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PUBLIC DEBT MATURITY STRUCTURE IN BRAZIL: A STOCK-FLOW CONSISTENCY (SFC) ANALYSIS

Rio de Janeiro
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Dissertação apresentada ao Programa de Pós-Graduação em Economia da Universidade Federal do Rio de Janeiro — UFRJ, como requisito parcial à obtenção do grau de Mestre em Economia.

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Rio de Janeiro
2025

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RESUMO

Essa dissertação investiga os impactos e a dinâmica da estrutura de maturação da dívida pública no âmbito do arcabouço institucional brasileiro. Com tal objetivo, construiu-se um modelo de consistência de fluxos e estoques (*SFC*, conforme acrônimo em língua inglesa) que reproduz, de maneira estilizada, o arcabouço institucional brasileiro no que tange a oferta e demanda por títulos públicos federais para explicitar a dinâmica de maturação da dívida, produzindo comportamento endógeno cíclico. Pelo nosso conhecimento, contabilizar explicitamente maturação de dívida é uma contribuição original à literatura *SFC*. O modelo é utilizado para avaliar efeitos secundários de três principais choques de política: (i) mudanças na política monetária — um corte (ou aumento) da taxa de juros de política monetária; (ii) expansão ou contração fiscal por meio do gasto público; (iii) expansão ou contração fiscal por meio de mudanças na alíquota de tributação. Também analisamos um choque no parâmetro de sensibilidade da escolha de portfólio dos detentores de títulos. Além disso, comparamos as dinâmicas da dívida-PIB sob cenários alternativos para a estrutura de maturação inicial. Os resultados mostram que choques de política monetária afetam apenas o preço dos títulos, sem impactar as variáveis reais ou a dívida; o crescimento do gasto público reduz a razão dívida-PIB, enquanto redução de tributação aumentam as razões dívida-PIB; e alterações na sensibilidade dos detentor à rentabilidade dos títulos são neutras. De forma geral os resultados indicam que contabilizar explicitamente as maturações de dívida introduz singificativa volatilidade ao sistema, mesmo considerando uma demanda por títulos públicos estável por parte do setor privado e sob a hipótese de banco central como comprador de última instância para os títulos públicos excedentes. Tal aspecto torna a modelagem proposta apropriada para analisar economias emergentes cujas estruturas de maturação exercem um papel importante na necessidade de financiamento do governo.

Keywords: dívida pública; estrutura de maturação; modelos de consistência fluxo-estoque; SFC; sustentabilidade da dívida; Brasil.

ABSTRACT

This dissertation investigates the impacts and dynamics of government debt maturity structure within Brazil's institutional framework. To accomplish that, a Stock-Flow Consistent (SFC) model that reproduces in a stylized manner the Brazilian institutional framework regarding supply and demand for federal government bonds is built to explicit debt maturities dynamics, producing endogenous cyclical behavior. As far as we are aware, explicitly accounting for debt maturities is an original contribution to SFC literature. The model is used to assess the secondary effects of three key policy shocks: (i) monetary policy changes – a cut (or increase) in monetary policy rate; (ii) fiscal expansion or contraction through government expenditure; (iii) fiscal loosening or tightening through tax rate changes. We also analyze a shock on bondholders' portfolio choice sensitivity parameter. Additionally, we compare debt-to-GDP dynamics under alternative scenarios for initial maturity structure. The results show that monetary policy shocks only affect the price of bills, without impacting real economic variables or debt; accelerating government spending lowers debt-to-GDP ratio, while tax rate cuts raise debt-to-GDP ratios, and altering bondholder's sensitivity to bill's return are neutral. Overall results indicate that explicitly accounting for debt maturities introduces significant volatility to the system, even considering a stable private sector demand for government bills and central bank as a last-resort buyer for excess government debt. This aspect renders the modelling appropriate for analyzing emerging market economies whose maturities structures play an important role in government borrowing needs.

Keywords: public debt; maturity structure; stock-flow consistent models; SFC; debt sustainability; Brazil.

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LIST OF EQUATIONS

PCMS model equations are in bold, the remaining being auxiliary equations.

$\boldsymbol{G = G_{-1} \cdot (1 + g)}$	**(1)**
$\boldsymbol{r_b = \overline{r_b}}$	**(2)**
$p_b = principal \cdot (1 + \Delta r_b) \cdot premium$	(3)
$p_b = 10^{-6} \cdot (1 + \Delta r_b)$	(3a)
$principal = 10^{-6}$	(4)
$\boldsymbol{p_b = p_{b_{-1}} \cdot e^{\frac{r_b}{252}}}$	**(5)**
or $\boldsymbol{p_b = p_{b_{-1}} \cdot \left(1 + \frac{r_b}{252}\right)}$	**(5a)**
$V = H + p_b \cdot B_h$	(6)
$D \equiv p_b \cdot B_h + B_{cb}$	(7)
$V = D$	(8)
$D = H + p_b \cdot B_h$	(9)
$\boldsymbol{B_{cb} = H}$ **(hidden equation)**	**(10)**
$\boldsymbol{Y \equiv C + G}$	(11)
$\boldsymbol{T = \theta \cdot \left(Y + p_{b_{-1}} \cdot B_m\right)}$	(12)
$\Delta H_s = \Delta H_d$	(13)
$\boldsymbol{\Delta B_{cb} = G - T + p_{b_{-1}} \cdot B_m - \left(p_b \cdot B_{net} - p_{b_{-1}} \cdot B_m\right)}$ **or** $\boldsymbol{\Delta B_{cb} = G - T + 2 \cdot p_{b_{-1}} \cdot B_m - (p_b \cdot B_{net})}$	**(14)**
$p_d \cdot D - p_{d_{-1}} \cdot D_{-1} = (\Delta p_d \cdot D_{-1}) + (\Delta D \cdot p_d)$	(15)
$\Delta B_s \equiv B_i - B_r - B_m + \Delta B_{bc} = B_{net} - B_m + \Delta B_{bc}$	(16)
$\Delta B_d \equiv \Delta B_h + \Delta B_{bc}$	(17)
$\Delta B_h \equiv B_h - B_{h_{-1}}$	(18)
$\Delta B_h \equiv B_{net} - B_m$	(19)
$B_h = B_{h_{-1}} + B_{net} - B_m$	(20)
$\boldsymbol{B_{net} = \max\left\{B_{h_d} - B_{h_{-1}} + B_m; B_m - B_{h_{-1}}\right\}}$	**(21)**
$\boldsymbol{CG = \Delta p_b \cdot \left(B_{h_{-1}} - B_m\right)}$	**(22)**
$\boldsymbol{YD = Y - T + p_{b_{-1}} \cdot B_m}$	**(23)**

$$V = V_{-1} + (YD - C) + CG$$	(24)
$$C = \alpha_1 \cdot YD + \alpha_2 \cdot V_{-1}$$	(25)
$$M = \{B_m; B_{m_1}; B_{m_2}; \dots; B_{m_n}\}$$	(26)
$$B_{m_n} = max(B_{net}; 0)$$	(27)
$$B_{m_7} = max\{B_{net}; 0\}$$	(27a)
$$B_m = \left(B_{m_1}\right)_{-1}$$	(28)
$$B_{m_1} = max\left\{\left(B_{m_2}\right)_{-1} - B_{nr}; 0\right\}$$	(29)
$$B_{m_2} = \begin{cases} \left(B_{m_3}\right)_{-1} \quad , \quad if\ B_{nr} \leq \left(B_{m_2}\right)_{-1} \\ max\left[\left(\left(B_{m_3}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1}\right)\right); 0\right] \quad , otherwise \end{cases}$$	(30)
$$B_{m_3} = \begin{cases} \left(B_{m_4}\right)_{-1} \quad , \quad if\ B_{nr} \leq \left(B_{m_2}\right)_{-1} + \left(B_{m_3}\right)_{-1} \\ max\left[\left(\left(B_{m_4}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1} - \left(B_{m_3}\right)_{-1}\right)\right); 0\right] \quad , otherwise \end{cases}$$	(31)
$$B_{m_4} = \begin{cases} \left(B_{m_5}\right)_{-1} \quad , \quad if\ B_{nr} \leq \left(B_{m_2}\right)_{-1} + \left(B_{m_3}\right)_{-1} + \left(B_{m_4}\right)_{-1} \\ max\left[\left(\left(B_{m_5}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1} - \left(B_{m_3}\right)_{-1} - \left(B_{m_4}\right)_{-1}\right)\right); 0\right] \quad , otherwise \end{cases}$$	(32)
$$B_{m_5} = \begin{cases} \left(B_{m_6}\right)_{-1} \quad , \quad if\ B_{nr} \leq \left(B_{m_2}\right)_{-1} + \left(B_{m_3}\right)_{-1} + \left(B_{m_4}\right)_{-1} + \left(B_{m_5}\right)_{-1} \\ max\left[\left(\left(B_{m_6}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1} - \left(B_{m_3}\right)_{-1} - \left(B_{m_4}\right)_{-1} - \left(B_{m_5}\right)_{-1}\right)\right); 0\right] \quad , otherwise \end{cases}$$	(33)
$$B_{m_6} = \begin{cases} \left(B_{m_7}\right)_{-1} \quad , \quad if\ B_{nr} \leq \left(B_{m_2}\right)_{-1} + \left(B_{m_3}\right)_{-1} + \left(B_{m_4}\right)_{-1} + \left(B_{m_5}\right)_{-1} + \left(B_{m_6}\right)_{-1} \\ max\left[\left(\left(B_{m_7}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1} - \left(B_{m_3}\right)_{-1} - \left(B_{m_4}\right)_{-1} - \left(B_{m_5}\right)_{-1} - \left(B_{m_6}\right)_{-1}\right)\right); 0\right] \quad , otherwise \end{cases}$$	(34)
$$B_{h_s} = \sum_{i=1}^{n} B_{m_i} = \underbrace{B_{m_1} + B_{m_2} + \cdots + B_{m_n}}_{n\ terms}$$	(35)
$$\frac{H}{V} = (1 - \lambda_0) - \lambda_1 \cdot ERr_b + \lambda_2 \left(\frac{YD}{V}\right)$$	(36)
$$\frac{B_{h_d} \cdot p_b}{V} = \lambda_0 + \lambda_1 \cdot ERr_b - \lambda_2 \cdot \left(\frac{YD}{V}\right)$$	(37)
$$H = V - p_b \cdot B_h$$	(38)
$$ERr_b = \ln\left(\frac{p_b}{p_{b_{-1}}}\right)$$	(39)
$$ERr_b = \frac{p_b}{p_{b_{-1}}} - 1$$	(39a)

LIST OF ABBREVIATIONS

BCB – Banco Central do Brasil or Central Bank of Brazil

DFPD – Domestic Federal Public Debt

DMO – Debt Management Office

EFPD – External Federal Public Debt

FPD – Federal Public Debt

FX – Foreign exchange rate

GDP – Gross Domestic Product

IBGE – Instituto Brasileiro de Geografia e Estatística

IMF – International Monetary Fund

IPCA – *Índice de Preços ao Consumidor Amplo* or broad consumer price index

LTN – Letras do Tesouro Nacional

LFT – Letras Financeiras do Tesouro

LRF – Lei de Responsabilidade Fiscal or Fiscal Responsibility Law

MDR – Monthly Debt Report

MMT – Modern Monetary Theory

NTN-B – Notas do Tesouro Nacional – Série B

NTN-F – Notas do Tesouro Nacional – Série F

PC – Portfolio Choice

PCMS – Portfolio Choice with Maturity Structure

SFC – Stock-Flow Consistent

VNA – Valor Nominal Atualizado or nominal accumulated value

LIST OF NOTATIONS

α_1	Propensity to consume out of regular income
α_2	Propensity to consume out of past wealth
b	Ratio of debt held by the Central Bank over GDP
$b1$	Ratio of federal public debt maturing within one year
$b2$	Ratio of federal public debt maturing between one and two years
$b3$	Ratio of federal public debt maturing between two and three years
$b4$	Ratio of federal public debt maturing between three and four years
$b5$	Ratio of federal public debt maturing between four and five years
$b6$	Ratio of federal public debt maturing between five and six years
$b7$	Ratio of federal public debt maturing later than six years ahead
B_{cb}	Bills held by the central bank
B_d	Total bills demanded, in quantity
b_h	Ratio of outstanding debt held by the market over GDP
B_h	Bills held by households
B_{h_d}	Bills demanded by households, in quantity
B_{h_s}	Bills supplied to households, in quantity
B_i	Issuance of bills to households, in quantity
B_r	Redemption of bills held by households, in quantity
B_m	Quantity of bills held by households maturing in the current period
B_{m_1}	Quantity of bills held by households maturing in the following period
B_{m_2}	Quantity of bills held by households maturing in the period two years ahead
B_{m_3}	Quantity of bills held by households maturing in the period three years ahead
B_{m_4}	Quantity of bills held by households maturing in the period four years ahead
B_{m_5}	Quantity of bills held by households maturing in the period five years ahead
B_{m_6}	Quantity of bills held by households maturing in the period six years ahead
B_{m_7}	Quantity of bills held by households maturing in the period seven years ahead
B_{m_n}	Quantity of bills held by households maturing in the latest maturity due
B_{net}	Net issuance of bills, in quantity or issuance minus redemption, in quantity
B_{nr}	Quantity of bills net redemptions absolute value
B_s	Total quantity of bills outstanding
C	Consumption
CG	Capital gains
D	Total public debt outstanding
ERr_b	Expected rate of return on bills
G	Pure government expenditure
g	Growth rate of pure government expenditure
H	High-powered money, or cash money
H_d	High-powered money, or cash money demanded by households
H_s	High-powered money, or cash money, supplied by the central bank
λ_0	Share of wealth households wish to maintain in the form of bills
λ_1	Sensitivity of the household demand to the return on bills
λ_2	Sensitivity of the household demand to the ratio of disposable income to total wealth
M	Maturity structure
p_b	Price of LFT bills
r_b	Domestic monetary policy interest rate
$\overline{r_b}$	Domestic monetary policy interest rate target

T	Taxes
θ	Tax rate
V	Wealth
Y	National income
YD	Disposable income

INDEX

1. INTRODUCTION

This dissertation investigates the role of government debt maturity structure dynamics within Brazil's institutional framework. To accomplish that, a Stock-Flow Consistent (SFC) model that reproduces in a stylized manner the Brazilian institutional framework regarding supply and demand for federal government bonds is built to include debt maturities dynamics, producing a cyclical behavior. As far as we know, explicitly accounting for debt maturities is an original contribution to SFC literature. The model is used to analyze the secondary effects of (i) a cut (or increase) in monetary policy rate; (ii) a cut (or expansion) of government expenditure; (iii) a cut (or expansion) in tax rates and (iv) a shift in investors sensitivity to interest rate. Finally, we compare debt-to-GDP dynamics under alternative scenarios for initial maturity structure. In the model, we find that monetary policy shocks only affect the price of bills without impacting real economic variables or debt levels. Similarly, shifts in investors' sensitivity to bills' returns are neutral. Higher government spending lowers the debt-to-GDP ratio, while tax rate cuts raise the debt-to-GDP ratio. Overall results indicate that explicitly accounting for debt maturities introduces significant volatility to the system, even considering a stable private sector demand for government bills and central bank as a last-resort buyer for excess government debt. This aspect renders the modelling appropriate for analyzing emerging market economies whose maturities structures exert an important role over government borrowing needs.

The focus is on domestic debt, which, throughout this work, unless otherwise stated, means debt issued in the domestic market and denominated in domestic currency. This choice is grounded in two reasons. First, it depicts Brazil's case more realistically, where external debt has, since 2007, presented a residual participation in Federal Public Debt (FPD)[1]. Secondly, this dissertation intends to contribute to the debate over the sustainability of public debt in cases where it is mainly denominated in the domestic sovereign currency.

On one hand, the arguments presented hereafter align with the heterodox literature by assuming that the Central Bank exogenously sets monetary policy interest rates and strongly influences debt dynamics. At this stance, heterodox economists challenge the orthodox view, which sees causality running in the opposite direction by attributing debt dynamics (particularly fiscal policy influence over it) a key role in yield curve behavior.

[1] According to the National Treasury Monthly Debt Report (MDR) December 2024 edition, the External Federal Public Debt (EFPD) amounts to R$349.19 billion, representing 5.01% of the volume of market-held FPD R$6,966.88.

On the other hand, from the theoretical standpoint, the framework presented hereafter follows a particular less-explored strand of the heterodox literature, which stands closer to what Aspromourgos (2018) identifies as Keynes's original thinking. Specifically, the claim that the ability of the government to control long-term interest rates depends on its success in handling debt markets' psychology. The developments in this dissertation focus on the initial debt maturity structure and the Treasury's limited freedom to set unilaterally debt securities benchmarks (terms).

Therefore, the arguments presented hereafter align with heterodox literature in the sense that they support the evidence that there are no *bond vigilantes*, once the demand for domestic sovereign debt, as a whole, is not likely to be subject to a sudden stop. However, we emphasize the effects of the structure of the demand, a topic somewhat neglected by this strand of literature. We argue that the National Treasury, mainly due to self-imposed institutional restrictions, has a restrained degree of freedom to control debt. In particular, debt maturity structure dynamics entails highly volatile and path-dependent debt dynamics.

Despite restraining the scope to Brazil, this analysis may help understand peer emerging countries in which a similar institutional framework may prevail. More generally, the insights may apply to any small[2] emerging country with a significant share of domestic debt over total government debt and a limited market for absorbing longer-term fixed-rate debt instruments.

In addition to this Introduction, this dissertation is composed of three other sections. Section 2 presents the institutional framework and the arguments sustaining the main assumptions adopted for the model. Section 3, after presenting a brief explanation for the adequacy of SFC models to analyze FPD dynamics in Brazil, describes the model and its calibration and presents its main results and test scenarios. Section 4 concludes.

