} + \kappa_{\max}\right) r^3 \left(1 - \frac{P_2}{\bar{P}_t}\right) \left(1 + \sin^2 \phi^i\right) \cos \phi^i \sin \theta = 0$$
(3.67)

To solve this fourth order differential equation, four boundary conditions are needed. Considering that the armor wire is pinned on the toroid in the positions of intrados and extrados in bending, the following boundary conditions can be used:

$$\Delta u^e \left(\theta = n\pi\right) = 0 \tag{3.68}$$

Besides, due to symmetry, the bending moment in the armor wire normal direction at the intrados and extrados in the equilibrium state should be approximately zero. According to Eq. 3.29b, additional boundary conditions can be obtained:

$$\frac{d\Delta^2 u^e}{d\theta^2} \left(\theta = n\pi\right) = 0 \tag{3.69}$$

Applying the periodic boundary conditions given by Eqs. 3.68-3.69, $\Delta u^{e}(\theta)$ can

be determined from Eq. 3.67 as:

$$\Delta u^{e}(\theta) = -\left(\kappa_{\min} + \kappa_{\max}\right) r^{2} \frac{\left(1 + \sin^{2}\phi^{i}\right)\cos\phi^{i}}{2\sin^{3}\phi^{i}} \frac{\bar{P}_{t} - P_{2}}{\bar{P}_{t} - P_{3}}\sin\theta \qquad (3.70)$$

In which P_3 is a constant corresponding to the armor wire characteristics:

$$P_{3} = \frac{\sin^{2} \phi^{i}}{r^{2}} \left[-EI_{n} \left(1 + \cos^{2} \phi^{i} \right) - 4EI_{b} \cos^{2} \phi^{i} + GJ \cos 2\phi^{i} \right]$$
(3.71)

It is interesting to note that $\Delta u^e = 0$ when $\kappa_{\min} = -\kappa_{\max}$. In other words, the armor wire tends to remain in the helix path under this circumstance.

Thereafter, with the equilibrium path determined, the curvatures, forces and moments on the armor wire in the equilibrium path can be promptly obtained. Substituting Eq. 3.70 into Eqs. 3.28a-3.28c and considering $\omega = 0$, the curvature components of the armor wire in the equilibrium path can be obtained as:

$$\kappa_g(\theta) = \kappa \left(1 + \sin^2 \phi^i\right) \cos \phi^i \sin \theta - \frac{1}{2} \frac{\bar{P}_t - P_2}{\bar{P}_t - P_3} \left(\kappa_{\min} + \kappa_{\max}\right) \left(1 + \sin^2 \phi^i\right) \cos \phi^i \sin \theta$$
(3.72a)

$$\kappa_n(\theta) = -\frac{\sin^2 \phi^i}{r} - \kappa \cos 2\phi^i \cos^2 \phi^i \cos \theta - \frac{\bar{P}_t - P_2}{\bar{P}_t - P_3} \left(\kappa_{\min} + \kappa_{\max}\right) \left(1 + \sin^2 \phi^i\right) \cos^2 \phi^i \cos \theta$$
(3.72b)

$$\tau(\theta) = -\frac{\sin 2\phi^{i}}{2r} + 2\kappa \sin \phi^{i} \cos^{3} \phi^{i} \cos \theta -\frac{1}{2} \frac{\bar{P}_{t} - P_{2}}{\bar{P}_{t} - P_{3}} \left(\kappa_{\min} + \kappa_{\max}\right) \left(1 + \sin^{2} \phi^{i}\right) \cot \phi^{i} \cos 2\phi^{i} \cos \theta$$
(3.72c)

Similarly, substituting Eq. 3.70 into Eqs. 3.29a-3.29c and considering $\omega = 0$, the armor wire cross-sectional moments in the equilibrium path can be obtained as:

$$M_t(\theta) = GJ \left[2\kappa \sin \phi^i \cos^3 \phi^i - \frac{1}{2} \frac{\bar{P}_t - P_2}{\bar{P}_t - P_3} \left(\kappa_{\min} + \kappa_{\max} \right) \left(1 + \sin^2 \phi^i \right) \cot \phi^i \cos 2\phi^i \right] \cos \theta$$
(3.73a)

$$M_n(\theta) = EI_n \left[\kappa - \frac{1}{2} \frac{\bar{P}_t - P_2}{\bar{P}_t - P_3} \left(\kappa_{\min} + \kappa_{\max} \right) \right] \left(1 + \sin^2 \phi^i \right) \cos \phi^i \sin \theta \qquad (3.73b)$$

$$M_b(\theta) = EI_b \left[-\kappa \cos 2\phi^i - \frac{\bar{P}_t - P_2}{\bar{P}_t - P_3} \left(\kappa_{\min} + \kappa_{\max} \right) \left(1 + \sin^2 \phi^i \right) \right] \cos^2 \phi^i \cos \theta$$
(3.73c)

Afterwards, substituting Eq. 3.70 into Eqs. 3.30, 3.62 and considering $\omega = 0$, the external moment in the axial direction and the external force in the bi-normal direction on the armor wire in the equilibrium path can be evaluated by:

$$m_t(\theta) = -\frac{\kappa}{r} \left[EI_n \left(1 + \sin^2 \phi^i \right) - 2GJ\cos^2 \phi^i \right] \sin^2 \phi^i \cos \phi^i \sin \theta + \frac{1}{2r} \left(\kappa_{\min} + \kappa_{\max} \right) \frac{\bar{P}_t - P_2}{\bar{P}_t - P_3} \left(EI_n \sin^2 \phi^i - GJ\cos 2\phi^i \right) \left(1 + \sin^2 \phi^i \right) \cos \phi^i \sin \theta$$
(3.74)

$$p_{b}(\theta) = \left\{ \kappa \left[\bar{P}_{t}\chi(\theta) - P_{2} \right] - \frac{1}{2} \left(\kappa_{\min} + \kappa_{\max} \right) \left(\bar{P}_{t} - P_{2} \right) \frac{\bar{P}_{t}\chi(\theta) - P_{3}}{\bar{P}_{t} - P_{3}} \right\} \left(1 + \sin^{2}\phi^{i} \right) \cos\phi^{i} \sin\theta$$

$$(3.75)$$

Subsequently, substituting Eq. 3.70 into Eqs. 3.31, 3.32 and employing $\omega = 0$, the armor wire shear forces in the normal and bi-normal directions in the equilibrium path can be obtained respectively as:

$$P_{n}(\theta) = \left\{ +\frac{\kappa}{r} \left[EI_{n} \left(1 + \sin^{2} \phi^{i} \right) - EI_{b} \cos 2\phi^{i} \right] -\frac{\kappa_{\min} + \kappa_{\max}}{2r} \frac{\bar{P}_{t} - P_{2}}{\bar{P}_{t} - P_{3}} \left(EI_{n} + 2EI_{b} \right) \left(1 + \sin^{2} \phi^{i} \right) \right\} \sin \phi^{i} \cos^{2} \phi^{i} \sin \theta$$

$$(3.76)$$

$$P_{b}(\theta) = \frac{\kappa}{2r} \left[EI_{n} \left(1 + \sin^{2}\phi^{i} \right) - EI_{b} \cos 2\phi^{i} \cos^{2}\phi^{i} - \frac{1}{2}GJ\sin^{2}2\phi^{i} \right] \sin 2\phi^{i} \cos \theta + \frac{\kappa_{\min} + \kappa_{\max}}{4r} \frac{\bar{P}_{t} - P_{2}}{\bar{P}_{t} - P_{3}} \left(EI_{n} - 2EI_{b}\cos^{2}\phi^{i} + GJ\cos 2\phi^{i} \right) \left(1 + \sin^{2}\phi^{i} \right) \sin 2\phi^{i} \cos \theta$$

$$(3.77)$$

Afterwards, substituting Eq. 3.70 into Eq. 3.25 yields:

$$\Delta\phi_2\left(\theta\right) = -\kappa r \frac{\sin 2\phi^i}{2} \cos\theta + \frac{\sin^2\phi^i}{r} \left(\kappa_{\min} + \kappa_{\max}\right) r^2 \frac{\left(1 + \sin^2\phi^i\right) \cos\phi^i}{2\sin^3\phi^i} \frac{\bar{P}_t - P_2}{\bar{P}_t - P_3} \cos\theta$$
(3.78)

