International Journal of Numerical Methods for Heat & Fluid Flow

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Received 17 August 2014
Revised 17 August 2014
Accepted 30 September 2014

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Abstract

Purpose – The purpose of this paper is to employ the Generalized Integral Transform Technique in the analysis of conjugated heat transfer in micro-heat exchangers, by combining this hybrid numerical-analytical approach with a reformulation strategy into a single domain that envelopes all of the physical and geometric sub-regions in the original problem. The solution methodology advanced is carefully validated against experimental results from non-intrusive techniques, namely, infrared thermography measurements of the substrate external surface temperatures, and fluid temperature measurements obtained through micro Laser Induced Fluorescence.

Design/methodology/approach – The methodology is applied in the hybrid numerical-analytical treatment of a multi-stream micro-heat exchanger application, involving a three-dimensional configuration with triangular cross-section micro-channels. Space variable coefficients and source terms with abrupt transitions among the various sub-regions interfaces are then defined and incorporated into this single domain representation for the governing convection-diffusion equations. The application here considered for analysis is a multi-stream micro-heat exchanger designed for waste heat recovery and built on a PMMA substrate to allow for flow visualization.

Findings – The methodology here advanced is carefully validated against experimental results from non-intrusive techniques, namely, infrared thermography measurements of the substrate external surface temperatures and fluid temperature measurements obtained through Laser Induced Fluorescence. A very good agreement among the proposed hybrid methodology predictions, a finite elements solution from the COMSOL code, and the experimental findings has been achieved. The proposed methodology has been demonstrated to be quite flexible, robust, and accurate.

Originality/value – The hybrid nature of the approach, providing analytical expressions in all but one independent variable, and requiring numerical treatment at most in one single independent variable, makes it particularly well suited for computationally intensive tasks such as in optimization, inverse problem analysis, and simulation under uncertainty.

Keywords Infrared thermography, Micro-channels, Conjugated problem, Hybrid methods, Integral transforms, Micro-LIF, Micro-heat exchangers, Conjugated heat transfer, Micro-laser induced fluorescence

Paper type Research paper

International Journal of Numerical Methods for Heat & Fluid Flow
Vol. 25 No. 6, 2015
pp. 1444-1462

0961-5539
DOI 10.1108/HFF-08-2014-0259

The authors would like to acknowledge the financial support provided by the Brazilian sponsoring agencies CNPq and FAPERJ. Diego C. Knupp is deeply indebted to Jovo Vidic (ETH-Zürich) for all the support in assembling the μ-LIF experimental setup.

Nomenclature

h_{eff} effective heat transfer coefficient at the heat exchanger external surface, in contact with the surrounding environment;
k thermal conductivity;
L_x, L_y, L_z height, width, and thickness of the heat exchanger;
M truncation order of the eigenvalue problem solution via integral transforms;
N normalization integrals;
T temperature field;
u flow velocity;
w thermal capacitance;
x longitudinal coordinate;
y transversal coordinate (width);
z transversal coordinate (thickness);

Greek letters
ψ eigenfunction of the eigenvalue problem with space variable coefficients;
μ eigenvalues corresponding to ψ;
Ω eigenfunction of the auxiliary eigenvalue problem;
λ eigenvalues corresponding to Ω;

Subscripts and Superscripts
in quantity corresponding to the entrance of the channel ($x = 0$);
i order of eigenquantities;
* filtered temperature field;
s quantity corresponding to the solid region;
f quantity corresponding to the fluid flow region;
\- integral transform;
~ normalized eigenfunction;

1. Introduction

The analytical treatment of conjugated conduction-convection problems has been pursued for many years since the pioneering works of Perelman (1961) and Luikov *et al.* (1971) requiring more involved mathematical approaches than either the pure conduction or the pure convection problems. In fact, conjugation effects are quite frequently neglected in heat transfer models for applications at the macro-scale, which allows the direct treatment of convective heat transfer with simplified boundary conditions. Nevertheless, for different micro-scale applications, it has been observed that the conjugation effects can be essential in appropriately quantifying heat transfer rates (Morini, 2004; Maranzana *et al.*, 2004; Hetsroni *et al.*, 2005; Nunes *et al.*, 2010; Horvat and Catton, 2003).