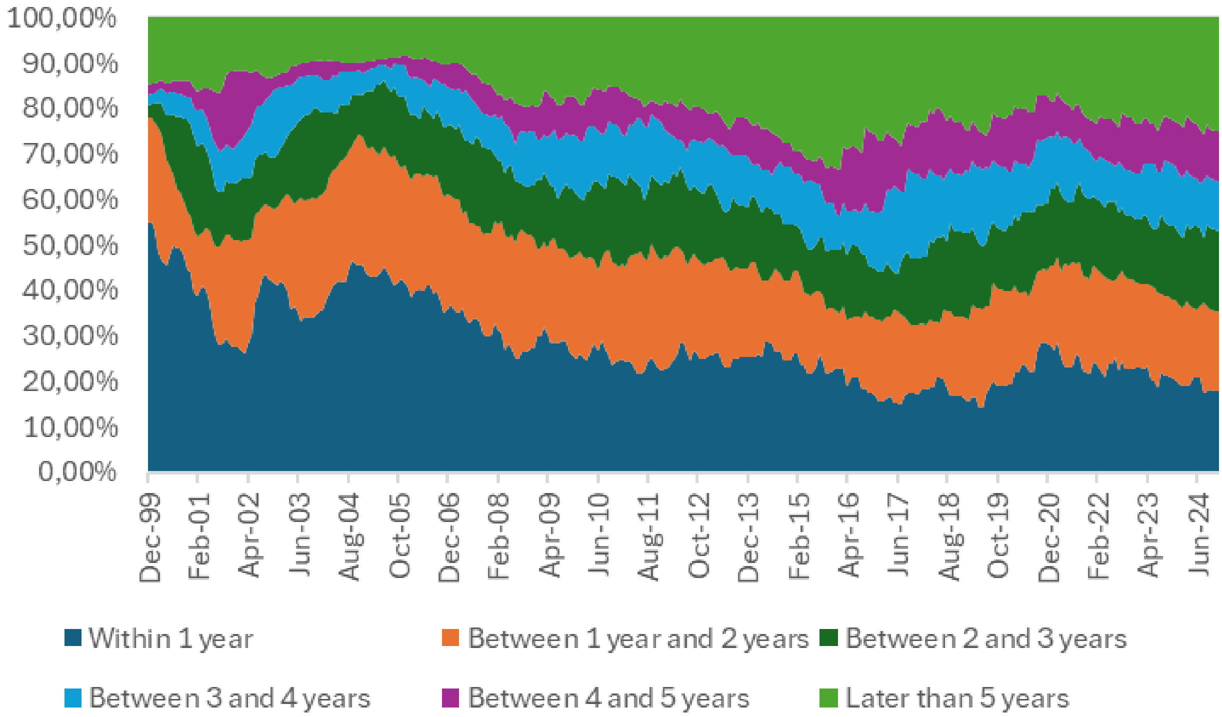
[2] We adhere to Raza et al (2019) definition of a small economy which is a country that although may be influenced by major economies (in case they have an open economy), their economies exert low or negligible impact over the remaining countries.

2. DESCRIPTION OF THE INSTITUTIONAL FRAMEWORK UNDERLYING GOVERNMENT DEBT MATURITY STRUCTURE IN BRAZIL

This section presents arguments and data that support the main institutional hypotheses adopted in our model.

Figure 1 depicts the historical behavior of the Domestic Federal Public Debt (DFPD) maturity structure from December 1999 through December 2024.

Figure 1 – Maturity structure for Brazil's DFPD outstanding



Source: Author's elaboration. National Treasury. Monthly Debt Report – December 2024

From Figure 1, we observe that the domestic sovereign debt maturity structure in Brazil has significantly changed during the last 25 years. The main aspect is the reduction of the share of DFPD maturing within one year and the expansion of the longer-term securities share. According to debt management best practices, longer-term securities tend to represent lower risk to the issuer compared to shorter-term debt instruments.

According to the most widely used debt management guidelines reference:

> "Poorly structured debt in terms of maturity, currency, or interest rate composition and large and unfunded contingent liabilities have been important factors in inducing or

propagating economic crises in many countries throughout history. For example, irrespective of the exchange rate regime, or whether domestic or foreign currency debt is involved, crises have often arisen because of an excessive focus by governments on possible cost savings associated with large volumes of short-term or floating rate debt. This has left government budgets seriously exposed to changing financial market conditions, including changes in the country s creditworthiness, when this debt has to be rolled over. Foreign currency debt also poses particular risks, and excessive reliance on foreign currency debt can lead to exchange rate and/or monetary pressures if investors become reluctant to refinance the governments foreign-currency debt. By reducing the risk that the governments own portfolio management will become a source of instability for the private sector, prudent government debt management, along with sound policies for managing contingent liabilities, can make countries less susceptible to contagion and financial risk Poorly structured debt portfolios, in terms of maturity, currency, or interest rate composition and large contingent liabilities, have been important factors in inducing or propagating economic crises in many countries throughout history. For example, irrespective of the exchange rate regime, or whether domestic or foreign currency debt is involved, crises have often arisen because of an excessive focus by governments on possible cost savings associated with short-term or floating rate debt. Issuance of large volumes of such debt instruments has left government budgets seriously exposed to changing growth and financial market conditions, including changes in the country's creditworthiness, when this debt has to be refinanced."

(IMF, World Bank, 2016, pp.5-6)

Following this reasoning, extending debt maturities is one of the guidelines of Brazil's National Treasury Debt Management Office (DMO). However, the National Treasury balances the pursuance of this objective with other guidelines, assessing cost-risk analyses in observance of market conditions[3]. This explains why after achieving a record of 33.7% of DFPD in February 2016 the share of bonds with maturities longer than 5 years fell and stood at 25.07% by the end of 2024. In the calibration section, we detail further the December 2024 maturity structure. In the testing sections, we test the behavior of the model for both the December 1999 and February 2016 maturity structures.

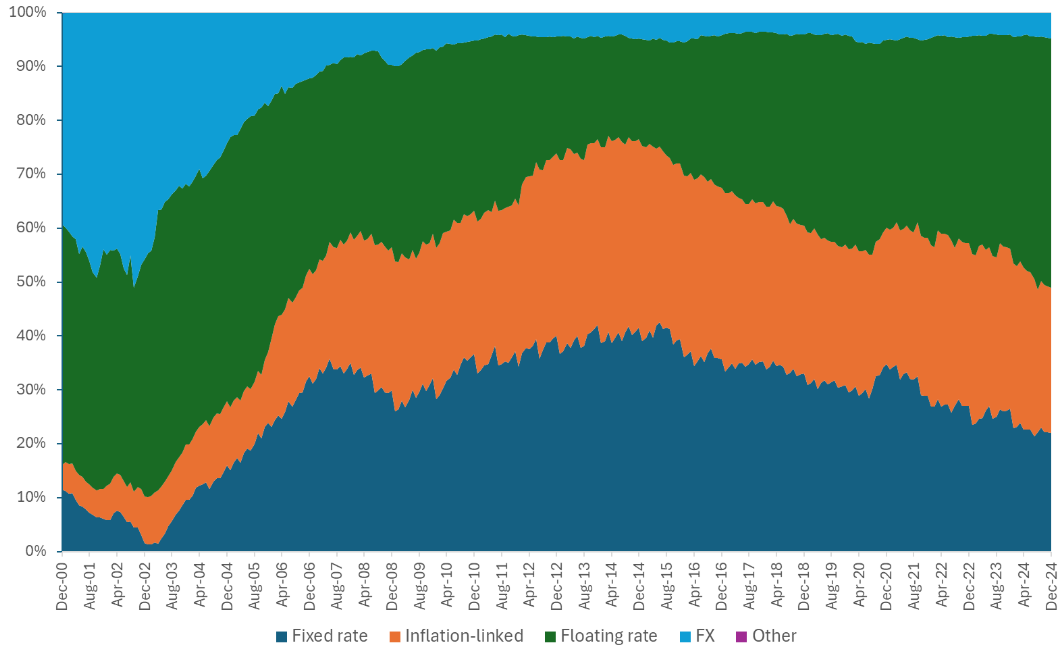
From the point of view of Modern Monetary Theory (MMT) and heterodoxy in general, the maturity structure of domestic sovereign debt *per se* is not very relevant (Ciccone, 2020; Bhering et al, 2019). However, the fact that the market follows and assigns relevance to

[3] The National Treasury of Brazil debt management objective is to efficiently meet federal government's financing needs by the lowest financing cost in the long term, respecting the maintenance of prudent levels of risk. Additionally, the DMO aims to contribute to the proper functioning of Brazil's government debt securities market.

these statistics may offer them a role in what Keynes called "market psychology" (Aspromourgos, 2018). Furthermore, the use of these metrics as parameters for the application of fiscal rules (Ligiéro, 2021; Godley and Lavoie, 2012, pp. 124-5; 160-5), debt management practices,[4] and monetary policy[5] choices, for instance, may end up producing self-imposed restrictions.

The first simplifying hypothesis adopted is that the Federal Public Debt (FPD) consists solely of domestic debt. We observe FPD's outstanding volume performance in Figure 2 to argue that this is a reasonable assumption.

Figure 2 - FPD outstanding volume by bond type



Source: Author's elaboration. National Treasury. Monthly Debt Report – December 2024

One of the main changes in FPD composition observed in Figure 2 is the significant reduction in bonds denominated in foreign exchange rate (FX) since the last quarter of 2002. This reflects the active movement of replacing external for domestic debt. Initially, external debt participation was replaced by domestic floating-rate and short-term fixed-rate debt instruments and, from 2006 onwards, also by inflation-linked instruments.

[4] This dissertation aims to contribute to this discussion.

[5] Lepers (2017), Aspromourgos (2007), Wray (2004) show how the views of central banks mostly align with the neoclassical orientation and how these views impose restrictions on policies that ultimately shape observed economic results.

There are four leading securities used by the Brazil's National Treasury as FPD instruments: the National Treasury Bills (*LTN*, for its acronym in Portuguese), the National Treasury Notes, F Series (*NTN-F*, for its acronym in Portuguese), the Financial Treasury Bills (*LFT*, for its acronym in Portuguese) and the National Treasury Notes, B Series (*NTN-B*, for its acronym in Portuguese). The first two are fixed-rate instruments, while the last two are floating-rate securities. *LFT* is indexed to the (effective) monetary policy rate (known as the *Selic* rate), and *NTN-B* combines a fixed-rate yield, defined at the time of the auction, with floating-rate inflation-linked remuneration. *NTN-F* and *NTN-B* are bonds with semiannual coupon payment flows, while *LTN* and *LFT* are zero-coupon bonds or bullets, *i.e.*, principal and interest are paid at maturity. *LTNs* tend to be issued at shorter maturities, while the other instruments are longer-term securities.

Figure 2 shows that, except for a small reversal in the 2008 crisis, from 2006 to 2014, there was an increasing trend in inflation-linked and fixed-rate bonds relative to floating-rate bonds. However, since 2015, the share of floating-rate bonds has been increasing relative to inflation-linked and fixed-rate bonds. At first sight, the COVID-19 crisis may appear as an exception to this trend, with a slight decrease in the floating-rate share of outstanding debt. However, during 2020, the increase in fixed-rate bonds was mainly based on issuances of short-term *LTN* bills (6- and 12-month maturity benchmarks). In 2021, the National Treasury conducted early redemptions of these bills in the face of a scenario more favorable than previously expected. As a result, by the end of 2021, the composition had improved compared to 2020 but continued to worsen in 2022. Even though the DMO pursues a benchmark portfolio (Table 1) that would recommend increasing fixed-rate relative to floating-rate bonds participation in FPD, as with lengthening debt maturities, it is constrained by market, macroeconomic, and institutional conditions[6].

[6] For instance, the strategy established in the Annual Borrowing Plan, the medium-term portfolio benchmark composition, the types of securities it may offer, the auctions frequency and modality, some of these practices although not being regulated by law, follow a standard pattern to provide market players predictability.

Table 1 - FPD long-term benchmark composition

	2024	Reference Limits for 2025		Benchmark Portfolio	
		Minimum	Maximum	Reference Value	Range
Outstanding Volume (R$ billions)					
FPD	7,316.1	8,100.0	8,500.0	--	--
Composition (%)					
Fixed-rate	22.0	19.0	23.0	35.0	+/- 2.0
Inflation-linked	27.0	24.0	28.0	35.0	+/- 2.0
Floating-rate	46.3	48.0	52.0	23.0	+/- 2.0
FX	4.8	3.0	7.0	7.0	+/- 2.0
Maturity Structure					
Maturing in 12 months (%)	17.9	16.0	20.0	20.0	+/- 2.0
Average Maturity (years)	4.0	3.8	4.2	5.0	+/- 0.5

Source: National Treasury. Annual Borrowing Plan – January 2025.

This leads to the next simplifying hypothesis of our model: marketable sovereign debt is represented solely by *LFT* bills. Although, as noted in Figure 2 and Table 1, *LFT* (floating rate bills) are currently the most representative FPD instrument, we recognize this is a strong assumption. This choice is made to reduce the number of assets in the model to the minimum necessary. Therefore, if it is necessary to choose only one debt instrument, *LFT* is the natural candidate. Additional reasons for that choice are that *LFT* is a zero-coupon bill, which simplifies the flows analysis, and that it is linked to the monetary policy rate, which is a variable already considered in canonical portfolio choice SFC models.

Regarding institutional sectors' representation, we choose to represent the National Treasury and the Central Bank separately. This choice of representation follows Lavoie (2013, 2022)'s argument that consolidating both institutions may lead to inadequate conclusions for economic policy discussions. We suggest further segregation for future developments between the Central Bank and the banking sector, which acts as a counterpart for repo operations. Throughout this work, for the sake of simplicity, we adopt the simplifying assumption that the Central Bank itself manages excess/scarcity of liquidity by buying/redempting government bills to maintain the Selic rate at the target.

In Brazil, financing of the Treasury by the Central Bank (*BCB*, for its acronym in Portuguese) has been ruled out since the 1988 Constitution[7],[8]. However, this **legal** prohibition[9]

[7] 1988 Constitution, article 164, §1º.

[8] See Monteiro (2016) for a legal analysis of this constitutional prohibition.

[9] The highlight serves to emphasize the difference between a legal boundary and a *de facto* prohibition. In the view of many MMT authors, even in the face of legal prohibitions of this nature, government financing through the central bank occurs.

was better defined only later, in 2000, by the Fiscal Responsibility Law[10] (*LRF*, for its acronym in Portuguese), which prevented the Central Bank from issuing government bills (or bonds). Since then, the main liquidity management instrument used by the *BCB* has been repo operations. Since the *LRF*, therefore, liquidity management has been carried out by the *BCB* using securities issued by the National Treasury, which currently holds the monopoly on issuing sovereign debt securities.

As of 2025, the National Treasury issues 7- and 10-year benchmarks of *NTN-F* bonds, a 6-year and a 3-year *LFT* benchmarks, and 6-, 12-, 24-, 48- and 72-month benchmarks of *LTN*[11]. Therefore, in terms of maturity, longer *LFT* and *NTN-F* are quite similar. However, while *LFT* has a one-day duration, NTN-F presents longer durations. Therefore, these bonds present very different sensitivities to interest rate variations. For the sake of simplicity, since we are modelling all the marketable debt as LFT instruments, we assume all the bills have a 7-year maturity at issuance, and we take the FPD maturity structure to calibrate the model. See Section 3.1.1 for more details.

[10] Complementary Law No. 101, of May 4, 2000.

[11] For more information, see National Treasury 2025 Annual Borrowing Plan and National Treasury auctions schedule for the first quarter of 2025.

3. MODELLING DOMESTIC DEBT COMPOSITION FOR BRAZIL

This dissertation adopts an SFC approach. The model built hereafter aligns with SFC premises by assuming that the object of our analysis qualifies as a modern capitalist economy whose dynamic is highly dependent on a complex institutional structure composed of firms, banks, governments, households, and their decisions and interactions (Godley and Lavoie, 2012, p. xxxiv). In this sense, the SFC approach, particularly the post-Keynesian institutionalist approach, draws special attention to the role of the institutional framework in modelling financial and real relationships between institutional sectors, seeking to represent as realistically as possible (and necessary) the institutional framework of current economic systems (Godley and Lavoie, 2012, p. 4).

Therefore, this approach seems appropriate to address the objectives stated in the previous sections. More specifically, it allows us to model the interactions between the institutional sectors (investors, National Treasury, Central Bank) and their influence over debt stocks and flows, as well as differentiate their sensitivity to real (output, debt-to-GDP ratios) and monetary (interest rate) variables.

SFC models contain four essential parts: (i) the specification of the relevant institutional sectors; (ii) the balance sheet or stocks matrix; (iii) the transactions and flow-of-funds matrix (and revaluation matrix); and (iv) the behavioral equations.

We will depart from Godley and Lavoie's (2012) canonic models, emphasizing the changes implemented in our version. We will then build a model named Portfolio Choice with Maturity Structure (PCMS) to implement the following changes with respect to Godley and Lavoie's (2012) Portfolio Choice canonical model: (i) maturity structure and payments, (ii) daily capitalization, and (iii) growth.

Following the methodology recommended by Godley and Lavoie (2012), p. 9, this section will be divided into three parts. The first part will describe the model, explaining its equations and accounting identities. The second part will expose the stylized facts that motivate the choices for the initial values of the stocks, flows, and behavioral parameters. The third part will present the results of the numerical simulations, focusing on the impacts over debt dynamics, in the baseline and five main alternative scenarios: (i) a cut (or expansion) of government expenditure; (ii) monetary policy shocks; (iii) alteration of demand sensitivity to bills' yield; (iv) cut (or increase) in tax rates; and (iv) short-run versus longer-run concentrated maturity schedules.

We assume away the production sector by considering a pure services economy.

3.1. Portfolio choice with maturity structure model (PCMS)

Model PCMS aims to represent in a stylized manner, government debt dynamics in Brazil. For that, it introduces three main features into Godley and Lavoie's (2012) Portfolio Choice (PC) model: (i) growth, (ii) daily capitalization, and (iii) maturity structure and payments.

To introduce growth, we assume that pure government expenditure (G) *i.e.* government expenditures net of interest payments grows by an exogenous constant real rate (g). Thus:

$$G = G_{-1} \cdot (1 + g) \tag{1}$$

Throughout the text the subscript (-1) denotes the variable in the previous period.

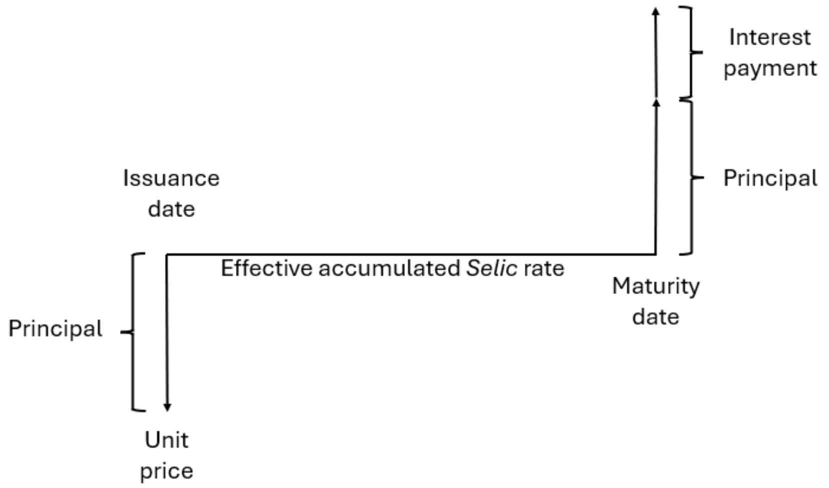
Bills' representation is modelled to approximate it to the reality of the National Treasury domestic debt securities, where maturity payments play an important role in government debt management and interest accrues daily. Thus, in model PCMS, bills represent schematically a floating-rate government debt security issued by the National Treasury of Brazil, indexed to the monetary policy interest rate (r_b), the *LFT*. As far as we are aware, the introduction of explicit maturity payments is an original contribution to SFC portfolio models.