Substituting Eqs. 3.22 and 3.78 into Eq. 3.26, employing $\kappa = 0$ and the average axial strain $\varepsilon = \bar{P}_t/EA$, and only keeping the lowest order of each small term, the difference between the new and initial helix lay angles can be approximately evaluated by:

$$\Delta\phi_1 = -\frac{\bar{P}_t \tan \phi^i}{EA} + \frac{1}{16} (\kappa_{\min} + \kappa_{\max})^2 r^2 \frac{\left(1 + \sin^2 \phi^i\right)^2}{\tan \phi^i} \left(\frac{\bar{P}_t - P_2}{\bar{P}_t - P_3}\right)^2 \tag{3.79}$$

Consequently, substituting Eq. 3.79 into Eq. 3.27, the flexible pipe longitudinal compressive strain, defined by the longitudinal shortening divided by the original pipe length in the straight pipe condition, in the equilibrium state can be obtained as:

$$\Delta \bar{L} = -\frac{\bar{P}_t}{EA\cos^2\phi^i} + \frac{1}{16}(\kappa_{\min} + \kappa_{\max})^2 r^2 \left(\frac{1}{\sin\phi^i} + \sin\phi^i\right)^2 \left(\frac{\bar{P}_t - P_2}{\bar{P}_t - P_3}\right)^2 \quad (3.80)$$

where the first term represents the effect of armor wire axial strain and the second term represents the effect of the armor wire deflection.

3.6 Lateral instability

On the contrary, for a flexible pipe subjected to large axial compressive loads combined to cyclic bending, each armor wire would migrate constantly on one side after each bending cycle and eventually result in the lateral instability failure due to material yield. Thus, for an armor wire in the unstable condition, there is no solution to the equilibrium state described by Eq. 3.66 and the equilibrium path given by Eq. 3.70 does not exist.

Note that, for a flexible pipe subjected to cyclic bending in the same bending manner, each armor wire migrates in one direction after each bending cycle, i.e., the sequence of $u_n(\theta)$ is monotonous. Thus, the following relation must hold:

$$\frac{\Delta u^e\left(\theta\right)}{\Delta u_1\left(\theta\right)} > 0 \tag{3.81}$$

where $\Delta u_1(\theta)$ represents the armor wire path relative to the new helix after the first bending cycle. Comparably, since the armor wire transverse deflection in the first bending cycle is small, its sliding rate with respect to bending curvature can be approximately evaluated by Eq. 3.65a employing $\Delta u(\theta) = 0$. Thus, substituting $\Delta u(\theta) = 0$ into Eq. 3.65a and integrating in terms of bending curvature for one bending cycle, $\Delta u_1(\theta)$ can be approximately given by:

$$\Delta u_1(\theta) = \left(\kappa_{\max}^2 - \kappa_{\min}^2\right) \frac{r^3}{\mu} \left(1 - \frac{P_2}{\bar{P}_t}\right) \left(1 + \sin^2 \phi^i\right) \sin \phi \cot^3 \phi^i \sin^2 \theta \qquad (3.82)$$

Subsequently, substituting Eqs. 3.70, 3.82 into Eq. 3.81 yields $\bar{P}_t > P_3$, which is corresponding to a state that the equilibrium state does not exist when $\bar{P}_t \leq P_3$. Note that in the present context the compressive loads are negative. Thus, it can be concluded that, when the axial compressive load \bar{P}_t equals or surpasses the critical value given by Eq. 3.71, the armor wire lateral instability may take place after numerous bending cycles. Thus, making \bar{P}_t in Eq. 3.57 substituted by Eq. 3.71, the critical longitudinal compressive load on a flexible pipe that may eventually result in the armor wire lateral instability after numerous bending cycles can then be approximately evaluated by:

$$P_{LI} = -N \frac{\sin^2 \phi^i \cos \phi^i}{r^2} \left[EI_n \left(1 + \cos^2 \phi^i \right) + 4EI_b \cos^2 \phi^i - GJ \cos 2\phi^i \right]$$
(3.83)

Subsequently, defining the non-dimensional parameters \bar{P}_{LI} , ξ , ζ as:

$$\bar{P}_{LI} = P_{LI}/(EA)$$

$$\xi = H/W$$

$$\zeta = W/r$$
(3.84)

and leting Ω represents the average filling factor of the inner and outer armor wire layers:

$$\Omega = \frac{NW}{4\pi r \cos \phi^i} \tag{3.85}$$

Eq. 3.83 can be rewriten in a non-dimensional form as:

$$\bar{P}_{LI} = -\pi\Omega\zeta\sin^2 2\phi^i \left[\frac{1}{8} + \frac{\xi^2}{6} + \left(\frac{1}{24} + \frac{\xi^2}{6}\frac{\nu}{1+\nu} + \frac{32\xi^3}{\pi^5}\frac{1}{1+\nu}\right)\cos 2\phi^i\right]$$
(3.86)

where ν is the poisson's ratio.

Chapter 4

Discussions

In this section, case studies are presented considering an armor wire with crosssectional area $10 \ mm \times 3 \ mm$ (width \times thickness), radius 0.1 m and initial lay angle 30° in the inner armor wire layer of a flexible pipe in the wet annulus condition. The friction coefficient between the armor wire and the anti-friction tape is set to 0.1. The Young's modulus and Poisson's ratio are respectively 210 *GPa* and 0.3. Using Eqs. 3.35b and 3.71, the constants P_2 and P_3 for this armor wire can be calculated respectively as -2182 N and -2576 N. The armor wire is considered subjected to a constant axial compressive load, which is smaller than the estimated lateral stability limit P_3 , combined to a significant number of bending cycles, so that the armor wire has reached the equilibrium path.

4.1 Equilibrium path

Consider that the minimum and maximum bending curvatures of the flexible pipe are respectively $\kappa_{\min} = 0.0 \ m^{-1}$ and $\kappa_{\max} = 0.5 \ m^{-1}$. Fig. 4.1 shows the plots of Eq. 3.70 with different axial compressive loads for half armor wire pitch $0 \le \theta \le \pi$. It is interesting to note that $\Delta u^e(\theta)$ equals zero when $\bar{P}_t = P_2 = -2182 \ N$. In other words, the armor wire tends to stay at the helix path in cyclic bending when $\bar{P}_t = P_2$. When \bar{P}_t is larger or smaller than P_2 , the armor wire tends to migrate in opposite directions. Fig. 4.2 depicts the equilibrium paths in the cylindrical coordinate system when $\bar{P}_t = -1500 \ N, -2182 \ N, -2500 \ N$, illustrating that the armor wire tends to migrate in the direction towards the intrados when $|\bar{P}_t| < |P_2|$ and migrate in the direction towards the extrados when $|\bar{P}_t| > |P_2|$.

Thereafter, employing the equilibrium path with $\bar{P}_t = -2500 N$, considering that the armor wire layer are constituted of 20 armor wires and assuming the deflection of each armor wire is the same, the armor wire layer configuration in the equilibrium state when the axial compressive load is close to the predicted lateral stability limit $P_3 = -2576 N$ can be approximately illustrated in Fig. 4.3 (left). It can be



Figure 4.1: Equilibrium paths of the inner armor wire in different axial compressive loads.



Figure 4.2: Equilibrium paths of the inner armor wire when $\bar{P}_t = -1500 N, -2182 N, -2500 N.$

observed that the armor wires tend to be squeezed in the extrados and loosened in the intrados, which is in accordance with a typical S-shape instability configuration as depicted in Fig. 4.3 (right).



Figure 4.3: The predicted configuration in the stable state when the compressive load is close to the stability limit (left) and a typical S-shape instability configuration (right) (CALEYRON et al. [1]).

Moreover, note that the maximum displacements are found at the neutral plane as depicted in Fig. 4.1. To illustrate the equilibrium path variation tendency, Fig. 4.4 shows the results of Eq. 3.70 using $\theta = 0.5\pi$ with different axial compressive loads and bending curvatures. It can be seen that the equilibrium path changes monotonically with increasing axial compressive loads. In all the cases, the equilibrium paths coincide with the helix path when $\bar{P}_t = P_2$ and change asymptotically and suddenly when \bar{P}_t approaches P_3 . As a result, large curvatures and displacements will be generated in the armor wire when \bar{P}_t is close to P_3 , which may result in large bending stresses and, eventually, the lateral contact between the adjacent armor wires.