The domestic monetary policy interest rate (r_b) is exogenously determined by the Central Bank.

$$r_b = \overline{r_b} \tag{2}$$

Figure 3 schematically represents the cash flow of a generic *LFT* bill:

Figure 3 - *LFT* bill cash flow



Source: Adapted from Brazil's National Treasury *"Cálculo da Rentabilidade dos Títulos Públicos ofertados no Tesouro Direto - Tesouro Selic (LFT)"*. Available at: https://www.tesourodireto.com.br/data/files/91/60/D8/C9/73B2D610393A62D6894D49A8/Calculo_Rentabilidade_TesouroSelic.pdf

The price of an *LFT* is given by:

$$p_b = principal \cdot (1 + \Delta r_b) \cdot premium \tag{3}$$

By convention, the *LFT*'s principal value (valor nominal) is set as *R$*1,000.00 at the base date of July 1, 2000. Since our model values are scaled in billions of Brazilian *Reais*, we represent the principal value at this date as:

$$principal = 10^{-6} \tag{4}$$

To simplify we assume *LFT* bills are always sold at *par*, thus there is no premium. Then, equation (3) becomes:

$$p_b = 10^{-6} \cdot (1 + \Delta r_b) \tag{3a}$$

where Δr_b is the accumulated Selic rate between the base date (July 1, 2000 and the given date).

Equation (3) elucidates that the current price of an *LFT* bill depends not only on the current interest rate but also on the whole history of monetary policy behavior. In the PCMS model, each period represents a 252-business-day year. We will find it useful to represent the current price in terms of the previous period's price ($p_{b_{-1}}$) and the current annual *Selic* rate (r_b). Accounting for daily interest capitalization:

$$p_b = p_{b_{-1}} \cdot e^{\frac{r_b}{252}} \tag{5}$$

where e denotes the Euler's constant.

For robustness check, we also test the model for a discrete daily compounding approximation:

$$p_b = p_{b_{-1}} \cdot \left(1 + \frac{r_b}{252}\right) \tag{5a}$$

Table 2 shows the balance sheet matrix of the PCMS model:

Table 2 – Balance sheet matrix for the PCMS model

	Households	Treasury	Central Bank	**Sum**
Money	$+H$		$-H$	**0**
Bills market	$+p_b \cdot B_h$	$-p_b \cdot B_h$		**0**
Bills BCB		$-B_{cb}$	$+B_{cb}$	**0**
Balance (net worth)	$-V$	$+V$	0	**0**
Sum	**0**	**0**	**0**	**0**

Notes:
(+) assets
(-) liabilities

Source: Author's elaboration

The first column means households' wealth (V) is allocated between two assets: high-powered money (H) and bills (B_h) which have a price p_b, thus:

$$V = H + p_b \cdot B_h \tag{6}$$

Total public debt outstanding (D) is defined as the sum of the value of bills held by the market ($p_b \cdot B_h$) and the value of bills held by the Central Bank (B_{cb}):

$$D \equiv p_b \cdot B_h + B_{cb} \tag{7}$$

Central Bank's net worth is assumed to be zero because we assume the Central Bank balance is fully transferred to the Treasury. Thus, using the equation (6) definition we have, from Table 2 column 'Treasury':

$$V = D \tag{8}$$

From equation (8) we conclude that public debt is the counterpart of the private sector's net worth in the PCMS model. Putting together equations (6) through (8) (see column 'Households' in Table 2), we have:

$$D = H + p_b \cdot B_h \tag{9}$$

In the 'Central Bank' column of Table 2, we have that the Central Bank purchases government bills B_{cb} (assets) and issues money to the households H (liability). As stated before, it is assumed that the Central Bank transfers its total profit immediately to the Treasury, which explains why its net worth (fourth row) equals zero. Thus, from Table 2, column (3), we have:

$$B_{cb} = H \tag{10}$$

which is a hidden equation in our model.

Table 3 shows the transactions and flow-of-funds matrix of the PCMS model:

Table 3 - Transactions and flow-of-funds matrix of the PCMS model

	Households	Production	Treasury	Central Bank		Sum
				Current	Capital	
Consumption	$-C$	$+C$				**0**
Government expenditure		$+G$	$-G$			**0**
Income = GDP	$+Y$	$-Y$				**0**
Maturity payments	$+p_{b_{-1}} \cdot B_m$		$-[p_{b_{-1}} \cdot B_m]$			**0**
Interest payments			$-r_{b_{-1}} \cdot B_{cb_{-1}}$	$+r_{b_{-1}} \cdot B_{cb_{-1}}$		**0**
Central Bank profit			$+r_{b_{-1}} \cdot B_{cb_{-1}}$	$-r_{b_{-1}} \cdot B_{cb_{-1}}$		**0**
Taxes	$-T$		$+T$			**0**
Variation in money	$-\Delta H$				$+\Delta H$	**0**
Net issuances market	$-[p_b \cdot B_{net} - p_{b_{-1}} B_m]$		$+[p_b \cdot B_{net} - p_{b_{-1}} B_m]$			**0**
Net issuances BCB			$+\Delta B_{cb}$		$-\Delta B_{cb}$	**0**
Sum	**0**		**0**	**0**	**0**	**0**
Capital gains	$-\Delta p_b \cdot (B_{h_{-1}} - B_m)$		$+\Delta p_b \cdot (B_{h_{-1}} - B_m)$			**0**

Notes:

(+) sources of funds, incoming flows of money

(-) uses of funds, outcoming flows

Source: Author's elaboration.

As stated before, the production sector is represented as simply as possible. We assume a closed pure service economy. As Godley and Lavoie (2012) explain, this assumption provides the model a series of simplifications:

> "The previous chapters described a service economy, where no capital goods were required for the firms to produce. This service economy was a kind of pure labour economy, where labour was the only input. Production did not require fixed capital, and it was assumed that all services could be produced on demand, without any need to hold inventories of goods or services. In a sense, production was instantaneous; it did not require time. These simplifying and artificial assumptions allowed us to move quickly forward in Chapters 3–6. They allowed us to circumvent relatively difficult accounting problems, such as cost accounting, inventory accounting, inflation accounting, as well as delicate economic questions, such as investment behaviour and the distribution of property income. It is now time to face some of these real-

world problems. In the real world, firms require fixed capital and working capital. As a result, they need to borrow from private banks."

Godley and Lavoie (2012), Ch. 7, p. 217

We also assume that firms always fully meet the demand stemming from households and the government sector. Likewise, households also fully meet firms' demand for labor. From the first column in Table 3, the national income (Y) does not include interest payments to the Central Bank ($r_{b_{-1}} \cdot B_{cb}$) nor interest and principal maturity payments to households ($p_{b_{-1}} \cdot B_m$). Therefore, the national income identity states that production equals the sum of consumption and government expenditures:

$$Y \equiv C + G \tag{11}$$

LFT bills are zero-coupon securities. Thus, payments only occur when they reach maturity, and households receive the principal plus interest back. Then, at each period, a quantity B_m of bills held by households matures, generating a flow of $p_b \cdot B_m$ from the government to households. From the price definition (equations 5 or 5a), we can verify that this flow includes interest and principal.

We assume that bills issued to the Central Bank are short-term government securities exactly like the ones in Godley and Lavoie's (2012) PC model. Thus, bills issued to the Central Bank have a fixed price of one unit and one-period (one-year) maturity. When they mature, they pay interest (equivalent to the annual monetary policy interest rate, $r_{b_{-1}}$) and the principal is automatically rolled over. Therefore, principal rollover of the Central Bank debt portfolio is implicit and does not appear in the transactions flow table. This modelling is coherent to the Brazil debt management practice, once the principal of maturing bonds in the *BCB* portfolio can be automatically rolled over by new government securities placements to the monetary authority balance sheet. Hence, these operations do not entail a financial counterpart flow. Differently, real interest derived from the bills in the Central Bank must be paid by the Treasury, generating a financial flow from the National Treasury to the monetary authority, represented by ($r_{b_{-1}} \cdot B_{cb}$). Although we consider this modelling appropriate to represent the Central Bank-National Treasury relationship in Brazil, we believe it is inadequate to represent the government debt management behavior towards the private sector, which is why we explicitly model debt maturities for the government bills held by households. This aspect will be clarified further when we discuss households' demand for government securities.

As stated before, we adopt the simplifying assumption that the Central Bank transfers its total profit to the National Treasury[12]. Looking at Table 2 we can verify the reason why this profit is necessarily positive: the only liability in the central bank balance sheet is cash (H), which we assume pays no interest, whereas BCB holds a non-negative portfolio of government bills (B_{cb}) which pays interest ($r_{b_{-1}}$).

The government demands services from the production sector generating a flow of pure government expenses (G) which we assume grows exogenously as given by equation (1). Government also taxes income generating a flow of tax revenues (T):

$$T = \theta \cdot \left(Y + p_{b_{-1}} \cdot B_m\right) \tag{12}$$

where θ is an exogenous fixed parameter representing the tax rate.

Observe that tax income includes maturity payments.

In the first row of the flow of funds part of Table 3 ('Variation in Money'), we assume, that the Central Bank provides all the cash that is demanded by households:

$$\Delta H_s = \Delta H_d \tag{13}$$

As usual, the government budget constraint is obtained by the sum of the elements in the 'Treasury' column of the transactions flow table (Table 3). The difference is that in model PCMS, the government buffer variable is not the total variation of the stock of bills but the flows B_{net} and ΔB_{cb}, which represent net issuance[13] of bills to the market and to the Central Bank respectively. The Treasury faces households' demand and defines B_{net}, as we will explain later. Then ΔB_{cb} meets the government budget constraint described by the 'Treasury' column in the flow of funds matrix (Table 3):

$$\Delta B_{cb} = G - T + \ p_{b_{-1}} \cdot B_m - \left(p_b \cdot B_{net} - p_{b_{-1}} \cdot B_m\right) \tag{14}$$
$$\text{or } \Delta B_{cb} = G - T + \ 2 \ \cdot p_{b_{-1}} \cdot B_m - (p_b \cdot B_{net})$$

In equation (14), $G - T$ represents the primary deficit, while $G - T + \left(p_{b_{-1}}\right) \cdot B_m$ represents overall balance (or net lending/net borrowing), *i.e.* government primary deficit plus interest payments. From equation (14), we obtain that the government issues to the Central

[12] Brochier and Pedrosa (2025) provide a detailed SFC analysis of the alterations in the framework regulating Central Bank and National Treasury relations in Brazil. As the authors argue, although the current legislation, namely Law No. 13.820/2019, defines that capital gains arising from international reserves in the Central Bank balance sheet accruals in a graphic account, eventually these values are transferred to the Treasury rendering the same result (in terms of debt issuances, repo operations and Treasury Single Account variation) as the 2008 regulation (Law No. 11.803/2008).

[13] Net issuance is obtained by total issuances minus total redemption and does not include maturities.

Bank the amount of bills that are necessary to cover for the overall government balance and that were not absorbed by net issuances to the market ($p_b \cdot B_{net} - p_{b_{-1}} B_m$). This explains the element $2 \cdot \left(p_{b_{-1}}\right) \cdot B_m$ which, at first sight, appears to be double accounting for the same flow. Looking at Table 3, we observe that this is not the case: to properly account for households' bills variation we must discount the value of maturities ($p_{b_{-1}} \cdot B_m$), since $p_b \cdot B_{net}$ denotes issuances net of redemptions. In the upper part of Table 3, we must account for the maturity payments, which appear on the right-hand side of equation (14).

To understand these flows it is important to recall that, in general, the difference between the stock of a generic asset between two periods is, by definition, given by:

$$\begin{aligned} & p_d \cdot D - p_{d_{-1}} \cdot D_{-1} \\ & = \left(p_{d_{-1}} + \Delta p_d\right) \cdot (D_{-1} + \Delta D) - \left(p_{d_{-1}} \cdot D_{-1}\right) \\ & = \left(p_{d_{-1}} \cdot D_{-1}\right) + (\Delta p_d \cdot D_{-1}) + \left(p_{d_{-1}} \cdot \Delta D\right) + (\Delta p_d \cdot \Delta D) - \left(p_{d_{-1}} \cdot D_{-1}\right) \\ & = (\Delta p_d \cdot D_{-1}) + \left(p_{d_{-1}} \cdot \Delta D\right) + (\Delta p_d \cdot \Delta D) \\ & = (\Delta p_d \cdot D_{-1}) + \Delta D \cdot \left(p_{d_{-1}} + \Delta p_d\right) \\ & = \underbrace{(\Delta p_d \cdot D_{-1})}_{capital\ gains\ or\ losses} + \underbrace{(\Delta D \cdot p_d)}_{issuances\ or\ redemptions} \end{aligned} \tag{15}$$

where D is the quantity of the given asset and p_d its price. The subscript -1 denotes the variable value in the previous period.

Therefore, the difference between stocks can be expressed in terms of the two parts in parentheses of the final expression in equation (15). The first one represents capital gains (or losses) while the second represents the difference in stock value due to the variation in the quantity of bonds held by bondholders.

At this point, we must be careful about what ΔB stands for in model PCMS. In Godley and Lavoie (2012), Chapter 5, bonds were perpetuities, so ΔB could only reflect National Treasury net issuances (if positive) or redemptions (if negative). Now, we have an additional component in ΔB: maturities, a part of the previous stock being paid back to investors, thus being deducted both from the Treasury liabilities and from the households' assets. We will also want to break down the variations in the stock that are issued to the Central Bank from variations in the stock that are due to issuances to the market. Thus, for each point t in time the variation in the **quantity** of bills is given by:

$$\Delta B_s \equiv B_i - B_r - B_m + \Delta B_{bc} = B_{net} - B_m + \Delta B_{bc} \tag{16}$$

where B_i stands for bills issuances to the market, B_r stands for bills redemptions to the market, B_m refers to bills' maturities (only marketable bills mature) and ΔB_{bc} represents net issuances to the Central Bank. We denote net issuances to the market for $B_{net} \equiv B_i - B_r$.

Equation (16) is a supply definition. From a demand perspective, the variation in the **quantity** of bills is given by the sum of the variation in the stock of bills held by households and the quantity of bills held by the Central Bank. Thus:

$$\Delta B_d \equiv \Delta B_h + \Delta B_{bc} \tag{17}$$

The variation in the **quantity** of bills held by households (ΔB_h) is defined by the difference between the final stock of bills held by households and the initial stock of bills held by households at the end of the previous period:

$$\Delta B_h \equiv B_h - B_{h_{-1}} \tag{18}$$

We may also represent this variation explicitly in terms of B_{net} and B_m:

$$\Delta B_h \equiv B_{net} - B_m \tag{19}$$

Equaling equations (18) and (19) and rearranging, we have:

$$B_h = B_{h_{-1}} + B_{net} - B_m \tag{20}$$

Equation (20) means that the final quantity of bills held by households (B_h) is equal to the initial quantity ($B_{h_{-1}}$) plus net issuances (B_{net}) minus the quantity of bills maturing in the current period (B_m). Considering $B_{h_{-1}}$ and B_m always assume values equal to or higher than zero, B_{net} may assume negative values (in case of a net redemption) but it is necessary to ensure that B_h will be non-negative. Thus equation (20) must respect the inequality:

$$B_h = B_{h_{-1}} + B_{net} - B_m \geq 0$$

Rearranging in terms of B_{net}:

$$B_{net} \geq B_m - B_{h_{-1}}$$

Ensuring $B_h \geq 0$, requires B_{net} to be defined as:

$$B_{net} = \max\{B_{h_d} - B_{h_{-1}} + B_m; B_m - B_{h_{-1}}\} \tag{21}$$

where B_{h_d} is the quantity of bills demanded by households.

This expression guarantees that:

(i) When $B_{h_d} - B_{h_{-1}} + B_m$ is already larger than $B_m - B_{h_{-1}}$ it takes the original value

(ii) If the net redemption $B_h - B_{h_{-1}} + B_m$ would result in $B_h < 0$ the maximum forces B_{net} to a value that guarantees $B_h \geq 0$.

Equation (21) implies a simple but important aspect of the National Treasury behavior. It means the National Treasury is the issuer of government bonds and never a net buyer from market securities. It may buy back government bills from the market, but only up to the limit of the outstanding existing stock. Equation (21) also implies that the final stock (in quantity) of bills held by households (B_h) is given by the previous stock ($B_{h_{-1}}$) minus the part of this stock that matured in the current period (B_m) plus the net purchases of new bills acquired by households (B_{net}). B_{net} is the left-hand variable because, as will become clearer later, $B_{h_{-1}}$ and B_m are defined in the previous period and B_h is defined by the demand that arises from households' portfolio choice equations. The subscript d in B_{h_d} is used to imply that, when choosing the amount of bills' net issuances, the Treasury supplies the necessary amount to meet market demand.

Table 4 presents the integration between the household sector's flow and stock accounts within Model PCMS.

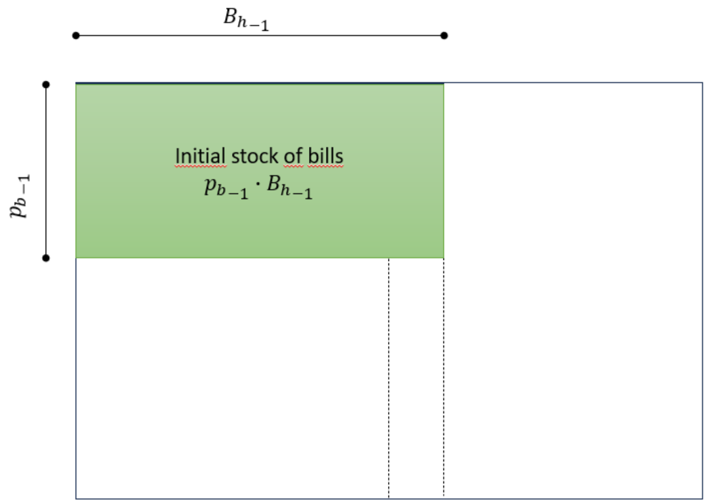
Table 4 -Integration of households' flow and stock accounts, within Model PCMS

		Money	Bills	Sum (assets)
Initial assets (end of previous period)		$\boldsymbol{H_{h_{-1}}}$	$\boldsymbol{p_{b_{-1}} \cdot B_{h_{-1}}}$	$\boldsymbol{V_{-1}}$
Consumption	$-C$			0
Income = GDP	$+Y$			0
Maturity payments	$+p_{b_{-1}} \cdot B_m$			0
Taxes	$-T$			0
Variation in Money	$-\Delta H_h$	$+\Delta H_h$		0
Net issuances (issuances-redemptions)	$-p_b \cdot B_{net}$		$+p_b \cdot B_{net}$	0
Maturities	$+p_{b_{-1}} \cdot B_m$		$-p_{b_{-1}} \cdot B_m$	0
Sum	0	H_h	$p_{b_{-1}} \cdot B_{h_{-1}} + [p_b \cdot B_{net}] - p_{b_{-1}} \cdot B_m$	$H_h + p_{b_{-1}} \cdot (B_{h_{-1}} - B_m) + [p_b \cdot B_{net}]$
Capital gains			$+\Delta p_b \cdot (B_{h_{-1}} - B_m)$	$+\Delta p_b \cdot (B_{h_{-1}} - B_m)$
Final assets (end of period)		$\boldsymbol{H_h}$	$\boldsymbol{p_b \cdot B_h}$	$\boldsymbol{V}$

Source: Author's elaboration.

To clarify the *LFT* bills behavior entailed in the integration described by Table 4 we will analyze it graphically. For simplicity we will illustrate the case where we have positive net issuances. Initially we have the situation described in Figure 4, the outstanding stock of bills held by households is given by the area in green that corresponds to the initial amount of bills multiplied by its initial price:

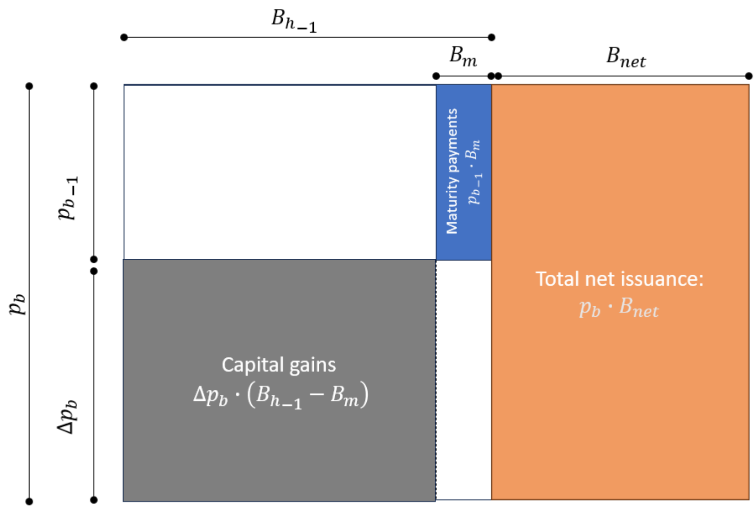
Figure 4 - Initial stock of bills held by households in model PCMS



Source: Author's elaboration. Adapted from Godley and Lavoie (2012).

Figure 5 illustrates the flows that occur between the initial and the final moment of each period. Part of the initial outstanding stock matures and is repaid to investors for the initial price generating a negative flow of $p_{b_{-1}} \cdot B_m$, corresponding to the area in blue. The portion of the initial stock that has not yet matured yields capital gains due to the difference between initial and final price. This flow corresponds to the grey area. Finally, the National Treasury issues an amount of B_{net} bills at current price p_b, which generates a flow of $p_b \cdot B_{net}$ corresponding to the area in orange.

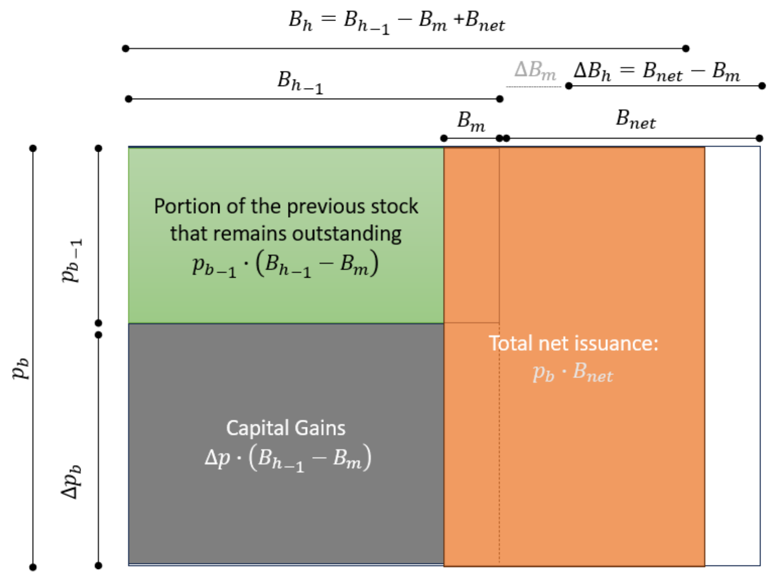
Figure 5 - Flows of bills held by households in model PCMS



Source: Author's elaboration. Adapted from Godley and Lavoie (2012)

Figure 6 presents the final stock of bills outstanding in the households' sector. This final stock ($p_b \cdot B_h$) is formed by the sum of the portion of the initial stock of bills that remains outstanding ($p_{b_{-1}} \cdot \left(B_{h_{-1}} - B_m\right)$), capital gains over this portion ($\Delta p_b \cdot \left(B_{h_{-1}} - B_m\right)$) and the total value of new issuances at the new price ($p_b \cdot B_{net}$).

Figure 6 - Final stock of bills held by households in model PCMS



Source: Author's elaboration. Adapted from Godley and Lavoie (2012)

Note that by including maturities ($p_{b_{-1}} \cdot B_m$), equation (12) incorporates realized capital gains. Thus, capital gains referring to maturities must be discounted from the capital gains equation (22). As evident by Figure 6, capital gains (CG) are calculated as:

$$CG = \Delta p_b \cdot \left(B_{h_{-1}} - B_m\right) \tag{22}$$

We assume these unrealized accrued capital gains (CG) are not taxed and do not count as disposable income (YD), which includes solely the flow of regular income, thus:

$$YD = Y - T + p_{b_{-1}} \cdot B_m \tag{23}$$

As noted in Table 4, household wealth accumulation is given by:

$$V = V_{-1} + (YD - C) + CG \tag{24}$$

Consumption (C) depends on disposable income (YD) and past wealth accumulated (V_{-1}):

$$C = \alpha_1 \cdot YD + \alpha_2 \cdot V_{-1} \tag{25}$$

where α_1 and α_2 represent respectively the propensities to consume out of disposable income and out of wealth, respecting the inequality $0 < \alpha_2 < \alpha_1 < 1$.

We assume bills have unchanging maturity of n periods. We also assume that if there are net redemptions, the government decides to execute early repayments from the earliest to the latest maturity due date. To implement this, we must set an initial exogenous maturity structure that represents the expected volume (in quantity) of bills that should mature from t until $t + n$. The maturity structure is defined by a set M of $n + 1$ variables.

$$M = \left\{B_m; B_{m_1}; B_{m_2}; \dots; B_{m_n}\right\} \quad (26)$$

From this initial exogenous maturity structure, the subsequent maturity structures will be defined endogenously as follows. The bills maturing at t equal the net issuance of bills in period $t - n$, unless in one of the periods between $t - n + 1$ and $t - 1$ a part of this stock of bills was repaid early due to a net redemption.

The first entry in M represents the bills that are maturing in the current period (B_m), the second (B_{m_1}) represents the bills that are maturing in the next subsequent period, the third (B_{m_2}) represents the bills that will mature in the period two years ahead, and so on, until (B_{m_n}) which represents the last maturity due. This last maturity payment is given by the quantity of bills issued in the current period (net of redemptions). In case net issuances are negative in the current period, which means redemptions are higher than issuances, then the maturity for n years ahead is zero. Thus:

$$B_{m_n} = max(B_{net}; 0) \quad (27)$$

The maturity structure is updated dynamically, since in the next period the whole maturity structure becomes one period closer to its maturity date. Therefore, the bills that are maturing in the current period (B_m) are the bills that were represented by B_{m_1} in the previous period:

$$B_m = \left(B_{m_1}\right)_{-1} \quad (28)$$

Analogously, the maturities expected for the next period (B_{m_1}) are the maturities that, in the period before, were due in two years. However, in case net issuances are negative in the current period, *i.e.*, issuances are lower than early redemptions, net redemptions should be subtracted from this value. Mathematically:

$$B_{m_1} = max\left\{\left(B_{m_2}\right)_{-1} - B_{nr}; 0\right\} \quad (29)$$

where B_{nr} is and auxiliary variable that takes the absolute value of net redemptions, that is $B_{nr} = |min(0; B_{net})|$.

For the following maturities entries in M it is important to consider the possibility that the net redemption exceeds the amount due for the closest period and was sufficient to pre-pay not only next year's maturity but also partially or fully the maturity expected for two years ahead. Thus:

$$B_{m_2} = \begin{cases} \left(B_{m_3}\right)_{-1} \ , & if \ B_{nr} \leq \left(B_{m_2}\right)_{-1} \\ max\left[\left(\left(B_{m_3}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1}\right)\right); 0\right] & , otherwise \end{cases} \tag{30}$$

The same logic applies to the following maturities entries in M, which must account for the possibility that between $t - n + 1$ and $t - 1$ there has been a net redemption large enough to prepay all the previous due maturities. Mathematical formalization is heavy, but the idea is simple: if the government has the possibility to prepay debt, it will do so departing from the maturities which due date is closer. We provide the formalization for the situation where $n = 7$, *i.e.* the Treasury only issues 7-year-maturity bills to households:

$$B_{m_3} = \begin{cases} \left(B_{m_4}\right)_{-1} \ , & if \ B_{nr} \leq \left(B_{m_2}\right)_{-1} + \left(B_{m_3}\right)_{-1} \\ max\left[\left(\left(B_{m_4}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1} - \left(B_{m_3}\right)_{-1}\right)\right); 0\right] & , otherwise \end{cases} \tag{31}$$

$$B_{m_4} = \begin{cases} \left(B_{m_5}\right)_{-1} \ , & if \ B_{nr} \leq \left(B_{m_2}\right)_{-1} + \left(B_{m_3}\right)_{-1} + \left(B_{m_4}\right)_{-1} \\ max\left[\left(\left(B_{m_5}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1} - \left(B_{m_3}\right)_{-1} - \left(B_{m_4}\right)_{-1}\right)\right); 0\right] & , otherwise \end{cases} \tag{32}$$

$$B_{m_5} = \begin{cases} \left(B_{m_6}\right)_{-1} \ , & if \ B_{nr} \leq \left(B_{m_2}\right)_{-1} + \left(B_{m_3}\right)_{-1} + \left(B_{m_4}\right)_{-1} + \left(B_{m_5}\right)_{-1} \\ max\left[\left(\left(B_{m_6}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1} - \left(B_{m_3}\right)_{-1} - \left(B_{m_4}\right)_{-1} - \left(B_{m_5}\right)_{-1}\right)\right); 0\right] & , otherwise \end{cases} \tag{33}$$

$$B_{m_6} = \begin{cases} \left(B_{m_7}\right)_{-1} \ , & if \ B_{nr} \leq \left(B_{m_2}\right)_{-1} + \left(B_{m_3}\right)_{-1} + \left(B_{m_4}\right)_{-1} + \left(B_{m_5}\right)_{-1} + \left(B_{m_6}\right)_{-1} \\ max\left[\left(\left(B_{m_7}\right)_{-1} - \left(B_{nr} - \left(B_{m_2}\right)_{-1} - \left(B_{m_3}\right)_{-1} - \left(B_{m_4}\right)_{-1} - \left(B_{m_5}\right)_{-1} - \left(B_{m_6}\right)_{-1}\right)\right); 0\right] & , otherwise \end{cases} \tag{34}$$

For the specific case where $n = 7$, which we will estimate in subsequent sections, then equation (27) assumes:

$$B_{m_7} = max\{B_{net}; 0\} \tag{27a}$$

By modelling maturity structure in this way, we are aware that the model presents a path dependency character, which we believe is appropriate to analyze not only Brazil, but also any other emerging market subject to constraints for altering debt maturity structure.

The maturity structure also results in a definition of the quantity of bills supplied to households:

$$B_{h_s} = \sum_{i=1}^{n} B_{m_i} = \underbrace{B_{m_1} + B_{m_2} + \cdots + B_{m_n}}_{n\ terms} \tag{35}$$

Portfolio choice equations follow an adaptation of Godley and Lavoie (2012) interpretation of Tobin (1969):

$$\frac{H}{V} = (1 - \lambda_0) - \lambda_1 \cdot ERr_b + \lambda_2 \left(\frac{YD}{V}\right) \tag{36}$$

$$\frac{B_{h_d} \cdot p_b}{V} = \lambda_0 + \lambda_1 \cdot ERr_b - \lambda_2 \cdot \left(\frac{YD}{V}\right) \tag{37}$$

where ERr_b stands for the expected rate of return on bills, and the parameters λ_0, λ_1 and λ_2 are reaction parameters in the portfolio choice of households, respecting the inequality $0 < \lambda_0, \lambda_1, \lambda_2 < 1$.

Equation (37) means that households wish to hold a certain share λ_0 of their wealth in the form of bills. Since there are only two assets in the model (bills and cash), equation (37) implies that by choosing to hold λ_0 of wealth in bills value $(B_d \cdot p_b)$, households are simultaneously choosing to hold a proportion $(1 - \lambda_0)$ of their wealth in the form of money in equation (36). These proportions are modulated by the sensitivities to the rate of return on government bills (λ_1) and to the ratio of disposable income to total wealth (λ_2). Evidently higher rate of return on bills increases the relative demand for bills while decreasing the relative demand for money[14]. This explains why λ_1 is preceded by a negative signal in equation (36) and by a positive signal in equation (37). The signals preceding λ_2 are due to the transactions demand for money.

Since the Central Bank is assumed to provide money endogenously, always meeting the households' demand for cash, equation (36) is omitted to avoid over-specification, and replaced by:

$$H = V - p_b \cdot B_h \tag{38}$$

The expected rate of return on bills is given by:

$$ERr_b = \ln\left(\frac{p_b}{p_{b_{-1}}}\right) \tag{39}$$

[14] Fang et al (2022) find evidence that in emerging countries, differently from advanced economies, all types of bondholders' demand respond positively to yield increases with the non-resident non-bank group having the higher elasticity.

When using the discrete price approximation alternative version (equation (5a)) we calculate expected return on bills as:

$$ERr_b = \frac{p_b}{p_{b_{-1}}} - 1 \tag{39a}$$

PCMS model contains a core of 23 equations and 23 unknown variables. Table 5 presents the list of endogenous and exogenous variables and parameters.

Table 5 - List of Variables for model PCMS

	Endogenous variables	Exogenous variables/parameters
(1)	Y	r_b
(2)	C	g
(3)	G	θ
(4)	YD	α_1
(5)	Tax	α_2
(6)	p_b	λ_0
(7)	B_m	λ_1
(8)	V	λ_2
(9)	CG	
(10)	B_h	
(11)	B_{net}	
(12)	H	
(13)	B_{h_d}	
(14)	ERr_b	
(15)	B_{cb}	
(16)	B_{nr}	
(17)	B_{m_1}	
(18)	B_{m_2}	
(19)	B_{m_3}	
(20)	B_{m_4}	
(21)	B_{m_5}	
(22)	B_{m_6}	
(23)	B_{m_7}	

Source: Author's elaboration.

3.1.1. Calibrating the PCMS model

Since model PCMS contains only one type of government security, we will consider the initial stock of bills in the market ($p_b \cdot B_h$) to be the total value of domestic federal public debt held by the market. We obtain this figure from the National Treasury Monthly Debt Report edition of December 2024. Therefore, in billions of reais (R$), the total value of outstanding domestic federal public debt held by the market equals:

$$p_b \cdot B_h = 6{,}966.88$$

We take p_b as the December 2024 *LFT* nominal accumulated value (*Valor Nominal Atualizado* or *VNA*)[15] which was R$15,682.35507. Since our model values are in R$ billion:

$$p_b = 0.0000156823507$$

We obtained the monetary policy interest rate (*Selic* rate) from the BCB: 12.15% per annum in December 2024. Since the PCMS model does not consider inflation, we deflate this nominal rate by the *IPCA* (acronym in Portuguese for *Índice de Preços ao Consumidor Amplo*, the main consumer price index in Brazil) inflation for the year 2024 to obtain the real rate (r_b). Thus:

$$r_b = 0.1215 - 0.0483 = 0.0732$$

Rearranging equation (5) and applying the values obtained above for p_b and r_b, we have:

$$p_{b_{-1}} = \frac{0.0000156823507}{e^{0.0732}/_{252}} = 0.00001567792$$

If we are dealing with the discrete approximation for price (equation 5a) then:

$$p_{b_{-1}} = \frac{0.0000156823507}{1 + \frac{0.0732}{252}} = 0.00001567792$$

As we can see both methods render the same value as a result for $p_{b_{-1}}$.

15 Available at: https://sisweb.tesouro.gov.br/apex/f?p=2501:9::::9:P9_ID_PUBLICACAO:48716

Using the results obtained for $p_{b_{-1}}$ from the expressions above, we have that the initial quantity of bills held by the market is:

$$B_h = \frac{6{,}966.88}{0.0000156823507} = 444{,}249{,}646.386957$$

We also obtain the initial outstanding volume of bills in the Central Bank portfolio from the National Treasury Monthly Debt Report edition of December 2024, thus we have, in R$ billion:

$$B_{cb} = 2{,}517.51$$

For the 2024 GDP, we took the 12-month accumulated GDP estimated by the BCB[16], since official figures by the *Instituto Brasileiro de Geografia e Estatística* (IBGE) were still not available by the time of submission of this dissertation. Then GDP in current R$ billion:

$$Y = 11{,}810.1012$$

For calibration purposes, we will set b as the ratio of debt held by the Central Bank over GDP:

$$b = \frac{B_{cb}}{Y} = \frac{2{,}517.51}{11{,}810.1012} = 0.2131658$$

Likewise, we will define b_h as the ratio of outstanding debt held by the market over GDP:

$$b_h = \frac{B_h \cdot p_b}{Y} = \frac{6{,}966.88}{11{,}810.1012} = 0.5899086$$

To represent the exogenous parameter θ that indicates tax rates in equation (12) we take a measure of tax burden. For that, we refer to the General Government Finance Statistics Statement published by the National Treasury[17], in Table 2.1.a. Statement of Government Operations by level of government revenues as a percentage of GDP:

$$\theta = 37.60\%$$

[16] Available at: https://www3.bcb.gov.br/sgspub/consultarvalores/consultarValoresSeries.do?method=getPagina series code No. 4382

[17] General Government Finance Statistics Statement. First quarter of 2024. Go to the Table 2.1. Statement of Government Operations by level of government - Brazil – Yearly, 2023, General Government. Even though our model does not include subnational governments, it was considered appropriate to take the total collection, since state and municipalities in Brazil collect a significant volume of taxes. Values are in current prices. We use the 2023 ratio since the 2024 value was not yet available.

Finally, we will set an initial maturity structure based on the maturities of domestic marketable federal public debt as described in Table 6. The first step is setting the total quantity of bills as B_h and equaling it to the value outlined in expression for it (*i.e.* 444,249,646.39). We assume that all the prices keep their proportionality and deal with the maturity structure only in terms of the quantity of bills. It is also assumed that bills have a 7-year maturity ($n = 7$).

Table 6 – Debt maturity structure calibration for baseline scenario

	DFPD maturities ratio		Maturity (in quantity of bills)	
Within 1 year	b1	17,89%	B_{m_1}	79.482.829,07
Between 1 year and 2 years	b2	17,43%	B_{m_2}	77.424.431,52
Between 2 and 3 years	b3	17,89%	B_{m_3}	79.486.148,54
Between 3 and 4 years	b4	10,67%	B_{m_4}	47.417.627,77
Between 4 and 5 years	b5	11,04%	B_{m_5}	49.061.718,91
Later than 5 years	b6 + b7	25,07%	$B_{m_6} + B_{m_7}$	111.376.890,58
Total	1	100,00%	B_h	444.249.646,39

Source: Author's elaboration. National Treasury Monthly Debt Report. December 2024

Since the National Treasury does not publishes a breakdown of maturities longer than 5 years it is assumed that this percentage of maturing debt is equally distributed between B_{m_6} and B_{m_7}.Thus:

$$b6 = b7 = \frac{25.07\%}{2} = 12.535\%$$

As we have seen in Section 2, the DFPD maturity structure of December 2024 is relatively balanced. In Subsection 3.1.3 we will test the model for other maturity structures and analyze results.