Figure 4.4: Δu^e at the neutral plane with different \bar{P}_t and bending curvatures.

4.2 Frictional forces

It needs to be emphasized that the present analytical model is based on an assumption that the armor wire sliding direction in cyclic bending is primarily axial. Nevertheless, in which conditions this assumption is valid needs to be carefully discussed. Note that frictional force acts in the opposite direction of relative sliding. Thus, according to Eq. 3.41, the question is equivalent to that in which conditions it is reasonable to consider the frictional force component in the tangential direction much larger than that in the bi-normal direction.

Using Eqs. 3.59, 3.75, Fig. 4.5 and Fig. 4.6 show respectively the frictional forces on the inner armor wire in the tangential and bi-normal directions for half armor wire pitch $0 \leq \theta \leq \pi$ when $\bar{P}_t = -2500 \ N$ and $\bar{P}_t = -1000 \ N$, where \hat{p}_t and \hat{p}_b represent respectively the frictional forces in the tangential and bi-normal directions when the bending curvature increases, \tilde{p}_t and \tilde{p}_b represent that when the bending curvature decreases. It can be observed that the frictional force in the tangential direction is much larger than that in the bi-normal direction when $\bar{P}_t = -2500 N$. It is thus reasonable to consider that the armor wire sliding is primarily axial under this circumstance. Nevertheless, for the case with $\bar{P}_t = -1000 N$, the frictional force in the bi-normal direction is approximately half of that in the tangential direction at the neutral plane when the bending curvature approaches the minimum or maximum values. Consequently, under this circumstance, the armor wire sliding cannot be considered primarily axial.



Figure 4.5: Frictional forces on the inner armor wire in the wet annulus condition in the equilibrium state when $\bar{P}_t = -2500 N$.

Based on the discussion above, it can be noted that the armor wire sliding direction in cyclic bending depends on the axial compressive load. To reveal the effect of axial compression on the armor wire sliding direction, Fig. 4.7 shows the ratio of the frictional force in the bi-normal direction to that in the tangential direction with different axial compressive loads. Since the frictional forces are symmetric when the bending curvature reaches the maximum and minimum values as depicted in Figs. 4.5-4.6, Fig. 4.7 only shows the results when the bending curvature approaches the maximum value. It can be observed that the frictional force in the bi-normal direction is much smaller than that in the tangential direction when the axial compressive load is close to P_2 . This is easy to understand since the armor wire tends to remain in the helix path when $\bar{P}_t = P_2$ so that its sliding tendency in the bi-normal direction is small. Besides, note that the estimated lateral stability limit P_3 is close to P_2 . Thus, it can be concluded that the present analytical model for a single armor wire is applicable when the axial compressive load is close to P_2 and smaller than P_3 .

Note that, for engineering application, the flexible pipe design should always be conservative. Thus, instead of using P_3 , the limit axial load that can be carried by a



Figure 4.6: Frictional forces on the inner armor wire in the wet annulus condition in the equilibrium state when $\bar{P}_t = -1000 N$.

single armor wire can be considered as P_2 for flexible pipe analysis since it represents the condition that the armor wire changes sliding direction.



Figure 4.7: Effect of axial compressive loads on the ratio of frictional force components on the inner armor wire in the wet annulus condition in the equilibrium state when the bending curvature approaches the maximum value.

4.3 Longitudinal shortening

Since large transverse deflection is generated in the armor wire when the axial compressive load is close to the estimated lateral stability limit P_3 as illustrated in section 4.1, intuitively, large longitudinal shortening should thus be generated. Fig. 4.8 shows the results of Eq. 3.80 with different axial compressive loads and bending curvatures. Evidently, large longitudinal shortenings are observed when the axial compressive load is close to P_3 in all the cases. Besides, it is also interesting to note that the longitudinal shortenings of those four cases coincide when $\bar{P}_t = P_2$. As previously stated, the armor wire tends to stay in the helix path when $\bar{P}_t = P_2$. Thus, according to Eq. 3.80, the effect of armor wire transverse displacement is ignorable under this circumstance, and the longitudinal shortening is solely due to the armor wire axial strain which is the same for those four cases.



Figure 4.8: Longitudinal shortening with different axial compression and bending curvatures.

4.4 Curvatures and moments

Using Eqs. 3.19a-3.19c, 3.72a-3.72c, Fig. 4.9 shows the variations of the geodesic curvature, normal curvature and torsion of the armor wire in the equilibrium path subjected to axial compression $\bar{P}_{\rm t} = -2500$ N and cyclic bending with the minimum and maximum bending curvatures $\kappa_{\rm min} = 0.0 \ m^{-1}$ and $\kappa_{\rm max} = 0.2 \ m^{-1}$, where the dotted and solid lines represent respectively the conditions when the bending curvature equals the minimum and maximum bending curvatures. It can be seen that when the bending curvature increases from the minimum bending curvature

to the maximum bending curvature the geodesic curvature and torsion increase but the normal curvature decreases.



Figure 4.9: Curvature components of the armor wire in the equilibrium path.



Figure 4.10: Cross-sectional moments in the armor wire in the equilibrium path.

Subsequently, using Eqs. 3.73a-3.73c, the corresponding cross-sectional moments are illustrated in Fig. 4.10. Clearly, the bending moment in the normal direction is much larger than the bending moment in the bi-normal and the torque, since the normal inertia is much larger than the transverse inertia and the stiffness of torsion. Moreover, Fig. 4.11 shows the external moment on the armor wire in the



Figure 4.11: Distributed torques on the armor wire in the equilibrium path.

axial direction using Eq. 3.74. It can be seen that maximum torques locate at the neutral plane. Due to symmetry, the armor wire has no tendency to rotate axially in the intrados and extrados, the torques in the intrados and extrados are thus zero.

4.5 Stress analysis

Moreover, it should be emphasized that the present analytical model is only applicable to the elastic condition. Thus, the stress analysis is needed to illustrate when the armor wire reaches the yield limit. The most critical yielding positions on an armor wire cross-section are one of the four corners, see Fig. 4.12. Ignoring the shear stress, the maximum compressive stress in each cross-section of the armor wire can be approximately evaluated by:

$$\sigma_{\max}\left(\theta\right) = \frac{P_t\left(\theta\right)}{A} - \left|\frac{aM_n\left(\theta\right)}{2I_n}\right| - \left|\frac{bM_b\left(\theta\right)}{2I_b}\right| \tag{4.1}$$

Consider that the minimum and maximum bending curvatures of the flexible pipe are respectively $\kappa_{\min} = 0.0 \ m^{-1}$ and $\kappa_{\max} = 0.2 \ m^{-1}$ and the average axial compressive load on the armor wire is $\bar{P}_t = -2500$ N. The maximum compressive stress on each cross-section of the armor wire can be evaluated by Eq. 4.1 with P_t , M_n , M_b substituted by Eqs. 3.60, 3.73b, 3.73c, which are illustrated in Fig. 4.13. The solid and dashed lines represent respectively the conditions when the bending curvature increases and decreases. It can be seen that the maximum compressive stresses locate at the positions above and below the neutral plane when the flexible



Figure 4.12: Critical yielding positions on an armor wire cross-section.

pipe is bent to the maximum bending curvature. Such maximum compressive stress can be easily identified numerically.



Figure 4.13: Maximum compressive stresses in the cross-sections of the inner armor wire in the equilibrium state in repeated bending.

Consequently, applying $\kappa = \kappa_{\text{max}}$ in Eq. 4.1 and searching the maximum value numerically, the maximum compressive stresses in the armor wire with different axial compressive loads and bending curvatures are illustrated in Fig. 4.14, in which the red dashed line represents the yield stress $\sigma_s = -1350 \ MPa$. It is very clear that the maximum compressive stresses grow suddenly and exceed the yield stress when \bar{P}_t is close to P_3 for all the cases.