We also obtain B_m from the December 2024 edition of the National Treasury Monthly Debt Report. For that we take the total value of DFPD maturities in 2024 (R$1,369.18988 billions) and divide for the $p_{b_{-1}}$ value obtained above. Thus:

$$B_m = \frac{1,369.18988}{0.00001567792} = 87,332,368$$

Then, exclusively for calibration purposes, we set equations as follows.

$$p_{b_{-1}} = \frac{p_b}{e^{r_b/252}}$$

$$C = Y - G$$

$$G = B_{cb} - \frac{B_{cb}}{(1+g)} + T - p_b \cdot B_m + (p_b \cdot B_{net} - p_b * B_m)$$

$$YD = Y - T + (p_b \cdot B_m)$$

$$T = \theta \cdot (Y + p_b \cdot B_m)$$

$$CG = (\Delta p_b)\left(\frac{B_h}{(1+g)} - B_m\right)$$

$$V = H + p_b \cdot B_h$$

$$H = B_{cb}$$

$$B_{h_d} = B_h$$

$$B_h = b_h \cdot \frac{Y}{p_b}$$

$$ERr_b = ln\left(\frac{p_b}{p_{b_{-1}}}\right) \text{ or } ERr_b = \left(\frac{p_b}{p_{b_{-1}}}\right) - 1$$

$$B_{cb} = b \cdot Y$$

$$B_{net} = \max\left\{B_h - \left({}^{B_h}/_{(1+g)}\right) + B_m; B_m - \left({}^{B_h}/_{(1+g)}\right)\right\}$$

$$B_{nr} = |min\{B_{net}; 0\}|$$

$$B_{m_1} = b1 \cdot B_h$$

$$B_{m_2} = b2 \cdot B_h$$

$$B_{m_3} = b3 \cdot B_h$$

$$B_{m_4} = b4 \cdot B_h$$

$$B_{m_5} = b5 \cdot B_h$$

$$B_{m_6} = b6 \cdot B_h$$

$$B_{m_7} = b7 \cdot B_h$$

By registering the values for the right-hand side variables from the equations above we will have initial values so we can run the model.

Finally, we set the remaining parameters, as they are usually specified in the SFC literature:

$$\alpha_1 = 0.6$$

$$\alpha_2 = 0.4$$

$$\lambda_0 = 0.635$$

$$\lambda_1 = 0.0500043$$

$$\lambda_2 = 0.01$$

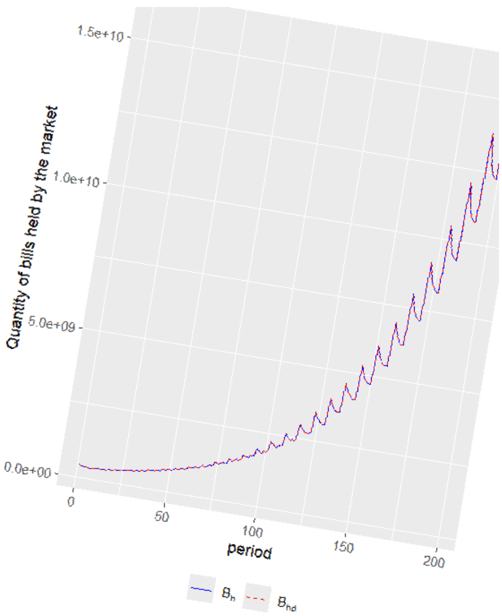
Annex 1 contains the R programming code both for calibration and for the model PCMS simulations.

3.1.2. Results of the PCMS model

The model was estimated using the *sfcr* package in R developed by Macalos (2021)[18].

In Figure 7 we observe that introducing maturities generates cycles in the PCMS model, which become increasingly pronounced over time. This progressive amplification is primarily due to the compounding effects of the debt maturity structure (equations 26-34), which introduces multiple lags and nonlinear adjustment mechanisms. As these nonlinearities accumulate, they reinforce cyclical deviations, leading to progressively sharper peaks and deeper troughs as the simulation advances.

Figure 7 – Dynamics of the quantity of bills held by the market



Source: Author’s elaboration.

Figure 8 depicts the dynamics of the debt stock ratios relative to GDP in the model: total debt, government debt held by the market, government debt held by the Central Bank. Notably, the ratio of debt held by the market relative to GDP consistently remains above that held by the Central Bank. This pattern is in line with our calibrated parameters, which reflect Brazil's institutional framework — where private market for government bonds (specially floating rate

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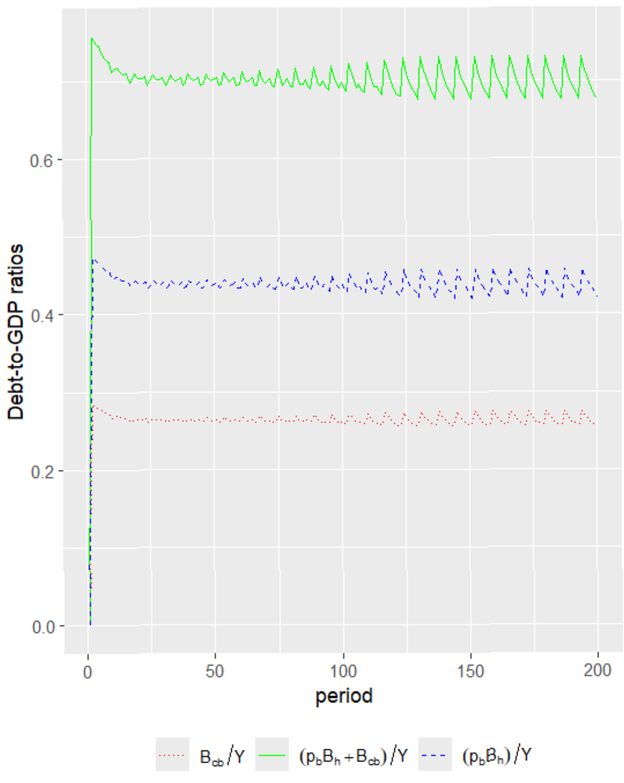
[18] Consistency tests values attended the parameters stablished for SFC models that incorporate growth dynamics.

debt instruments) is liquid and deep, while issuances to the Central Bank have the residual role of providing the monetary authority tools for managing liquidity. In addition, the series exhibit a distinctive behaviour over time. In the early periods, all debt-to-GDP ratios adjust sharply, likely reflecting the model's convergence from calibrated initial conditions toward a dynamic equilibrium. After this initial adjustment, the ratios' trends stabilize at lower levels. For example, the total debt-to-GDP ratio declines from an initial average of approximately 75% to a final average of 67% over the simulation horizon, while the market-held debt ratio and the Central Bank-held debt ratio falls from 47% and 28% to 42% and 25%, respectively[19]. Cyclical patterns deepen through time — mirroring the increasing volatility observed in other key variables of the model.

Figure 8– Total debt, government debt held by the market, and government debt held by the Central Bank over GDP ratios

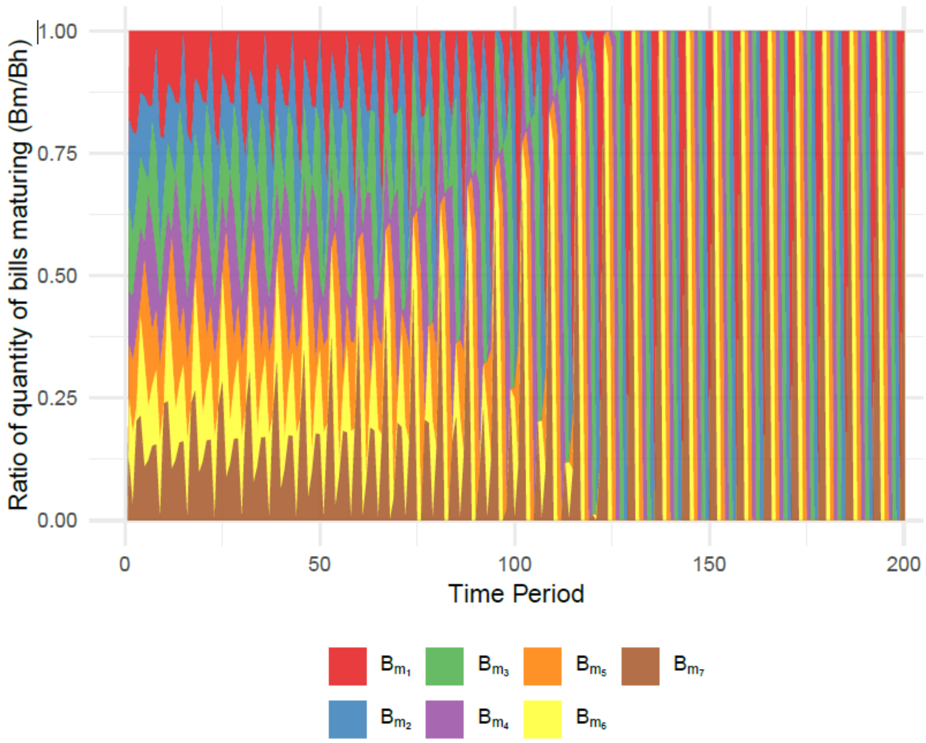


Source: Author's elaboration.

[19] For context, the calibration targets for total debt/GDP, market-held debt/GDP and Central Bank-held debt/GDP were set at 80%, 58.9% and 21% respectively. The initial simulated values — approximately 75% for total debt, 47% for market-held debt, and 28% for Central Bank-held debt —differ from these targets due to the inherent transitional dynamics in SFC models.

Figure 9 illustrates the evolution of the debt maturity structure in the model, showing the relative shares of each quantity of bills in each maturity bucket (B_{m_1}through B_{m_7}) as a portion of total government bills held by the private sector (B_h). As we can observe by the model (particularly equations 27-34), each quantity of bills in the maturity structure is not only linked to the previous maturity value but also to the net National Treasury redemption (in quantity of bills), B_{nr}, creating a cascading dynamic across the debt maturities. Furthermore, since issuances in the model have a fixed 7-year maturity, periods with pronounced issuances build up a proportional pronounced maturity for seven years ahead (see equations 27 and 27a). In the early periods, the distribution across the maturity schedule is relatively balanced, reflecting stable net issuance dynamics. However, as the simulation progresses, the figure reveals nearly vertical stripes — alternating concentrations in specific maturities. In the final periods the maturities approximate the form of stripes. These stripes suggest that it is likely that, in the long term, at each period, transient episodes of pronounced issuances or redemptions cause the model to channel a larger share of debt into a particular due date (*e.g.*, B_{m_1} in one period, then B_{m_2} in the next, and so on. Figure 10 investigates further this hypothesis.

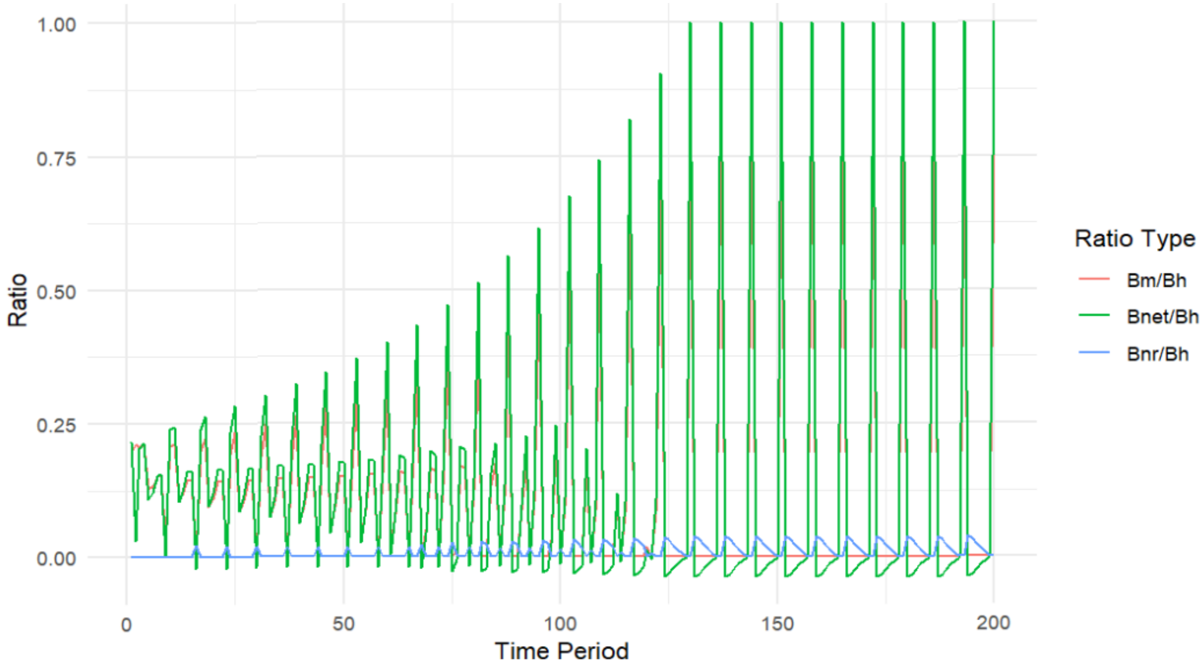
Figure 9 - Evolution of debt maturity structure as a share of total bills held by the private sector (B_h)



Source: Author’s elaboration.

Figure 10 presents the dynamics of three key ratios to total government bills held by the private sector (B_h): (i) net market issuances (*i.e.*, issuances minus redemptions) ($^{B_{net}}/_{B_h}$); (ii) net redemptions ($^{B_{nr}}/_{B_h}$); and current quantity of bills maturing ($^{B_m}/_{B_h}$). Recall from equation (21) that the market debt rollover process is characterized by the relationship between net issuances and maturing debt. When the Treasury is rolling debt at a rate above 100%, we have $B_{net} > B_m$ (net issuances exceed the maturing debt, and the overall debt stock increases, $B_h > B_{h_{-1}}$). Conversely, when the rollover rate is below 100% (*i.e.*, $B_{net} > B_m$), net issuances fall short of the maturing debt, leading to a reduction in the debt stock (*i.e.*, $B_h < B_{h_{-1}}$). Thus, Figure 11 captures both the funding dynamics – via B_{net}/B_h or B_{nr}/B_h – and the refinancing process – via B_m/B_h. In the early periods cycles are smoother and the difference between B_{net}/B_h and B_{net}/B_h is small, which explains the relative persistence of the balanced maturity structure observed in Figure 9. However, in the long run, B_{net}/B_h fluctuates sharply upward, largely exceeding B_m/B_h and systematically reaching 1. These peaks imply that in those periods net issuances are equal to 100% of the outstanding debt. Since debt maturities are fixed, these episodes of strong issuances accumulate in specific maturity buckets creating the alternating, nearly vertical stripes observed in the debt maturity structure (Figure 9).

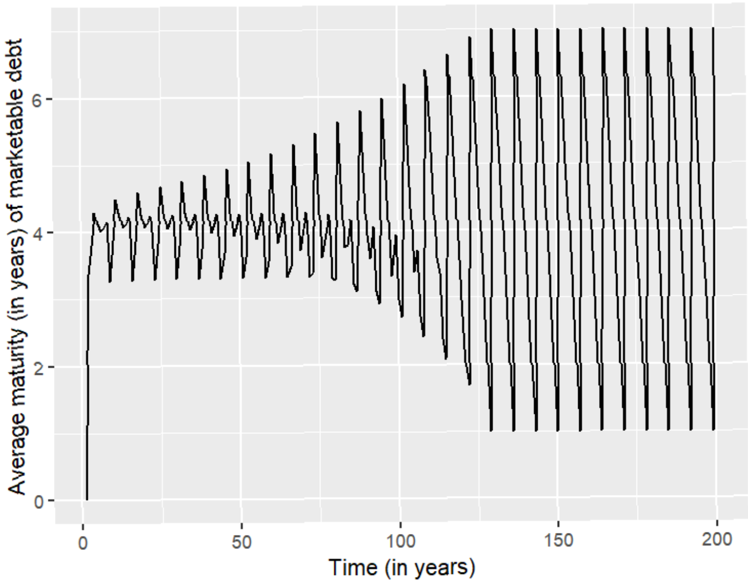
Figure 10 – Maturing bills, net issuances and net redemptions ratios to private-sector-held bills (B_h)



Source: Author’s elaboration.

Figure 11 presents the dynamics of the average time to maturity of privately held government debt.

Figure 11 – Average time to maturity of government debt dynamics

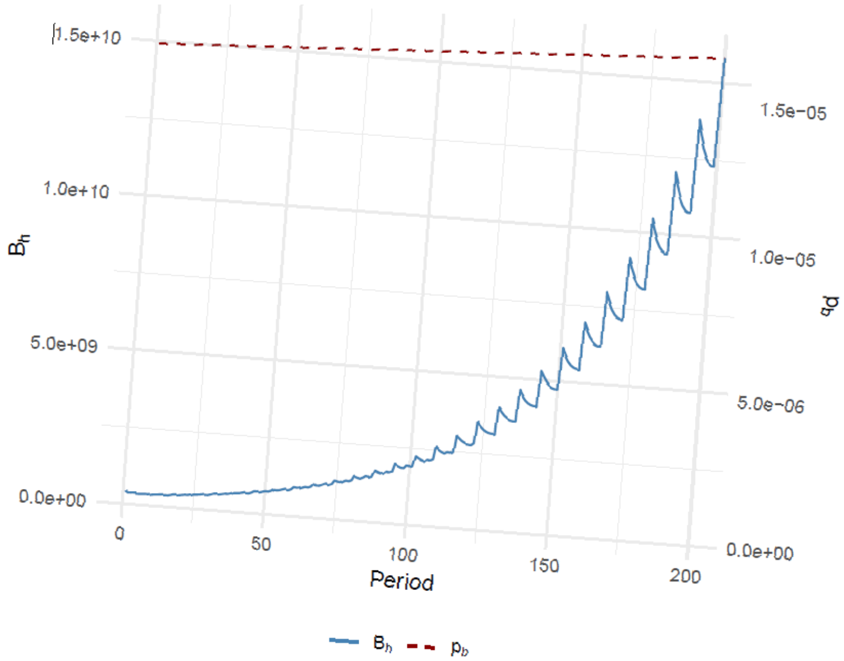


Source: Author's elaboration.

When analyzing the average life of privately held government debt (we assume Central Bank debt is automatically rolled over), we observe that in the early periods, this measure is relatively stable, reflecting the balanced issuance and redemption dynamics observed in the maturity structure (Figure 9) and the smoother issuance behavior by the Treasury (Figure 10). Here also, as the model progresses, the oscillations become more pronounced, fluctuating between its theoretical lower and upper bounds (one and seven years, respectively), mirroring the sharp fluctuations in net issuances (Figure 10).