4.6 Lateral contact

Additionally, it needs to be noted that the present analytical model is based on the assumption that the adjacent armor wires do not contact each other. Nevertheless, since large transverse deflections are generated in the armor wires when the axial



Figure 4.14: Maximum compressive stresses in the inner armor wire in the equilibrium state with different axial compressive loads and bending curvatures.

compressive load is close to the lateral stability limit, they may be in contact before the lateral instability takes place.



Figure 4.15: Schematic of the adjacent armor wire segments in the initial (left) and contact (right) conditions.

Fig. 4.15 shows configurations of the adjacent armor wire segments in the initial and contact conditions, where g denotes the gap between the adjacent armor wires in the initial straight pipe condition and $\Delta \phi^c$ represents the lay angle variation when the armor wires are in contact. Since the deflections of the armor wires within the same layer are deemed identical, the distance between the central lines of adjacent armor wires in the longitudinal direction should satisfy the following relation:

$$\frac{W+g}{\sin\phi^i} \left\| \partial \mathbf{X} / \partial u \right\| = \frac{W}{\sin\left(\phi^i + \Delta\phi^c\right)} \tag{4.2}$$

Applying Eq. 3.1 into Eq. 4.2 and ignoring the higher order small terms, the lay angle variation in the contact section can be approximately evaluated by:

$$\Delta\phi^{c}\left(\theta\right) = \frac{W}{W+g} \frac{\tan\phi^{i}}{1+\kappa r\cos\theta} - \tan\phi^{i} \tag{4.3}$$

In other words, the armor wire equilibrium path given by Eq. 3.70 is not valid if the armor wire lay angle variations surpass $\Delta \phi^c$.

Using Eqs. 3.78-3.79, Fig. 4.16 shows the lay angle variations of the armor wire in the equilibrium paths with different cyclic bending amplitudes. Evidently, the minimum armor wire lay angle locates at the extrados ($\theta = 0, 2\pi$) when the bending curvature reaches the maximum bending curvature. Thus, if the adjacent armor wires are in contact, the first contact locations are the extrados.



Figure 4.16: Armor wire lay angle variations in equilibrium paths with different bending curvatures.

Subsequently, Fig. 4.17 shows the lay angle variation at the extrados when the bending curvature reaches the maximum value with different axial compressive loads and bending amplitudes. Substituting $\theta = 0$ and $\kappa = \kappa_{max}$ into Eq. 4.3 and considering that the filling factor of the armor wire layer in a flexible pipe is generally around 90%, i.e.: $W/(W + g) \approx 0.9$, the lay angle variations causing lateral contact at the extrados when the bending curvature reaches the maximum value for those four cases with $\kappa_{max} = 1/20 \ m^{-1}$, $1/15 \ m^{-1}$, $1/10 \ m^{-1}$, $1/5 \ m^{-1}$ are obtained respectively as $\Delta \phi^c = -0.060 \ rad$, $-0.061 \ rad$, $-0.063 \ rad$, $-0.068 \ rad$ and marked by the dots in Fig. 4.17. Accordingly, the critical axial compressive loads resulting in the armor wire lateral contact for those four cases are obtained respectively as $-2544 \ N$, $-2534 \ N$, $-2516 \ N$, $-2473 \ N$. Substituting those critical axial compressive loads into Eq. 3.80, the longitudinal shortenings corresponding to the first contact condition of those cases are all approximately 0.17%. Besides, substituting those critical axial compressive loads into Eq. 4.1, applying $\kappa = \kappa_{\text{max}}$ and searching the maximum value numerically, the corresponding maximum compressive stresses for each case can be obtained respectively as -484 *MPa*, -501 *MPa*, -536 *MPa*, -644 *MPa*, which are much smaller than the yielding stress. Thus, it can be concluded that the armor wire is in the elastic stage when the first lateral contact takes place. Nevertheless, how much more compressive load the armor wires are capable of carrying after the lateral contact takes place is still unknown.



Figure 4.17: Armor wire lay angle variation at the extrados with different axial compression and bending curvatures.



Figure 4.18: Schematic of the armor wire lateral contact with a small gap (left) and a big gap (right).

Consider that the radial constraint is very strong so that the gap between the armor wire and the inner core formed by axial compressive loads is very small. Under this circumstance, the armor wire has nearly no space to rotate axially and move laterally in the contact sections as depicted in Fig. 4.18 (left). Thus, the armor

wire lateral stabilization may thus be increased due to the lateral contact, and the lateral stability limit estimated by Eq. 3.83 may not be applicable in this case. In contrary, if the radial constraint is not very strong so that the gap is relatively big, the lateral contact may push the armor wire to rotate axially, which generates some spaces for the armor wire to continue to move laterally, see Fig. 4.18 (right). Thus, the effect of the lateral contact on the armor wire lateral stabilization may thus not be significant. Under this circumstance, the lateral stability limit may be approximately estimated by Eq. 3.83.

Chapter 5

Experimental reconstruction of the armor wire lateral instability

To obtain a better understanding of the flexible pipe armor wire lateral instability mechanism, as well as to validate the developed analytical model, a bending hyperbaric chamber was constructed in the laboratory Núcleo de Estruturas Oceânicas (NEO) of the Federal University of Rio de Janeiro, see Fig. 5.1. The hyperbaric chamber was made of a 16-inch flexible pipe which is capable of applying pressures up to 2500 *psi* with bending radius up to 6.53 m. The bending curvature was applied in the horizontal plane since no out-of-plane bending needs to be worried. Using this rig, a 6-inch flexible riser sample and a 6-inch flexible flowline sample have been tested, whose armor wire characteristics are presented in Table 5.1, where the last row shows the lengths of the flexible pipe samples in terms of the number of the pitches of the inner armor wire.



Figure 5.1: Schematic drawings of the test setup.

| | Parameters | 6" riser | 6" flowline |
|-----------------------------|-----------------------|-------------------|----------------|
| Outer armor | Outer diameter (mm) | 242.5 | 203.4 |
| | Lay angle (deg) | -30 | -30 |
| | Wire size (mm) | 6.2×14.1 | 2.5×7 |
| | Number of wires | 44 | 73 |
| Inner armor | Outer diameter (mm) | 230.1 | 198.4 |
| | Lay angle (deg) | 30 | 30 |
| | Wire size (mm) | 6.2×14.1 | 2.5×7 |
| | Number of wires | 41 | 71 |
| Fluid barrier | Outer diameter (mm) | 182.5 | 193.4 |
| Pitch number of inner armor | | 4.12 | 5.88 |

Table 5.1: Armor wire layers characteristics of the flexible pipe samples.

5.1 Test of the 6" flexible riser sample

The original sample of the 6" flexible riser has a total length of 5.725 m, which is smaller than the required length for the flex-compression test. To make full use of the sample length, the connectors were prepared as short as possible as depicted in Fig. 5.2. The armor annulus was not sealed by the connectors so that it was flooded during the test. Thus, the equivalent longitudinal compressive loads generated by the hydrostatic end-cap effect can be calculated by multiplying the hydrostatic pressure by the cross-sectional area of the fluid barrier. After the assembly of both connectors, the sample has a total length of 5.020 m (flange to flange). Besides, to compensate for the length of the sample, an adaptation piece was constructed which consists of a suitably dimensioned steel tube. One side of the steel tube was connected to one flange of the sample through a steel stud and the other side was connected to the chamber cover.



Figure 5.2: Preparation of the connectors for the 6-inch flexible riser sample.

Since the large axial rotation is an important sign for the armor wire lateral instability, a straight line was marked on the sample surface so that the sample axial rotation after the test can be easily observed. Moreover, note that the outer diameter of the sample is smaller than the inner diameter of the hyperbaric chamber so that they may not deflect in the same manner during the test. To transfer the bending curvature of the chamber to the sample, several centralizers, which are constituted of several layers of rubber tapes, were installed on the sample. The outer diameters of those centralizers are slightly smaller than the inner diameter of the chamber so that the bending loads can be efficiently transferred to the sample. To facilitate the sample installation into the chamber as well as releasing the sample longitudinal degree of freedom during cyclic bending, the outer surfaces of the centralizers were lubricated with solid petroleum grease. Fig. 5.3 shows the layout of the centralizers, connectors and adaption piece.