Figure 12 illustrates the evolution of the two components of privately held government debt: the price of bills (p_b) and its outstanding quantity (B_h). Due to differences in scale, these series are displayed on separate axes for clearer visualization.

Figure 12 – Government debt held by the private sector: bills' price and quantity

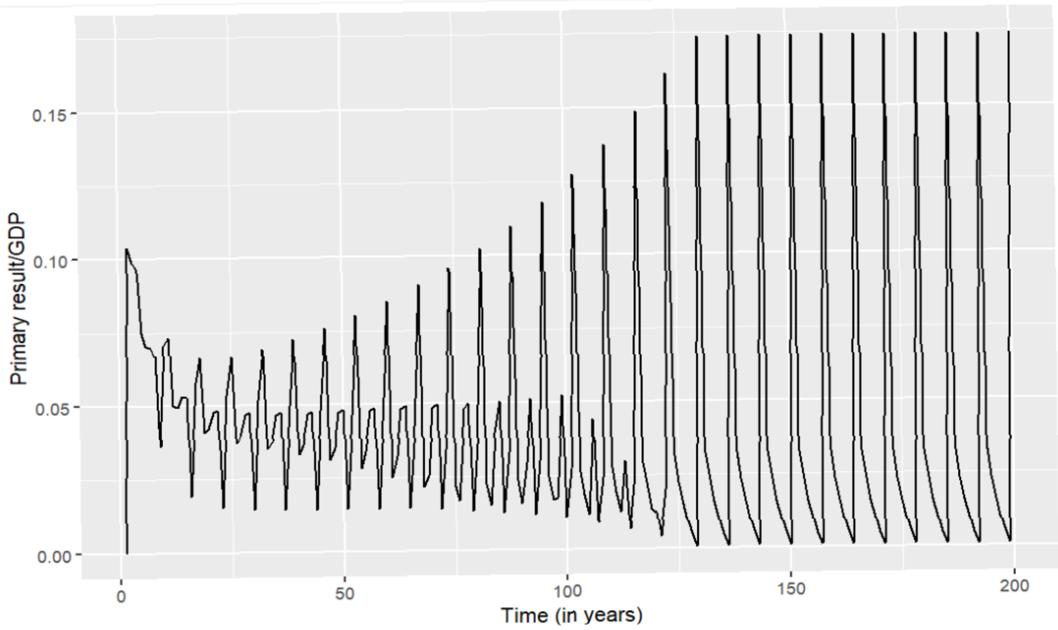


Source: Author's elaboration.

The distinct growth pattern for these variables reflects their specification in the model. The quantity of bills (B_h) follows an exponential trajectory. In contrast, the bills' price (p_b) exhibits a more linear trend, in line with its specification by equation (5), which is influenced by the exogenously set interest rate assumption and the absence of short-term price shocks.

Figure 13 presents the evolution of the primary balance-to-GDP ratio, defined as the difference between tax revenues and government expenditures relative to GDP.

Figure 13 - Primary result-to-GDP ratio dynamics

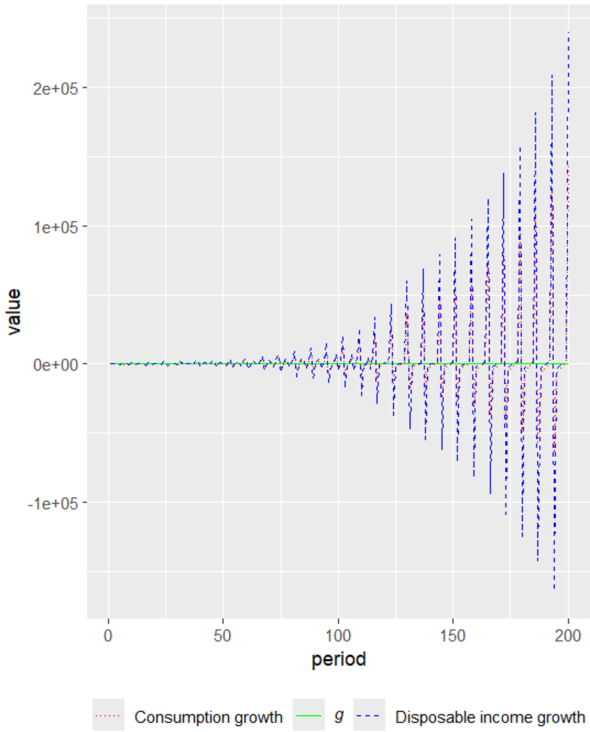


Source: Author's elaboration.

The series, as the other previously analyzed, exhibits two distinct phases: an initial adjustment period followed by a long-run cyclical pattern with increasing volatility. In the early periods, the primary result declines sharply from an initial 10% of GDP to approximately 5% of GDP, reflecting the model's transition dynamics as fiscal balances adjust to the evolving stock-flow relationships. This initial drop is consistent with the stabilization process observed in other macroeconomic aggregates, where the system shifts from its initial conditions toward an endogenous dynamic equilibrium. Over time, the series exhibits pronounced cyclical fluctuations, with volatility increasing significantly. The primary result oscillates within a bounded range (from 0% to 17% of GDP), mirroring the amplifying cycles seen in other key fiscal and debt-related variables (Figures 8 through 11). These fluctuations stem mainly from the influence of the maturities in the tax function (equation 12). Since maturities become increasingly volatile, tax recollection from these payments respond to this effect.

Figure 14 presents the baseline simulation results for the growth rates of key macroeconomic variables: the exogenously defined GDP growth rate (g), along with the endogenous growth rates of disposable income and consumption . In PCMS model, government spending—and hence overall GDP—is designed to grow at the constant rate g. However, while GDP growth (g) remain steady by construction, the growth rates of disposable income and consumption exhibit cyclical fluctuations. These cycles emerge from the interplay between fiscal policy, wealth accumulation, and the complex debt maturity structure, which introduces lags and nonlinear adjustments in the system. Notice how the smoother trajectory of consumption growth, compared to the more volatile disposable income growth, reflects the mitigating effect of past wealth on current consumption decisions (equation 25).

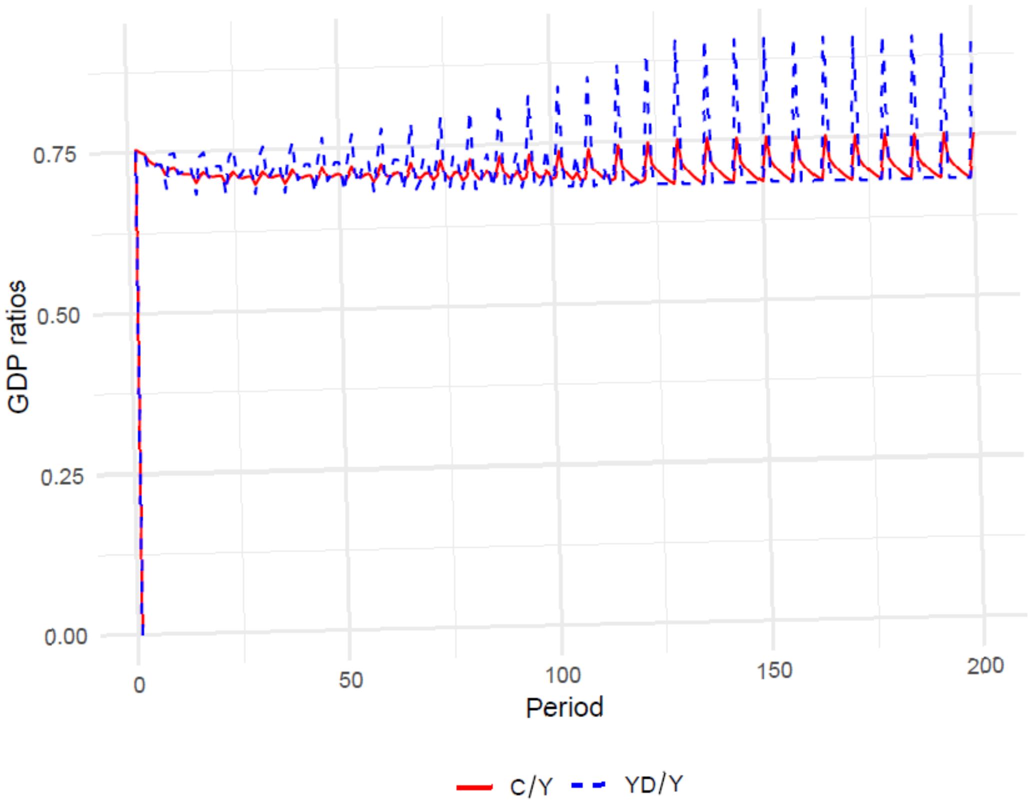
Figure 14 – Growth rates for output, consumption and disposable income



Source: Author's elaboration.

Investigating further the consumption and disposable income series behaviour, Figure 15 displays the cyclical dynamics of two key ratios in the model: the consumption-to-output ratio (C/Y) and the disposable income-to-output ratio (YD/Y). As expected, both ratios follow cyclical patterns; with, YD/Y exhibiting notably sharper fluctuations over time. The heightened volatility in YD/Y probably stems from its direct exposure to immediate fiscal variables—such as taxes, government spending, and debt maturity payments—which drive the endogenous fluctuations in the model. In contrast, as stated above, the consumption-to-output (C/Y) ratio is moderated by the consumption function's reliance on lagged wealth, thereby smoothing the impact of the endogenous fluctuations.

Figure 15 - Consumption and Disposable Income as output ratios



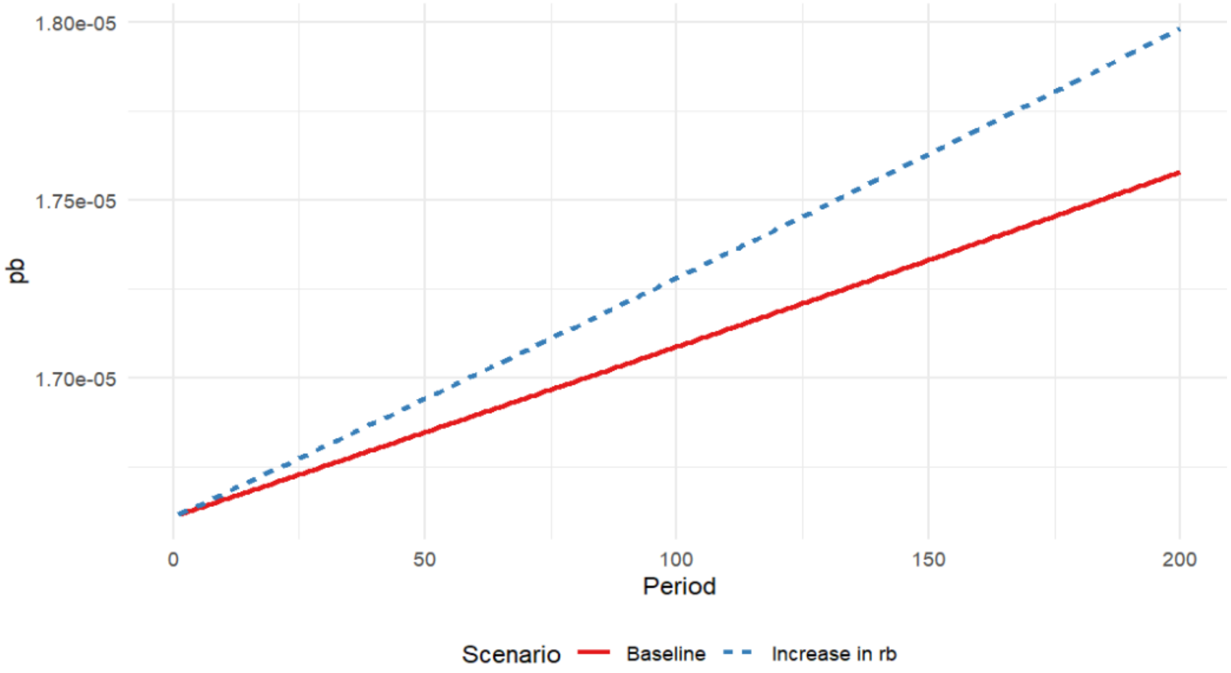
Source: Author's elaboration

Combined results of the PCMS model allow us to observe that explicitly accounting for maturities introduces significant volatility to the model, even assuming a stable private sector demand for government bills and Central Bank as a last-resort buyer for excess government debt. This aspect renders model PCMS appropriate to analyze emerging market economies whose maturities structures exert an important role over government debt management.

3.1.3. Alternative scenarios for the PCMS model

We begin by introducing a monetary policy tightening shock, specifically an increase in the real monetary policy interest rate from 7.13% per annum in the baseline scenario to 10% per annum in the shocked scenario. Figure 16 presents the effect of the monetary policy shock on bills' prices.

Figure 16 – Monetary policy tightening shock effect on bills' price

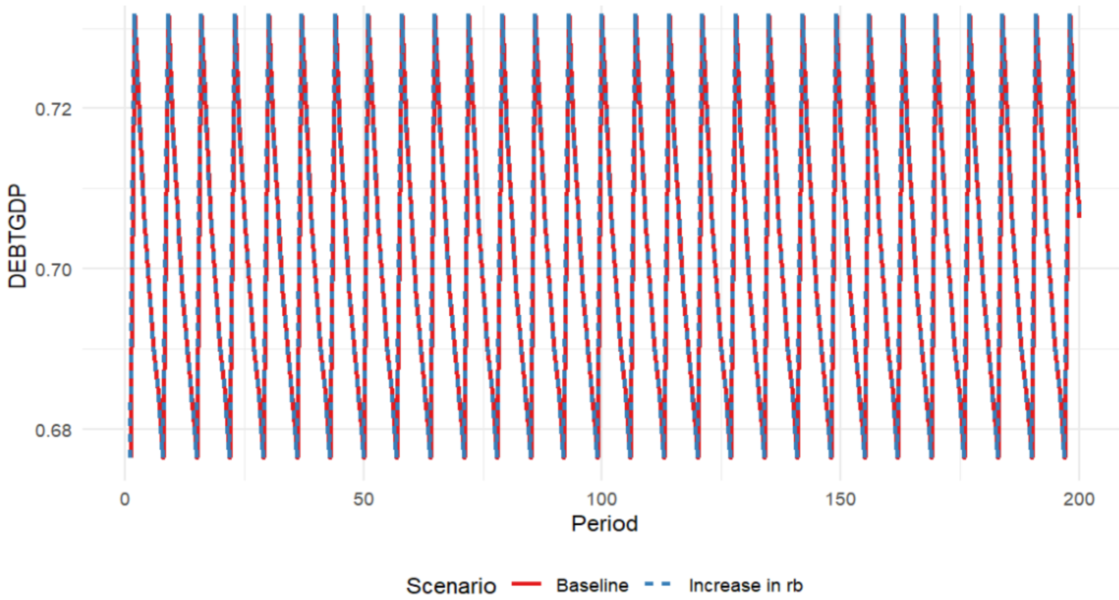


Source: Author's elaboration

Contrary to conventional portfolio choice models, where an interest rate increase reduces bond prices, in this model, the shock leads to a rise in bills' prices. This occurs because government securities in the PCMS model are floating-rate bills, whose yield fully adjusts to the policy rate.

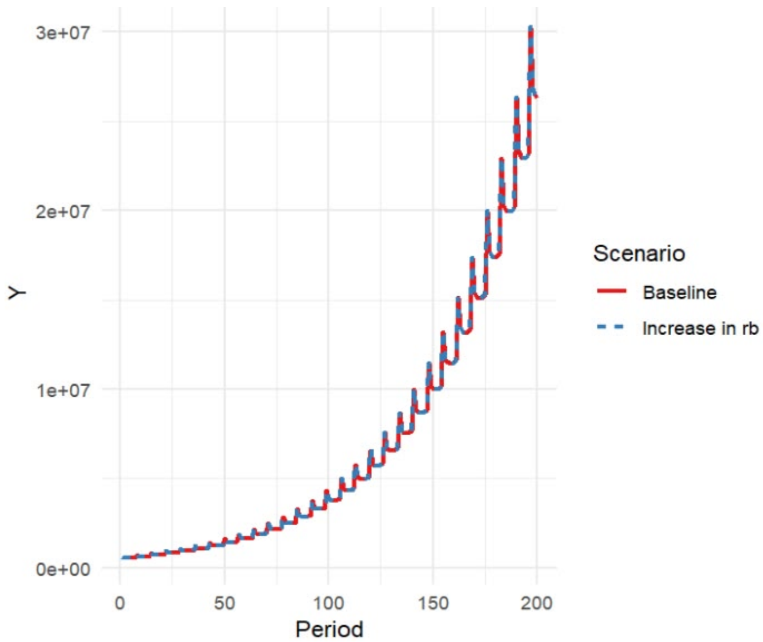
However, as observed in Figures 17 through 19, the monetary policy shock does not generate any significant effects on real macroeconomic variables such as GDP and consumption, nor does it alter debt ratios (*e.g.*, debt-to-GDP). These results arise because, in the current model specification, all government securities are floating-rate bills indexed to the monetary policy rate, so that interest payments on public debt adjust fully and immediately to the new rate. As a result, no wealth effects or liquidity constraints emerge, preventing changes in household consumption or aggregate output.

Figure 17– Debt-to-GDP ratio dynamics under monetary policy tightening shock vs baseline scenario



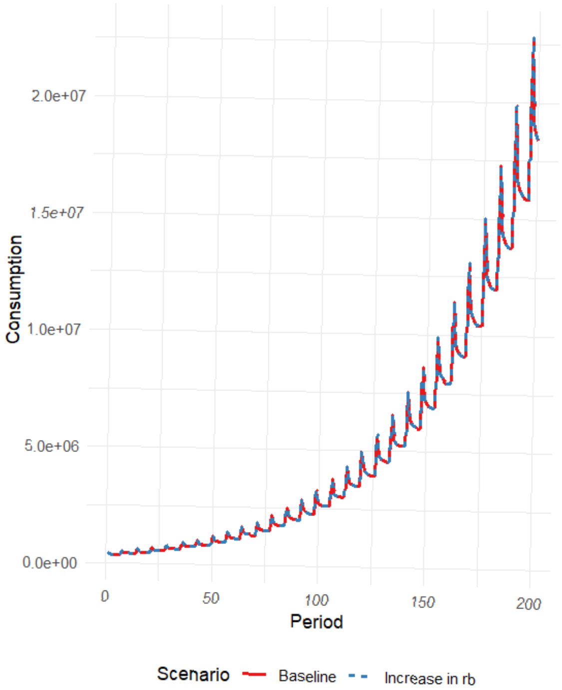
Source: Author’s elaboration.

Figure 18 – GDP Dynamics under monetary policy tightening shock *vs.* baseline scenario



Source: Author’s elaboration

Figure 19 – Consumption Dynamics under a monetary policy tightening shock vs. baseline scenario



Source: Author's elaboration.

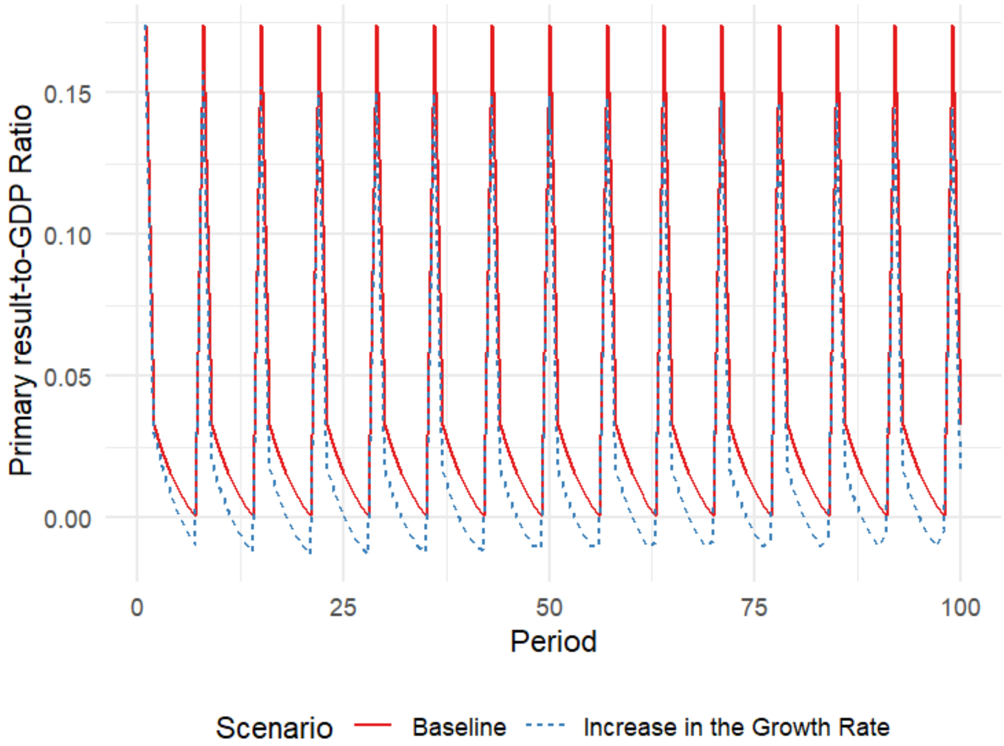
Monetary loosening shocks of the same magnitude produce symmetric effects, reinforcing the neutrality of monetary policy in the current model. Since government securities automatically adjust to the interest rate, monetary policy shifts do not create financial frictions or portfolio effects that would typically drive changes in fiscal or real economic dynamics. These results highlight a key limitation of the PCMS model in its current form — since it only includes one type of government security (floating-rate bills), monetary policy primarily affects the price of bills without altering debt composition or broader economic variables. Future extensions will introduce fixed-rate bonds, allowing us to assess how monetary policy changes influence the relative allocation between fixed- and floating-rate debt and their impact on public debt composition and dynamics.

Similar results are obtained for the case where we increase the interest rate sensitivity of household demand for bills (λ_1) from 0.0500043 to 0.0600043. There are no observed effects over debt (neither in level nor in GDP ratios) or any other relevant variable in the model dynamics. Even with a substantially larger shock (increasing λ_1 from 0.05 to 0.15) the debt-to GDP ratios remain unchanged, confirming that the interest rate sensitivity of household demand for bills does not significantly impact macroeconomic or fiscal outcomes in the PCMS model.

Since in the PCMS model household demand for bills (equation 37) responds to the return on bills (ERr_b) and these government securities are floating rate bills linked to the *Selic* rate, changes in λ_1 do not lead to shifts in household asset allocation. Again, introducing fixed-rate bonds and a more complex private debt demand for government securities would then provide the model potential to analyze whether interest rate sensitivity would matter under a more complex debt structure.

We will now analyze a fiscal shock in which the growth of pure government expenditure increases from 2% in the baseline scenario to 4% (shock scenario). As expected, the average primary result to GDP decreases with the shock (Figure 20). Since government spending increases at a faster rate than in the baseline, while tax revenues adjust endogenously, the fiscal balance deteriorates as expenditures rise more rapidly than revenues.

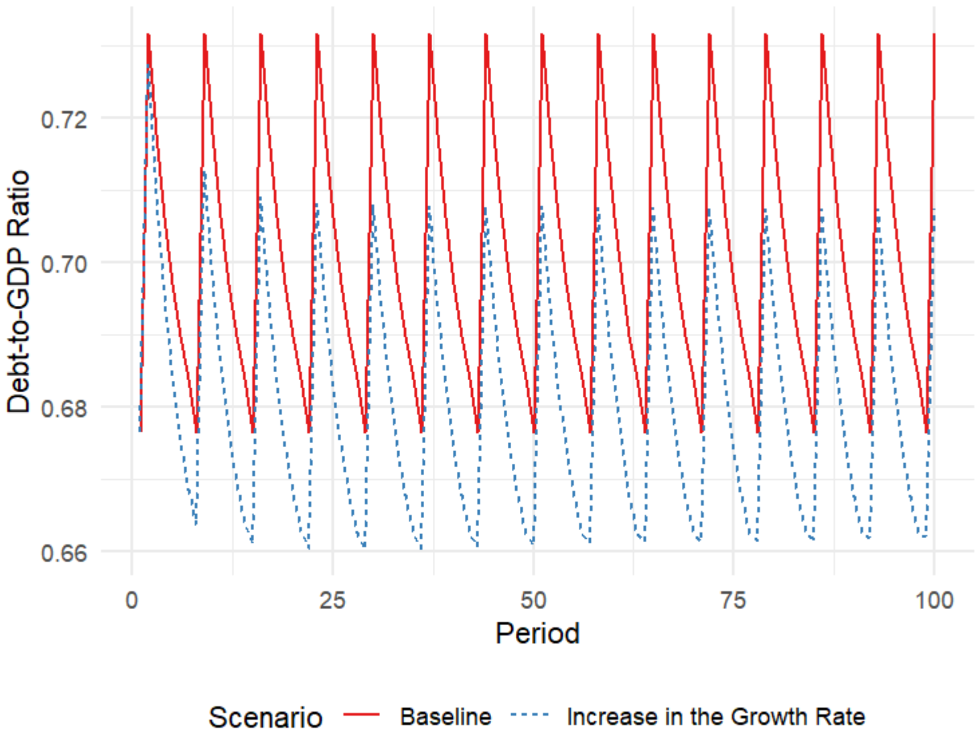
Figure 20 – Fiscal result dynamics under fiscal policy shock vs baseline scenario



Source: Author's elaboration

Despite the deterioration in the primary fiscal result, Figure 21 shows that debt-to-GDP ratios actually decline over time following a shock that increases permanently government expenditure growth ratio. This result is consistent with the demand-driven growth mechanism in the model, where higher government expenditures lead to faster economic growth (equation 1 combined with equation 11), effectively reducing the relative burden of debt. The short-term debt ratio response is relatively muted, but as GDP expands, the debt-to-GDP ratio falls to a significantly lower average range in the long run. This highlights an important insight: higher government spending does not necessarily lead to rising debt-to-GDP ratios, particularly when economic growth is responsive to fiscal stimulus.

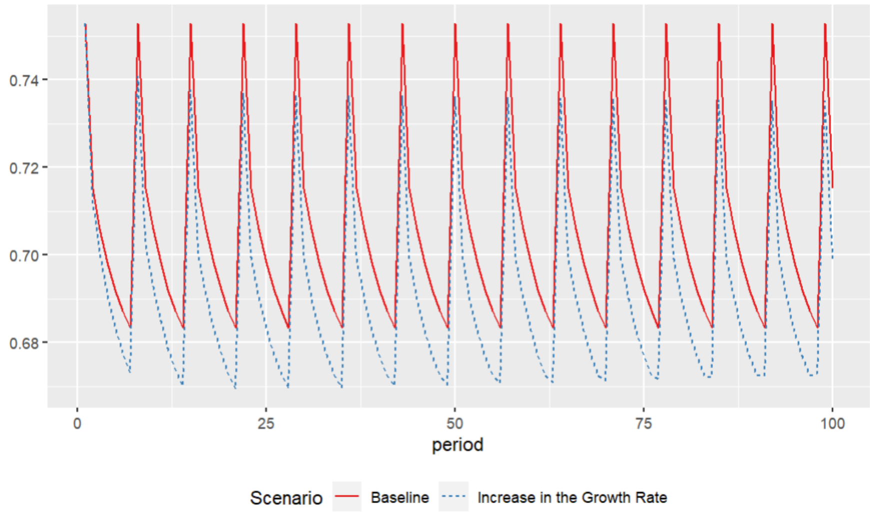
Figure 21 - Debt-to-GDP dynamics under fiscal policy shock vs baseline scenario



Source: Author's elaboration.

Interestingly, Figure 22 shows that the consumption-to-GDP ratio decreases in response to the shock, despite the economy experiencing higher growth. This outcome emerges due to the relative expansion of government expenditures compared to private sector activity: since GDP is demand-driven, the shock increases total output, but a larger share of this output is now composed of government spending rather than private consumption. Households' disposable income (YD) increases, but since government spending plays a dominant role in the expansion, the proportion of consumption in GDP declines. This reflects a reallocation of economic activity, where government expenditures crowd-in growth but shift the composition of aggregate demand.

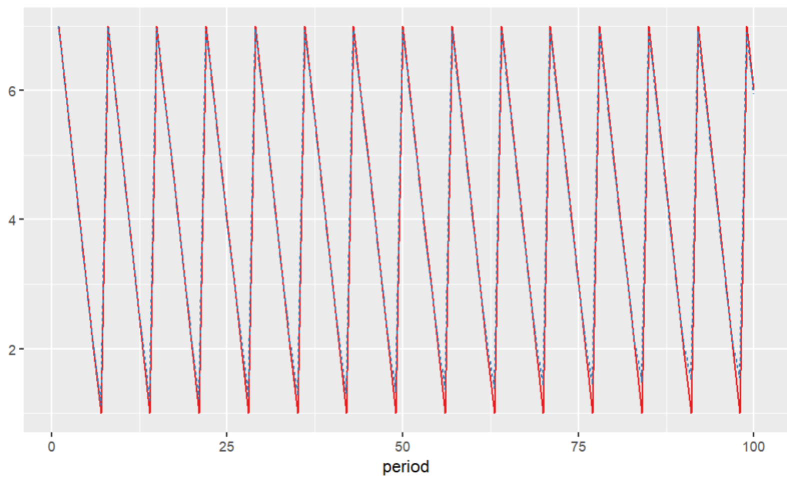
Figure 22 - Consumption-to-GDP ratio under fiscal shock



Source: Author’s elaboration.

Unlike other model variables, the average life of public debt remains unchanged despite increased government spending (Figure 23). This result suggests that variations in public expenditure growth do not endogenously affect the debt maturity profile. There are three key reasons for this behavior. Public debt issuance and redemption patterns remain stable, meaning that the share of debt maturing on different horizons does not shift significantly. Since the government continues to roll over debt consistently, the maturity structure is not affected by higher spending. The floating-rate structure of bills in the model prevents market-driven changes in the maturity structure, as there is no direct reallocation across maturities in response to fiscal expansions. Thus, the fiscal policy stance in the PCMS model primarily influences overall debt levels rather than its maturity composition.

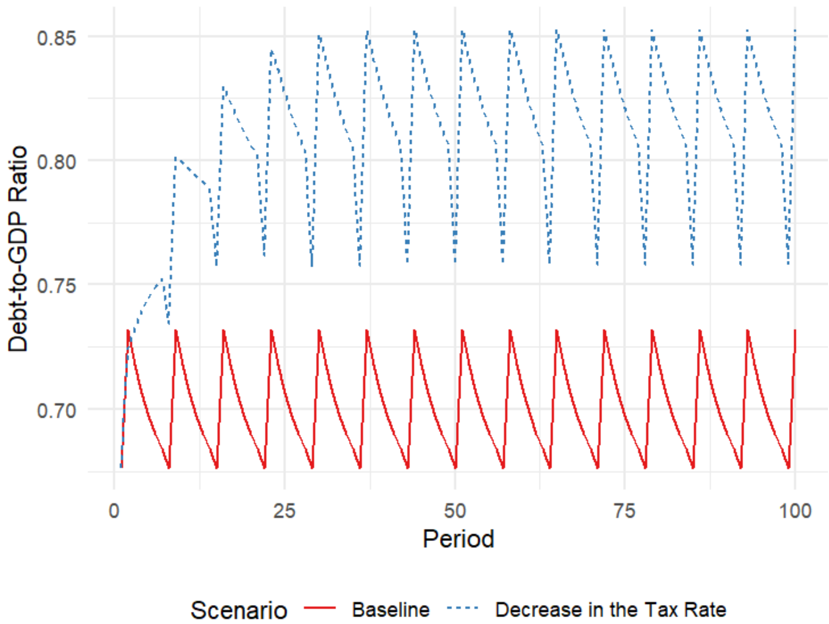
Figure 23 – Average life of public debt under fiscal shock



Source: Author’s elaboration

We now move to analyze a fiscal loosening shock through the decrease in the tax rate (θ) from 31.7% in the baseline scenario to 21% after the shock. We observe in Figure 24 that, under the shock, the debt-to-GDP ratio increases significantly. It acquires a new cyclical dynamic, creating an intermediary step between the bottom and upper part of the cycle. The emergence of this intermediary step in the debt cycle can be attributed to several interacting effects in the model. The first one is that higher fiscal deficits drive debt accumulation. With lower tax revenues, the government's primary fiscal balance deteriorates, leading to a steady debt accumulation. Since spending remains constant in this scenario, the increase in deficit financing directly contributes to higher debt levels over time. The second one is that increased net issuances introduce an additional feedback loop through two channels: (i) higher issuance volumes that increase outstanding debt stock; (ii) debt servicing costs that rise as the stock of debt expands, reinforcing the need for further issuances. The model preserves cyclical fiscal behavior but at higher debt levels.

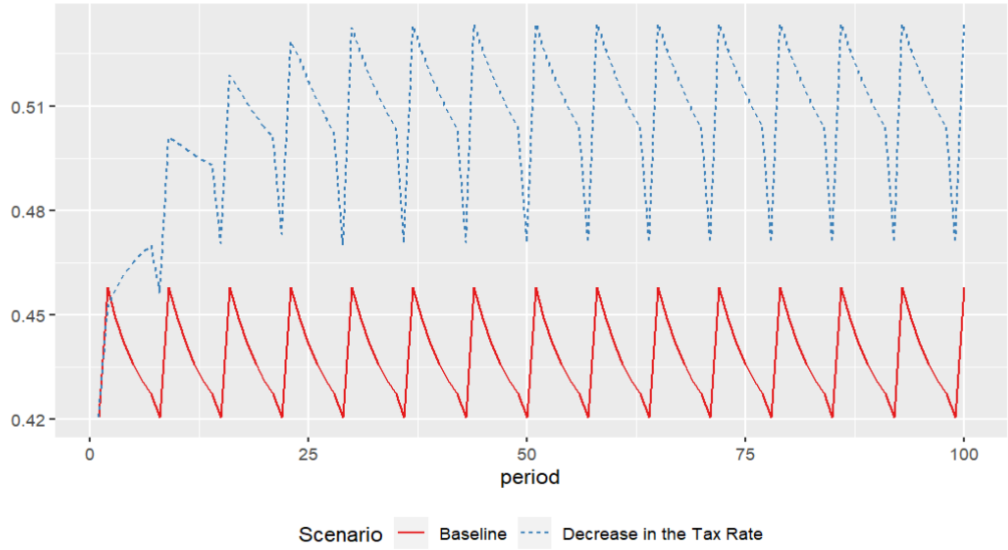
Figure 24 – Debt-to-GDP dynamics under tax loosening shock vs. baseline scenario



Source: Author's elaboration.

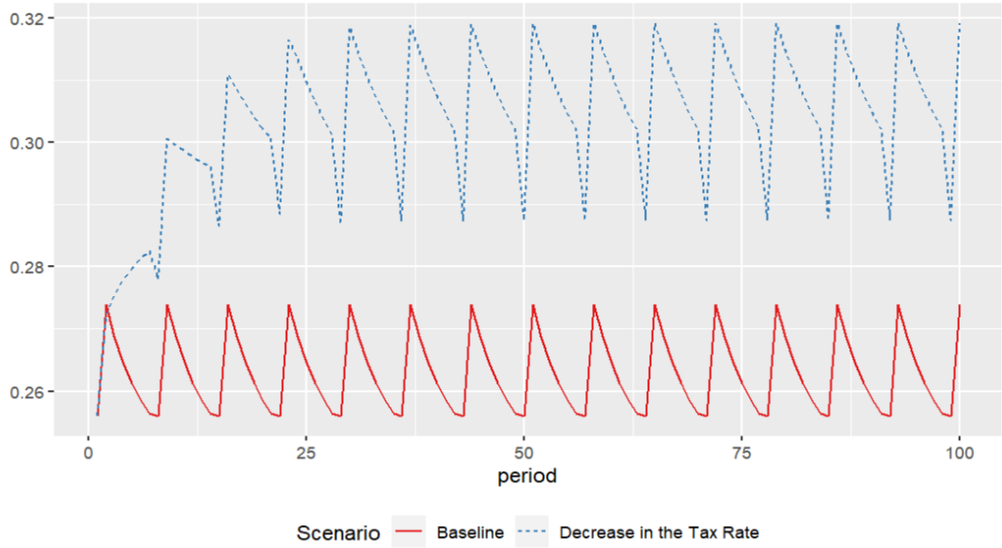
Figures 25 and 26 show that privately- and BCB held- debt-to-GDP ratios behave symmetrically following a tax loosening shock, indicating that debt composition remains unaltered. Regarding maturity structure, the average life of debt is also unchanged.

Figure 25 - Privately-held government debt-to-GDP ratio under tax loosening shock



Source: Author's elaboration

Figure 26 – BCB-held government debt-to-GDP ratio under tax loosening shock



Source: Author's elaboration.

Finally, we test the model for two alternative initial debt maturity schedules, one mirroring the December 1999 FPD maturity structure and one mirroring the February 2016 FPD maturity structure, presented in Table 7, below. These dates were chosen because they represent two extreme situations: while December 1999's context had a strong concentration in short-term debt, February 2016 was when the share of securities longer than five years peaked. For a broader view of the historical evolution of debt maturities, see Figure 1 in Section 2.