Figure 5.3: Schematic of sample preparation.

Additionally, few pairs of strain gauges were installed along the sample in the intrados and extrados correspondingly. Using the strain difference measured by the strain gauges in the intrados and extrados of each cross-section, the bending curvature of the corresponding cross-section can be calculated. Meanwhile, to verify if the hyperbaric chamber and the sample experience similar bending curvatures, the curvatures of the hyperbaric chamber during the test were measured as well through an optical system. After all the preparations finished, the sample was installed into the chamber with one end fixed on the chamber cover and the other end free to move. The cables of the strain gauges exit the chamber through the penetrators installed in the chamber cover in the movable side.

After the accomplishment of the sample installation, the pressure in the chamber was increased to 2500 *psi* and cyclic bending was applied in the horizontal plane between the straight and the maximum bent positions. According to the measurements of the strain gauges and the optical system, the bending curvatures experienced by the sample and the hyperbaric chamber are very close and the maximum bending curvature is approximately $1/10 \ m^{-1}$ which locates near the fixed end. After the accomplishment of 1500 bending cycles, the sample was removed from the hyperbaric chamber and no obvious twist nor radial deformation were observed in the sample, indicating that neither the armor wire lateral instability nor the birdcage took place.

5.2 Test of the 6" flexible flowline sample

Similar to the previous test, the connectors installed on the previous sample were reutilized on the 6" flexible flowline sample and six centralizers were placed to transfer the bending curvature of the chamber to the sample. A series of strain gauges were installed on this sample in the same manner as the previous sample. Since this sample has the required length for the flex-compression test, the adaption piece was not needed. It needs to be noted that, at the beginning of the test, the datasheet of the 6" flexible flowline was not received and it was supposed that the collapse pressure is higher than 2500 psi as the previously tested 6" flexible riser sample. Thereafter, the sample was installed in the hyperbaric chamber and a trial test was carried out with 2500 psi. However, before the hydrostatic pressure reached 2500 psi, a sudden drop in the pressure was detected. Then, the sample was removed from the chamber and the collapse was observed close to one sample end as depicted in Fig. 5.4. After the occurrence of collapse, the datasheet was obtained showing that the collapse pressure is 2420 psi, which is lower than the applied hydrostatic pressure.



Figure 5.4: Collapse of the carcass.

Since the collapse was located at one end of the sample, it was then decided to cut the collapsed section and re-terminate the sample. After the cut, the sample has a total length of $6.264 \ m$ (flange to flange). The previously constructed adaption piece was reutilized to compensate for the sample length. Thereafter, the sample was installed into the hyperbaric chamber again and pressurized with 1000 *psi*, and 1000 bending cycles were applied in the same manner as the previous test. After the test, the sample was removed from the chamber. While no obvious dislocations were found in the radial direction of the sample, a significant twist was observed in the sample close to the fixed end, see the marked line in Fig. 5.5, indicating that the armor wire lateral instability may have taken place.

Subsequently, to check the deflections of the armor wires, the sample was dissected. To preserve the instability shape of the armor wires as much as possible, a



Figure 5.5: Sample appearance after the test (close to the fixed end).

number of steel straps were installed on the sample which evenly divides the sample into 9 sections. After the dissection of the outer sheath and the anti-birdcage tapes, the outer armor wire layer was exposed where no significant dislocations were observed. Thereafter, the outer armor layer was measured and removed and the inner armor layer was exposed. As expected, severe transverse deflections were observed in the inner armor wire layer, evidencing that the flexible flowline sample had failed due to the lateral instability in this layer. Fig. 5.6 shows the inner armor wire layer configuration of each section where the up and down directions are respectively the intrados and extrados during the test.



Figure 5.6: Configuration of the inner armor layer in the 6" flowline sample after the test.

5.3 Discussions

5.3.1 Deflection shape

Despite the fact that the present analytical model is not capable of describing the armor wire layer configuration after the formation of the lateral instability failure, it is conceivable that the armor wire deflection in the post-instability stage should follow the deflection tendency in the equilibrium state when the axial compressive load is close to the stability limit. Comparing Fig. 5.6 and Fig. 4.3, however, it seems that the armor wire lateral instability of the 6" flexible flowline sample does not follow the S-shape failure mode as shown in Fig. 4.3. For instance, the armor wires are squeezed in the intrados and loosened in the extrados in section 3 of Fig. 5.6, which is contrary to the analytical prediction. Nevertheless, in fact, the lateral instability modes depicted in Fig. 5.6 and Fig. 4.3 are the same. The configuration difference is due to the reason that they reveal different stages after the formation of S-shape failure. In Fig. 4.3 (right), the armor wires were in the initial stage of post-instability that the deflections of the armor wires and the pipe axial rotation were relatively small. Thus, a good correlation was obtained between the analytical prediction and the experimental observation. In contrast, severe deflections were generated in the inner armor wires in sections 1-4 of Fig. 5.6 and a large axial rotation along the helix direction of the inner armor wires had thus been triggered as illustrated in Fig. 5.5, indicating that the armor wire deflection had been fully developed in the post-instability stage. Due to the large axial rotation, the locations of squeezed and loosened regions were thus transferred as shown in section 3 of Fig. 5.6. However, in the section close to the fixed end, see section 1 in Fig. 5.6, it is very clear that the armor wires were squeezed in the extrados and loosened in the intrados, which agrees with the deflection tendency revealed by the present analytical model.

Besides, it is also interesting to observe from Fig. 5.6 that severe armor wire deflections were concentrated in sections 1-4 while the armor wire deflections in the other sections were relatively small. This is due to the fact that the hyperbaric chamber was not uniformly bent and the maximum bending curvature located near the fixed end. This is corresponding to a state that the armor wires slide faster when experiencing larger bending curvatures.

5.3.2 Lateral contact

Since large deflections were generated in the inner armor wires in the post-instability stage, as expected, lateral contacts between the neighboring armor wires were generated. Fig. 5.7 (left) and (right) show respectively the partial enlargements of the

inner armor wire configurations in sections 1 and 3 of Fig. 5.6, which correspond respectively to the conditions depicted in Fig. 4.18 (left) and (right). Section 1 is close to the connector so that the radial expansions of the armor wires are strongly restricted. Consequently, the armor wires have no sufficient space to rotate around their own axes when reaching in contact, see Fig. 5.7 (left). On the other hand, section 3 is far from the connector so that the radial restriction from the connector was negligible. Also, the radial stiffness of the anti-birdcage tapes in this sample was not sufficiently big so that a relatively big gap between the armor wires and the pipe core was generated in this section when the sample was subjected to large axial compressive loads. This gap allows the armor wires to twist and thus results in the overlap of the armor wires as depicted in Fig. 5.7 (right). Thus, the armor wires were still able to move laterally when they were in contact. Since the connectors only affect the sections close to the ends, the majority of the sample sections were in a similar condition as section 3. As discussed in section 4.6, under this circumstance, the effect of the lateral contact on the armor wire lateral stabilization may not be significant. Thus, the lateral stability limit may be approximately estimated by Eq. 3.83, which will be verified in the following section.



Figure 5.7: Lateral contact of armor wires in section 1 (left) and section 3 (right).

5.3.3 Lateral stability limit

Hereafter, the analytical prediction of the flexible pipe armor wire lateral stability limit given by Eq. 3.83 will be compared with the results from the present tests and other existing tests available in the literature. BRAGA [52] conducted a series of tests using mechanical rigs concerning flexible pipe armor wire lateral instability in three different loading conditions: monotonic longitudinal compression without bending, cyclic longitudinal compression without bending, and constant longitudinal compression combined to cyclic bending. Since the present analytical model concerns the last loading condition, only the results of the tests in the last loading condition are used for the analytical model verification. Similar tests were also conducted by ØSTERGAARD [22] through mechanical rigs at atmospheric pressure. The armor layers characteristics of the flexible pipe samples tested by them are presented in Table 5.2. Their test conditions and results, together with that of the present tests, are presented in Table 5.3, in which the axial compressive loads for the present tests are calculated by multiplying the hydrostatic pressure by the cross-sectional area of the fluid barrier.