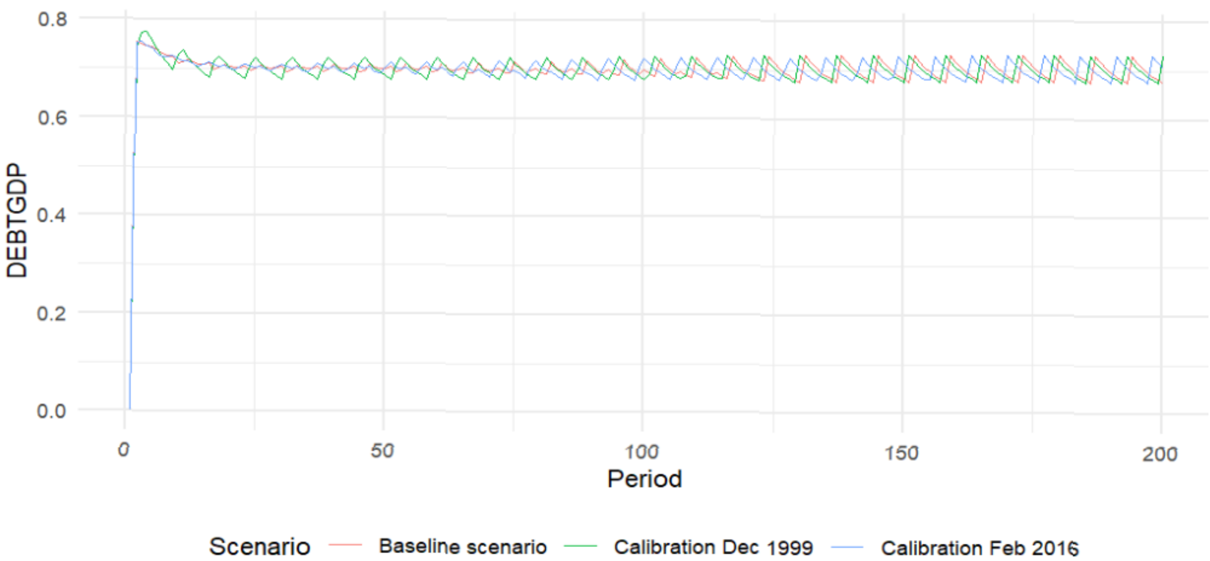
Table 7 – Alternative DFPD maturity structures

	DFPD maturities ratio December 1999		DFPD maturities ratio February 2016	
Within 1 year	b1	54,94%	b1	22,88%
Between 1 year and 2 years	b2	23,26%	b2	12,28%
Between 2 and 3 years	b3	2,47%	b3	13,73%
Between 3 and 4 years	b4	2,26%	b4	7,66%
Between 4 and 5 years	b5	2,15%	b5	10,12%
Later than 5 years	b6 + b7	14,92%	b6 + b7	33,34%
Total	1	100,00%	1	100,00%

Source: Author's elaboration. National Treasury Monthly Debt Report. December 2024.

Figure 27 reveals that the initial debt composition scenario plays an important role in debt dynamics even in the medium term. Although all three calibrations eventually converge to similar fluctuation patterns and average level of debt-to-GDP ratios in the long-run, the short- and medium-term behavior differs considerably depending on the initial maturity structure. It appears that the stronger the concentration of maturities in the short term, the stronger the volatility and the higher the debt-to-GDP ratio. Since *LFT* bills accrue daily interest, a larger share of short-term debt structure forces the government to refinance debt more frequently, which, under daily capitalization, amplifies debt accumulation due to compounding effects. Longer-term concentrated maturity structures probably reduce the frequency of debt rollovers and capitalized debt growth, leading to lower debt levels in the medium term.

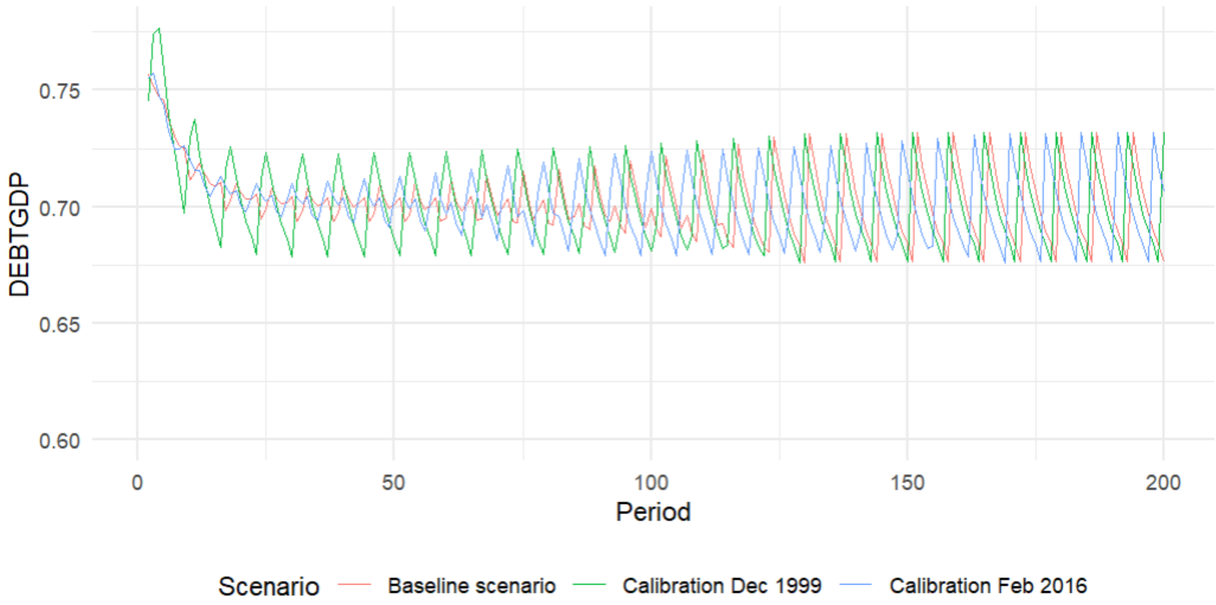
Figure 27 – Debt-to-GDP ratios under three alternative calibrations for maturity structure



Source: Author's elaboration

Finally, Figure 28 zooms in on the Figure 27 graph for a clearer visualization.

Figure 28 – Debt-to-GDP ratios under three alternative calibrations for maturity structure



Source: Author's elaboration.

FINAL REMARKS

This dissertation builds an SFC model to analyse the role of sovereign debt maturity structure within Brazil's institutional framework. As far as we are aware, explicitly incorporating debt maturities in an SFC portfolio choice framework is an original contribution of this study to the literature. The model was built as simply as possible to allow for assessment of the effects of considering debt maturity structure explicitly. Our findings show that debt maturities introduce significant cyclical fluctuations, even under stable private demand for government bills and a central bank acting as a last-resort buyer. This characteristic renders the PCMS model particularly relevant for analysing emerging markets, where debt maturity structures may play a key role in government borrowing dynamics.

Monetary policy shocks (whether tightening or loosening) do not impact any model variables other than the price of bills. Similarly, shifts in the interest rate sensitivity of household demand for bills does not produce effects. On one hand these results stem from a setback of the model: the simplified structure of debt instruments, which includes as a single debt instrument floating-rate government bills linked to the monetary policy rate. On the other hand, this finding suggest that when government debt is primarily composed of floating-rate instruments, monetary policy changes may have limited macroeconomic effects, as interest rate adjustments are largely absorbed by debt yields.

In contrast, we encounter that fiscal policy shocks yield notable effects within the model. An increase in government expenditures reduces the debt-to-GDP ratio, while a tax rate reduction leads to a rise in the debt-to-GDP ratio, introducing changes to the cyclical fluctuations.

Finally, we observe that the initial debt maturity structure plays a crucial role in short- and medium-term debt dynamics. A higher concentration of short-term debt leads to greater volatility and a higher average debt-to-GDP ratio, as frequent refinancing requirements amplify debt accumulation under daily interest capitalization.

While our findings challenge the orthodox concept of *bond vigilantes* — as the model assumes that demand for domestic sovereign debt is unlikely to face a sudden stop — we emphasize a relatively underexplored issue in heterodox debates: the structure of debt maturities. Institutional constraints, particularly self-imposed restrictions on debt issuance and maturity benchmarks, may limit the Treasury's ability to manage debt composition actively, reinforcing the idea that debt dynamics are not purely policy-driven but also shaped by institutional and market-imposed constraints.

Introducing fixed-rate government bonds, relaxing the hypothesis that bills are always sold at *par*, and modelling a more complex private sector or a flexible maturity structure are extensions that may potentially provide deeper insights into the role of public debt composition in emerging markets.

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ANNEX 1

R Programming code for PCMS model

```
###### PORTFOLIO CHOICE WITH MATURITY STRUCTURE (PCMS) Model #####

#Package for simulation and analysis of SFC models
library(sfcr)
#Additional package for data manipulation and visualization
library(tidyverse)
#Package for Sankey diagram
library(networkD3)
#Package for DAGs
library(ggraph)

#Before calibrating, some calculation

PVTDEBT_cal <- 6966.88 #debt held by the market in R$ bn according to the MDR Dec-24
pb_cal  <- 0.000015682353507 #LFT nominal value in R$ bn
Bh_cal <- PVTDEBT_cal/pb_cal #initial quantity of bills held by the market
Bcb_cal <- 2517.51 #debt held by the BCB in R$ bn according to the MDR Dec-24
rb_cal <- 0.0713
pbminus1_cal <- pb_cal/(exp(rb_cal/252))
pbminus1_cal_disc <- pb_cal/(1+(rb_cal/252))
pbminus1_cal_disc
Bm_cal <- 1369.18988/0.00001567792
Bm_cal


##### Calibrating model PCMS #####
pcms2.calib<-sfcr_set(
  eq1 = pb[-1] ~ pb/(exp((rb/252))),
  eq2 = C ~ Y - G,
  eq3 = G ~ Bcb - (Bcb/(1+g)) + Tax - pb * Bm + (pb * Bnet - pb * Bm ),
  eq4 = YD ~ Y - Tax + (pb * Bm),
```

```
  eq5 = Tax ~ theta * (Y + pb * Bm ),
  eq7 = CG ~ (pb - pb[-1])*((Bh/(1+g)) - Bm),
  eq8 = V ~ H + pb * Bh,
  eq9 = H  ~ Bcb,
  eq10 = Bhd ~ Bh,
  eq11 = Bh ~ bh * Y / pb,
  eq12 = Errb ~ log(pb/pb[-1]),
  eq13 = Bcb ~ b * Y,
  eq14 = Bnet ~ max(Bh - (Bh/(1+g)) + Bm, Bm - Bh/(1+g)),
  eq15 = Bnr ~ abs( min(Bnet,0) ),
  eq16 = Bm1 ~ b1 * Bh,
  eq17 = Bm2 ~ b2 * Bh,
  eq18 = Bm3 ~ b3 * Bh,
  eq19 = Bm4 ~ b4 * Bh,
  eq20 = Bm5 ~ b5 * Bh,
  eq21 = Bm6 ~ b6 * Bh,
  eq22 = Bm7 ~ b7 * Bh,
  )


pcms2.calib.input<-sfcr_set(
  b ~ 0.2131658,
  bh ~ 0.5899086,
  rb ~ 0.0732,
  pb ~  0.000015682353507,
  g ~ 0.02,
  b1~0.1789,
  b2~0.1743,
  b3~0.1789,
  b4~0.1067,
  b5~0.1104,
  b6~0.12535,
  b7~0.12535,
  theta ~ 0.3173578,
```

```
  alpha1 ~ 0.6,
  alpha2 ~ 0.4,
  lambda0 ~ 0.635,
  lambda1 ~ 0.0500043,
  lambda2 ~ 0.01,
  Y ~ 11810.1012,
  Bm ~ 87332368
)

pcms2.calib.output <- sfcr_baseline(
  equations = pcms2.calib,
  external = pcms2.calib.input,
  periods = 2,
  method = "Gauss"
)

pcms2.calib.output <- pcms2.calib.output[-1, ]



## Defining the model equations
pccg_model <- sfcr_set(
  eq1 = G ~ G[-1] * (1+ g),
  #eq2 = rb ~ rb_hat,
  eq3 = pb ~ pb[-1]* exp(rb/252),
  eq4 = H ~ V - (pb * Bh),
  eq5 = Y ~ C + G,
  eq6 = Tax ~ theta * (Y + pb[-1] * Bm ),
  eq7 = Bcb ~ Bcb[-1] + G - Tax + pb[-1] * Bm - (pb * Bnet - pb[-1] * Bm),
  eq8 = Bnet ~ max(Bhd - Bh[-1] + Bm, Bm - Bh[-1]),
  eq9 = CG ~ (pb - pb[-1])*(Bh[-1] - Bm),
  eq10 = YD ~ Y - Tax + (pb[-1] * Bm),
  eq11 = V ~ V[-1] + (YD - C) + CG,
  eq12 = C ~ alpha1 * YD + alpha2 * V[-1],
  eq13 = Bnr ~ abs( min(Bnet,0) ),
```

```
  eq14 = Bm ~ Bm1[-1],
  eq15 = Bm1 ~ max(Bm2[-1] - Bnr, 0),
  eq16 = Bm2 ~ ifelse(Bnr <= Bm2[-1], Bm3[-1], max(Bm3[-1] - (Bnr - Bm2[-1]), 0)),
  eq17 = Bm3 ~ ifelse(Bnr <= Bm2[-1] + Bm3[-1], Bm4[-1], max(Bm4[-1] - (Bnr - Bm2[-
1] - Bm3[-1]), 0)),
  eq18 = Bm4 ~ ifelse(Bnr <= Bm2[-1] + Bm3[-1] + Bm4[-1], Bm5[-1], max(Bm5[-1] -
(Bnr - Bm2[-1] - Bm3[-1] - Bm4[-1]), 0)),
  eq19 = Bm5 ~ ifelse(Bnr <= Bm2[-1] + Bm3[-1] + Bm4[-1] + Bm5[-1], Bm6[-1], max(
Bm6[-1] - (Bnr - Bm2[-1]- Bm3[-1] - Bm4[-1] - Bm5[-1]), 0)),
  eq20 = Bm6 ~ ifelse(Bnr <= Bm2[-1] + Bm3[-1] + Bm4[-1] + Bm5[-1] + Bm6[-1], Bm7[-
1], max( Bm7[-1] - (Bnr - Bm2[-1]- Bm3[-1] - Bm4[-1] - Bm5[-1] - Bm6[-1]), 0)),
  eq21 = Bm7 ~ max(Bnet, 0),
  eq22 = Bh ~ Bm1 + Bm2 + Bm3 + Bm4 + Bm5 + Bm6 + Bm7,
  eq23 = Bhd ~ (V/pb) * (lambda0  + lambda1 * Errb - lambda2 * (YD/V)) ,
  eq24 = Errb ~ log(pb/pb[-1]),
  #auxiliary variables
  DEBTGDP ~ ((Bh * pb)+ Bcb) / Y,
  PVTDEBTGDP ~ (Bh *pb)/ Y,
  PVTDEBT ~ (Bh *pb),
  BCBDEBTGDP ~ Bcb / Y,
  GGDP ~ G/Y,
  BMDEBT ~ (pb*Bm)/V, # current maturities/total outstanding debt
  vhy ~ V/Y,
  hy ~ H/ Y,
  bv ~ (pb * Bh)/V,
  hv ~ H/ V,
  PRGDP ~ (Tax-G)/Y,
  AVGLIFE ~ (Bm1 * 1 + Bm2 * 2 + Bm3 * 3 + Bm4 * 4 + Bm5 *5 + Bm6 * 6 + Bm7 *
7)/Bh,
  VC12MONTHS ~ Bm1/Bh,
  pby ~ pb/Y,
  byperc ~ Bh/Y
)
```

```
pcms2.pars <- sfcr_set( # exógenas e parâmetros
  rb ~ pcms2.calib.output$rb,
  g ~ pcms2.calib.output$g,
  theta ~ pcms2.calib.output$theta,
  alpha1 ~ pcms2.calib.output$alpha1,
  alpha2 ~ pcms2.calib.output$alpha2,
  lambda0~pcms2.calib.output$lambda0,
  lambda1~pcms2.calib.output$lambda1,
  lambda2~pcms2.calib.output$lambda2
)

pcms2.init<- sfcr_set( #endogenous variables
  pb~pcms2.calib.output$pb,
  G~pcms2.calib.output$G,
  C~pcms2.calib.output$C,
  Y~pcms2.calib.output$Y,
  YD~pcms2.calib.output$YD,
  Tax~pcms2.calib.output$Tax,
  CG~pcms2.calib.output$CG,
  V~pcms2.calib.output$V,
  Bh~pcms2.calib.output$Bh,
  Bm~pcms2.calib.output$Bm,
  Bnet~pcms2.calib.output$Bnet,
  Bnr~pcms2.calib.output$Bnr,
  Bm1~pcms2.calib.output$Bm1,
  Bm2~pcms2.calib.output$Bm2,
  Bm3~pcms2.calib.output$Bm3,
  Bm4~pcms2.calib.output$Bm4,
  Bm5~pcms2.calib.output$Bm5,
  Bm6~pcms2.calib.output$Bm6,
  Bm7~pcms2.calib.output$Bm7,
  Bhd~pcms2.calib.output$Bhd,
  Bcb~pcms2.calib.output$Bcb,
```

```
  H~pcms2.calib.output$H,
  Errb~pcms2.calib.output$Errb
)

pcms2 <- sfcr_baseline(
  equations = pccg_model,
  external = pcms2.pars,
  initial = pcms2.init,
  periods = 200,
  tol = 1e-15,
  method = "Broyden"
)

#Writing down balance sheet matrix

bs_pcms <- sfcr_matrix(
  columns = c("Households", "Firms", "Treasury", "Central bank", "sum"),
  codes = c("h", "f", "g", "cb", "s"),
  r1 = c("Money", h = "+H", cb = "-H"),
  r2 = c("Bills market", h = "+ pb * Bh", g = "-pb * Bh"),
  r3 = c("Bills BCB", g = "-Bcb", cb = "+Bcb"),
  r4 = c("Balance (net worth)", h = "-V", g = "+V")
)

sfcr_matrix_display(bs_pcms, "bs")

#Writing down transactions flow matrix

tfm_pcms <- sfcr_matrix(
  columns = c("Households", "Production", "Treasury", "CB current", "CB capital"),
  codes = c("h", "f", "g", "cbc", "cbk"),
  c("Consumption", h = "-C", f = "+C"),
  c("Govt. Expenditures", f = "+G", g = "-G"),
  c("Income", h = "+Y", f = "-Y"),
```

```
  c("Maturity payments", h = "+pb[-1] * Bm", g = "-pb[-1] * Bm"),
  c("Interest payments", g = "-rb[-1] * Bcb[-1]", cbc = "+rb[-1] * Bcb[-1]"),
  c("Central Bank profit", g = "+rb[-1] * Bcb[-1]", cbc = "-rb[-1] * Bcb[-1]"),
  c("Taxes", h = "-Tax", g = "+Tax"),
  c("Ch. Money", h = "-d(H)", cbk = "+d(H)"),
  c("Net issuances market", h = "-(pb * Bnet - pb[-1]* Bm)", g = "+(pb * Bnet - pb[-1] *
Bm)"),
  c("Net issuances BCB", g = "+d(Bcb)", cbk = "-d(Bcb)"),
  c("Capital gains", h = "-d(pb) *(Bh[-1] - Bm)", g = "+d(pb) *(Bh[-1] - Bm)")
)

sfcr_matrix_display(tfm_pcms)

sfcr_validate(tfm_pcms, pcms2, which = "tfm", tol = 1e-7, rtol = TRUE )
```