Using Eq. 3.83, the lateral stability limits for each sample are evaluated and listed in Table 5.3 and compared with the test results as shown in Fig. 5.8, where the area above the dashed line represents the conditions for which the longitudinal compressive loads are larger than the predicted lateral stability limits. Fig. 5.8 shows excellent agreement between the analytical predictions and the test data as all the cases above the dashed line failed while no failures were observed in the cases below the dashed line.

| | Parameters | 4" flowline | 6" riser [22] | 8" riser [22] | 14" jumper |
|------------------|-------------------------|----------------|---------------|-----------------|---------------|
| | | [52] | | | [22] |
| Inner armor | Outer diameter (mm) | 141 | 201 | 276 | 442 |
| | Lay angle (deg) | 35 | 26.2 | 30 | 31.5 |
| | Wire size (mm) | 2.5×7 | 3×10 | 5×12.5 | 4×15 |
| | Number of wires | 49 | 52 | 54 | 70 |
| Outer armor | Outer diameter (mm) | 146 | 209 | 289 | 452 |
| | Lay angle (deg) | -35 | -26.2 | -30.3 | -31 |
| | Wire size (mm) | 2.5×7 | 3×10 | 5×12.5 | 4×15 |
| | Number of wires | 50 | 54 | 56 | 72 |
| Steel properties | Young's Modulus (GPa) | 210 | 210 | 210 | 210 |
| | Yield stress (MPa) | 1350 | 650 | 1350 | 1350 |
| | Poisson's ratio | 0.3 | 0.3 | 0.3 | 0.3 |
| Pitch number | | 3.74-3.91 | 3.96 | 3.39 | 3.34 |
| of inner armor | | | | | |

Table 5.2: Tensile armors characteristics.
| Pipe ID | Case number | $\kappa_{max} \times r$ (10^{-2}) | Applied compression | Results | $ P_{LI} \ (kN)$ |
|-------------------------|----------------|-------------------------------------|------------------------|------------|-------------------|
| | indifficient | (10) | $P_{test} (kN)$ | | |
| 4" flowline [52] | 1 | 1.4 | 243 | Failure | 164 |
| | 2 | 1.6 | 242 | Failure | |
| | 3 | 2.6 | 242 | Failure | |
| | 4 | 2.6 | 174 | Failure | |
| 6" riser (Li et al.) | 5 | 1.1 | 451 | No failure | 993 |
| 6" flowline (Li et al.) | 6 | 1.0 | 203 | Failure | 100 |
| 6" riser [22] | 7 | 2.0 | 265 | Failure | 201 |
| | 8 | 0.9 | 265 | Failure | |
| | 9 | 0.9 | 80 | No failure | |
| | 10 | 0.9 | 210 | Failure | |
| | 11 | 0.9 | 160 | No failure | |
| | 12 | 1.2 | 265 | Failure | |
| 8" riser [22] | 13 | 1.1 | 700 | Failure | 474 |
| | 14 | 1.1 | 300 | No failure | |
| | 15 | 1.1 | 400 | No failure | |
| 14" jumper [22] | 16 | 1.2 | 277 | No failure | 308 |
| | 17 | 3.1 | 269 | No failure | |
| | 18 | 2.4 | 411 | Failure | |
| | 19 | 1.8 | 950 | Failure | |

Table 5.3: Tests conditions and results.



Figure 5.8: Analytical predictions versus test data.

Chapter 6

Design recommendations

Based on the present analytical model, measures to improve the flexible pipe design against the armor wire lateral instability failure are discussed in this chapter.

Note that armor wires in flexible pipes are made of steel and the radii of armor wires depend on the flexible pipe functional design. Thus, except for the armor wire radius and material property, the remaining factors that we can play with include the armor wire lay angle, cross-sectional dimension and the number of armor wires. Note that the armor wire layer filling factor is generally around 90%. Thus, the armor wire width, lay angle and the number of armor wires within a layer are related. Besides, solely increasing the armor wire thickness without modifying the filling factor will increase the flexible pipe weight and outer diameter, which will in return raise the costs and operational difficulty. Based on the discussion above, to modify the armor wire layer design without increasing the flexible pipe weight and outer diameter, a feasible method can be employed by fixing the armor wire thickness, modifying the armor wire lay angle and width and correspondingly adjusting the number of armor wires to keep the armor wire layer filling factor approximately fixed.

Firstly, the effect of the armor wire lay angle on the lateral stability limit is discussed. It is considered that the inner and outer armor wires have opposite lay angles. Using the armor wire layer properties of those six flexible pipes presented in Table 5.1 and Table 5.2, varying the lay angle of the inner armor wire from 20° to 60° and adjusting the number of armor wires in both inner and outer armor wire layers to keep the filling factors approximately fixed, the corresponding lateral stability limit variations are shown in Fig. 6.1. It can be observed that, for all those six flexible pipes, the maximum lateral stability limits are obtained when the armor wire lay angle is approximately 40° . Either increasing or decreasing the armor wire lay angle from 40° reduces the lateral stability limit.

Subsequently, the effect of the armor wire width on the lateral stability limit is investigated. Still using the armor wire layer properties of those six flexible pipes presented in Table 5.1 and Table 5.2, modifying the armor wire widths and adjusting



Figure 6.1: Effect of armor wire lay angle on the lateral stability limit.

the number of armor wires in both inner and outer armor wire layers to keep the filling factors approximately fixed, the lateral stability limit variations are shown in Fig. 6.2. It can be seen that, for all those six flexible pipes, the lateral stability limits increase monotonically as the armor wire widths increase. However, it needs to be noted that the armor wire width cannot be too large as it would be difficult to fit the armor wires into the toroidal wall formed by the neighboring layers.



Figure 6.2: Effect of armor wire width on the lateral stability limit.

Based on the discussions above, it can be concluded that the flexible pipe ar-

mor wire lateral stability limit can, to some extent, be improved by adjusting the armor wire lay angle and increasing its width. Meanwhile, the number of armor wires needs to be adjusted correspondingly to keep the filling factor approximately fixed. Besides, it needs to be noted that modifying the design of the armor wire layers affects the responses of flexible pipes in bending and tension which should be carefully counterbalanced.

Chapter 7

Concluding remarks

The goal of this Ph.D. thesis has been to improve the understanding of the armor wire lateral instability mechanism in unbonded flexible pipes, as well as to evaluate the critical loading conditions that may cause the armor wire lateral instability, and most importantly, to provide the guidance for flexible pipe design against the armor wire lateral instability.

In the present context, a flexible pipe in the wet annulus condition subjected to a constant axial compressive load combined to cyclic bending with uniform curvatures has been investigated. A single armor wire within the wall of the flexible pipe has been modeled as a thin curved beam within a frictional toroidal wall through six coupled Love's differential equations. Thereafter, a perturbation technique has been proposed to approximately linearize this coupled differential equation system based on the following observations and assumptions:

- (i) The bending radius of a flexible pipe is usually much larger than the flexible pipe radius;
- (ii) The deflection of the armor wire, as well as the flexible pipe axial rotation and shortening, are small when no lateral instability takes place;
- (iii) The deflection of the armor wire is deemed periodical in the section far away from the pipe ends;
- (iv) The lateral contact of neighboring armor wires is ignored;
- (v) The shear deformation of the underlying polyamide anti-wear tape is ignored;
- (vi) The entire armor wire is assumed sliding immediately when bending is applied. This is equivalent to an assumption that full dynamic frictional forces are applied on the entire armor wire;
- (vii) The armor wire sliding is assumed primarily axial in dynamic bending.

Through this perturbation technique, the frictional force components in the tangential and bi-normal directions on each section of the armor wire are identified. Based on the geometrical relation between the armor wire and the underlying toroid, and considering the sliding direction is opposite to the direction of frictional force, the armor wire instantaneous sliding directions and rates with respect to the bending curvature are approximately evaluated. Instead of describing the armor wire path evolution in cyclic bending, the present study focuses on the armor wire ultimate state, stable or unstable, after a significant number of bending cycles. By discussing the convergence of the recursive formula of the armor wire paths after each bending cycle, the armor wire ultimate equilibrium path after numerous bending cycles in the converged state, as well as the critical axial compressive load causing the non-converged state, both are obtained. If the axial compressive load surpasses this critical load, the armor wire would migrate in the lateral direction constantly after each bending cycle, and eventually, develop into a lateral instability failure due to material yielding. Thereafter, considering that the axial compressive load carried by each armor wire is approximately the same due to symmetry, the critical axial compressive load on a flexible pipe that may cause the armor wire lateral instability has been evaluated by multiplying the limit load for a single armor wire by the total number of armor wires.

According to the present analytical model, it is interesting to note that, despite the fact that the magnitudes of frictional forces and cyclic bending curvatures may have significant effects on the armor wire marching towards the ultimate state, their effects on the lateral stability limit are negligible. The armor wire lateral stability limit depends solely on the geometrical and material properties of the armor wire layers.

To calibrate and validate the present analytical model, two 6-inch flexible pipe samples were tested through a hyperbaric chamber where high hydrostatic pressure and numerous bending cycles were applied. While no failure was observed in the pipe sample constituted of relatively bigger armor wires, the lateral instability failure was successfully reconstructed in another sample constituted of smaller armor wires. Excellent agreement was observed when comparing the analytical estimation of the lateral stability limit with the present test data, as well as the available test data in the literature.

Additionally, on the basis of the present analytical model, some useful suggestions for improving the armor wire layer design against the lateral instability failure have been proposed, including adjusting the armor wire lay angle, increasing the armor wire width, and correspondingly adjusting the number of armor wires to keep the filling factor approximately fixed.

7.1 Recommendations for future research

To extend and improve the understanding of the armor wire lateral instability mechanism in unbonded flexible pipes, the following proposals are suggested for future research:

- The present model is only applicable for a flexible pipe subjected to a constant axial compressive load combined to cyclic bending with uniform curvatures. Nevertheless, in practical applications of flexible pipes, cyclic axial compression and bending with non-uniform curvatures are usually experienced in the touchdown zone, which may affect the armor wire progression towards the ultimate state as well as the critical conditions that may cause the armor wire lateral instability failure. Thus, further research is needed to investigate flexible pipe armor wire lateral instability mechanism in such a general loading condition;
- Besides, the number of bending cycles for the triggering of armor wire lateral instability also deserves further investigations. In practical applications of flexible pipes, the varying amplitudes of the cyclic bending curvatures in the touchdown zone are usually very small. Thus, even though the axial compressive load is larger than the critical load, the lateral instability failure may not take place after a limited number of bending cycles. For a flexible pipe subjected to an axial compressive load larger than the critical load, the evaluation of the necessary bending cycles for the triggering of lateral instability would be useful for the estimation of the safe installation/operation time based on the sea condition;
- Moreover, note that the neighboring armor wires within the same layer may be in contact due to large displacements before reaching the yielding limit. Such lateral contact may, to some extent, increase the armor wire stabilization, especially when the anti-birdcage tapes are significant stiff since it prevents the overlap of the armor wires in the contact sections and thus retards them to continuously move in the lateral direction. Further research is needed to investigate the effect of lateral contact on the armor wire stabilization.

Bibliography

- CALEYRON, F., LEROY, J.-M., GUITON, M., et al. "Stresses in tensile armour layers of unbounded flexible risers loaded with external pressure: application to lateral buckling mode". In: *Proceedings of the 36th International Conference on Ocean, Offshore and Arctic Engineering*, p. V05AT04A006, Trondheim, Norway, 2017. American Society of Mechanical Engineers. OMAE2017-61133.
- [2] HOFFMAN, D., ISMAIL, N., NIELSEN, R., et al. "The design of flexible marine risers in deep and shallow water". In: Offshore Technology Conference, Houston, Texas, USA, 1991. OTC-6724-MS.
- [3] API 17J. "Specification for Unbonded Flexible Pipe", 2008.
- [4] API 17B. Recommended practice for flexible pipe, latest ed. American Petroleum Institute, Washington, 2014.
- [5] API 17L1. "Specification for Flexible Pipe Ancillary Equipment", 2013.
- [6] API 17L2. "Recommended Practice for Flexible Pipe Ancillary Equipment", 2013.
- [7] GAUTAM, M., KATNAM, K., POTLURI, P., et al. "Hybrid composite tensile armour wires in flexible risers: a multi-scale model", *Composite Structures*, v. 162, pp. 13–27, 2017.
- [8] DO, A. T., LAMBERT, A. "Qualification of unbonded dynamic flexible riser with carbon fibre composite armors". In: Offshore Technology Conference, Houston, Texas, USA, 2012. OTC-23281-MS.
- [9] HANONGE, D., BERNARD, G., DO, A. T. "Carbon Fiber Armors Applied to Presalt Flexible Pipe Developments". In: Offshore Technology Conference, Houston, Texas, USA, 2013. OTC-24036-MS.
- [10] LAMBERT, A., DO, A.-T., FELIX-HENRY, A., et al. "Qualification of Unbonded Dynamic Riser With Carbon Fiber Composite Armours". In: Pro-

ceedings of the 31st International Conference on Ocean, Offshore and Arctic Engineering, v. 3, pp. 117–125, Rio de Janeiro, Brazil, 2012. American Society of Mechanical Engineers.

- [11] VIDIGAL DA SILVA, J., DAMIENS, A. "3000 m Water Depth Flexible Pipe Configuration Portfolio". In: Offshore Technology Conference, Houston, Texas, USA, 2016. OTC-26933-MS.
- [12] SOUSA, J. R. M., VIERO, P. F., MAGLUTA, C., et al. "An experimental and numerical study on the axial compression response of flexible pipes", *Journal of Offshore Mechanics and Arctic Engineering*, v. 134, n. 3, pp. 031703, 2012.
- [13] LI, X., VAZ, M. A., CUSTÓDIO, A. B. "Analytical study of a repair methodology for flexible pipes anti-birdcage tapes", *Marine Structures*, v. 63, pp. 289–303, 2019.
- [14] RABELO, M. A., PESCE, C. P., SANTOS, C. C. P., et al. "An investigation on flexible pipes birdcaging triggering", *Marine Structures*, v. 40, pp. 159– 182, 2015.
- [15] SÆVIK, S., THORSEN, M. J. "An analytical treatment of buckling and instability of tensile armors in flexible pipes", Journal of Offshore Mechanics and Arctic Engineering, v. 139, n. 4, pp. 041701, 2017.
- [16] BORGES, M. F., TALGATTI, O. L., MOSQUEN, A. "Radial instability of flexible pipes with defects in the high resistance bandage and external sheath". In: Proceedings of the 36th International Conference on Ocean, Offshore and Arctic Engineering, v. 5A, p. V05AT04A025, Trondheim, Norway, 2017. American Society of Mechanical Engineers. OMAE2017-61850.
- [17] MALTA, E. R., DE ARRUDA MARTINS, C. "Finite element analysis of flexible pipes under compression". In: Proceedings of the 33rd International Conference on Ocean, Offshore and Arctic Engineering, v. 6A, p. V06AT04A010, San Francisco, California, USA, 2014. American Society of Mechanical Engineers. OMAE2014-23192.
- [18] BRAGA, M. P., KALEFF, P. "Flexible pipe sensitivity to birdcaging and armor wire lateral buckling". In: Proceedings of the 23rd International Conference on Offshore Mechanics and Arctic Engineering, v. 1, pp. 139– 146, Vancouver, British Columbia, Canada, 2004. American Society of Mechanical Engineers. OMAE2004-51090.

- [19] BECTARTE, F., COUTAREL, A. "Instability of tensile armour layers of flexible pipes under external pressure". In: Proceedings of the 23rd International Conference on Offshore Mechanics and Arctic Engineering, v. 3, pp. 155–161, Vancouver, British Columbia, Canada, 2004. American Society of Mechanical Engineers. OMAE2004-51352.
- [20] CUSTÓDIO, A. B., LEMOS, C. A. D., TROINA, L. M. B., et al. "Recent research on the instability of flexible pipe's armours". In: *Proceedings of the* 17th International Offshore and Polar Engineering Conference, Lisbon, Portugal, 2007.
- [21] SECHER, P., BECTARTE, F., FELIX-HENRY, A. "Lateral buckling of armor wires in flexible pipes: reaching 3000 m water depth". In: Proceedings of the 30th International Conference on Ocean, Offshore and Arctic Engineering, v. 4, pp. 421–428, Rotterdam, The Netherlands, 2011. American Society of Mechanical Engineers. OMAE2011-49447.
- [22] ØSTERGAARD, N. H. On lateral buckling of armouring wires in flexible pipes (Ph.D. thesis). Department of Mechanical and Manufacturing Engineering, Aalborg University, 2012.
- [23] GARCÍA, R., TYRBERG, A., ALVARADO, L. F., et al. "Testing submarine cables for combined axial compression and bending loads". In: *Proceedings* of the 9th International Conference on Insulated Power Cables, Jicable'15, Versailles, France, 2015.
- [24] OUT, J. M. M., VON MORGEN, B. J. "Slippage of helical reinforcing on a bent cylinder", *Engineering Structures*, v. 19, n. 6, pp. 507–515, 1997.
- [25] SÆVIK, S. "A finite element model for predicting stresses and slip in flexible pipe armouring tendons", *Computers & structures*, v. 46, n. 2, pp. 219– 230, 1993.
- [26] WITZ, J. A., TAN, Z. "On the axial-torsional structural behaviour of flexible pipes, umbilicals and marine cables", *Marine Structures*, v. 5, n. 2-3, pp. 205–227, 1992.
- [27] KRAINCANIC, I., KEBADZE, E. "Slip initiation and progression in helical armouring layers of unbonded flexible pipes and its effect on pipe bending behaviour", *The Journal of Strain Analysis for Engineering Design*, v. 36, n. 3, pp. 265–275, 2001.
- [28] SÆVIK, S. "Theoretical and experimental studies of stresses in flexible pipes", Computers & Structures, v. 89, n. 23-24, pp. 2273–2291, 2011.

- [29] CUSTÓDIO, A. B. Modelo analítico para avaliação de instabilidade nas armaduras de dutos flexíveis (Ph.D. thesis). Department of Ocean Engineering, Federal University of Rio de Janeiro, 2005, (in Portuguese).
- [30] BRACK, M., TROINA, L. M. B., SOUSA, J. R. M. "Flexible riser resistance against combined axial compression, bending, and torsion in ultra-deep water depths". In: *Proceedings of the 24th International Conference on Offshore Mechanics and Arctic Engineering*, v. 1, pp. 821–829, Halkidiki, Greece, 2005. American Society of Mechanical Engineers. OMAE2005-67404.
- [31] TAN, Z., LOPER, C., SHELDRAKE, T., et al. "Behavior of tensile wires in unbonded flexible pipe under compression and design optimization for prevention". In: Proceedings of the 25th International Conference on Offshore Mechanics and Arctic Engineering, v. 4, pp. 43–50, Hamburg, Germany, 2006. American Society of Mechanical Engineers. OMAE2006-92050.
- [32] VAZ, M. A., RIZZO, N. A. S. "A finite element model for flexible pipe armor wire instability", *Marine Structures*, v. 24, n. 3, pp. 275–291, 2011.
- [33] LI, X. Application of finite element method for axisymmetric analysis of flexible pipe armor wires instability (Master's thesis). Department of Ocean Engineering, Federal University of Rio de Janeiro, 2014.
- [34] YANG, X., SÆVIK, S., SUN, L. "Numerical analysis of buckling failure in flexible pipe tensile armor wires", *Ocean Engineering*, v. 108, pp. 594– 605, 2015.
- [35] ØSTERGAARD, N. H., LYCKEGAARD, A., ANDREASEN, J. H. "A method for prediction of the equilibrium state of a long and slender wire on a frictionless toroid applied for analysis of flexible pipe structures", *Engineering Structures*, v. 34, pp. 391–399, 2012.
- [36] ØSTERGAARD, N. H., LYCKEGAARD, A., ANDREASEN, J. H. "Imperfection analysis of flexible pipe armor wires in compression and bending", *Applied Ocean Research*, v. 38, pp. 40–47, 2012.
- [37] ØSTERGAARD, N. H., LYCKEGAARD, A., ANDREASEN, J. H. "On modelling of lateral buckling failure in flexible pipe tensile armour layers", *Marine Structures*, v. 27, n. 1, pp. 64–81, 2012.
- [38] PAIVA, L. F., VAZ, M. A. "An empirical model for flexible pipe armor wire lateral buckling failure load", *Applied Ocean Research*, v. 66, pp. 46–54, 2017.

- [39] GONZALEZ, G. M. Investigação do mecanismo de instabilidade lateral dos arames das armaduras de tração de dutos flexíveis (Ph.D. thesis). Department of Civil Engineering, Federal University of Rio de Janeiro, 2018, (in Portuguese).
- [40] MALTA, E. R., DE ARRUDA MARTINS, C. "Finite element analysis of flexible pipes under axial compression: influence of the sample length", *Journal* of Offshore Mechanics and Arctic Engineering, v. 139, n. 1, pp. 011701, 2017.
- [41] MALTA, E. R., DE ARRUDA MARTINS, C. "Finite element analysis of flexible pipes under compression: influence of the friction coefficient", *Journal of Offshore Mechanics and Arctic Engineering*, v. 141, n. 6, pp. 061601, 2019.
- [42] SÆVIK, S., THORSEN, M. J. "Techniques for predicting tensile armour buckling and fatigue in deep water flexible risers". In: Proceedings of the 31st International Conference on Ocean, Offshore and Arctic Engineering, v. 3, pp. 469–482, Rio de Janeiro, Brazil, 2012. American Society of Mechanical Engineers. OMAE2012-83563.
- [43] SÆVIK, S., LI, H. "Shear interaction and transverse buckling of tensile armours in flexible pipes". In: Proceedings of the 32nd International Conference on Ocean, Offshore and Arctic Engineering, v. 4A, p. V04AT04A013, Nantes, France, 2013. American Society of Mechanical Engineers.
- [44] SÆVIK, S., JI, G. "Differential equation for evaluating transverse buckling behavior of tensile armour wires". In: Proceedings of the 33rd International Conference on Ocean, Offshore and Arctic Engineering, v. 6B, p. V06BT04A021, San Francisco, California, USA, 2014. American Society of Mechanical Engineers.
- [45] LI, X., VAZ, M. A., CUSTÓDIO, A. B. "Analytical prediction for lateral buckling of tensile wires in flexible pipes", *Marine Structures*, v. 61, pp. 268–281, 2018.
- [46] ØSTERGAARD, N. H., LYCKEGAARD, A., ANDREASEN, J. "Simulation of frictional effects in models for calculation of the equilibrium state of flexible pipe armouring wires in compression and bending", *Rakenteiden Mekaniika (Journal of Structural Mechanics)*, v. 44, n. 3, pp. 243–259, 2011.
- [47] ZHOU, C., SÆVIK, S., YE, N., et al. "Effect of lay angle of anti-buckling tape on lateral buckling behavior of tensile armors". In: *Proceedings of the*

34th International Conference on Ocean, Offshore and Arctic Engineering, v. 5A, p. V05AT04A040, St. John's, Newfoundland, Canada, 2015. American Society of Mechanical Engineers. OMAE2015-41527.

- [48] CALEYRON, F., GUITON, M., LEROY, J.-M., et al. "A multi-purpose finite element model for flexible risers studies". In: *Proceedings of the 33rd International Conference on Ocean, Offshore and Arctic Engineering*, v. 6A, p. V06AT04A015, San Francisco, California, USA, 2014. American Society of Mechanical Engineers. OMAE2014-23250.
- [49] LUKASSEN, T. V. Constitutive behavior of tensile armor wires in unbonded flexible pipes (Ph.D. thesis). Department of Mechanical Engineering, Technical University of Denmark, 2019.
- [50] LOVE, A. E. A treatise on the mathematical theory of elasticity. Cambridge: Cambridge University Press, 1944.
- [51] TIMOSHENKO, S., GOODIER, J. N. Theory of elasticity. McGraw-Hill book Company, 1951.
- [52] BRAGA, M. J. P. Instabilidade de armaduras de tração de linhas flexível (Ph.D. thesis). Department of Ocean Engineering, Federal University of Rio de Janeiro, 2003, (in Portuguese).