Even though modern numerical methods may readily handle conjugated problems within different platforms and through different methodologies, the availability of analytical solutions for such class of problems remains of major interest, not only in the context of validation and verification, but also for handling more computationally intensive tasks, such as optimization, inverse analysis, and more recently, stochastic simulation due to properties, geometric, or source terms uncertainties. The method of integral transforms for solving partial differential equations dates back to Fourier's classical treatise on heat conduction first published in 1822, and has been widely used and developed since then in the analytical solution of linear heat and mass diffusion problems (Koshlyakov, 1936; Luikov, 1968, 1980; Ozisik, 1968, 1980; Mikhailov and Özisik, 1994/1984). It is essentially based on the proposition of eigenfunction expansions, associated with the eigenvalue problems that naturally appear after application of the method of separation of variables to the homogeneous versions of the originally posed problems. The integral transformation process is in general applied to all but one independent variable. Then, in such transformable cases, it yields a decoupled linear system of ordinary differential equations for the transformed potentials, for which analytical solutions can be readily obtained. By mid 1980s this classical approach gained a hybrid numerical-analytical structure, known as the Generalized Integral Transform Technique (GITT) (Cotta and Ozisik, 1986, 1987; Aparecido *et al.*, 1989; Cotta, 1990, 1993, 1994a, b, 1998; Serfaty and Cotta, 1990, 1992; Diniz *et al.*, 1990; Perez Guerrero and Cotta, 1992; Machado and Cotta, 1995; Cotta and Mikhailov, 1997) for overcoming the barrier of handling non-transformable problems, such as in the cases of nonlinear diffusion problems, irregular geometries, moving boundaries, boundary layer equations, Navier-Stokes equations, etc., as reviewed in Cotta and Mikhailov (2006).

Concerning the conception and design of thermal microsystems, numerous works are still aimed at the investigation of the most adequate models and solution methodologies for describing the physical phenomena that take place in such micro-scale heat transfer problems. For instance, it has been observed for different micro-scale applications that conjugation effects can be essential in appropriately quantifying heat transfer rates, while usual macro-scale based correlations can result in considerable deviations from both measurements and simulations accounting for the participation of the whole micro-system structure (Morini, 2004; Maranzana *et al.*, 2004; Hetsroni *et al.*, 2005; Nunes *et al.*, 2010; Horvat and Catton, 2003). In this context and quite recently, a hybrid numerical-analytical approach has been advanced for the solution of conjugated heat transfer problems, based on the GITT and on a single domain formulation of the solid-fluid coupling (Knupp *et al.*, 2012, 2013a, b). This same approach has also been further advanced in order to allow for the analysis of complex physical and geometrical configurations, such as heat and fluid flow in multiple and irregular sub-regions (Knupp *et al.*, 2014a, b; Cotta *et al.*, 2014).

The present work is thus aimed at the theoretical and experimental analysis of conjugated heat transfer in a multi-stream micro-heat exchanger, designed for waste heat recovery applications. The experimental investigation is undertaken for validation purposes, employing non-intrusive optical techniques for both temperature and velocity measurements. The temperature measurements of the micro-heat exchanger external surface are obtained through infrared thermography. Fluid temperature measurements within the micro-channels are also provided, obtained through a micro Laser Induced Fluorescence (μ-LIF) system. The analyzed configuration and associated experimental setups provide a complete verification and validation framework for the proposed combined hybrid approach based on integral transforms.

2. Problem formulation and solution methodology

The conjugated heat transfer problem here considered comes from applications related to waste heat recovery employing multi-stream micro-heat exchangers. A model for flow visualization made on PMMA through laser ablation is shown in Figure 1. The heat exchanger section is graphically represented in Figures 2(a) and (b) for the whole set of channels and for a single micro-channel.

The conjugated problem, involving conduction in the acrylic substrate and laminar forced convection in the water flow within the six triangular shapped micro-channels, is thus combined into a single domain formulation. Motivated by the fairly high values of the Péclet number and relatively low thermal conductivity of the substrate, the longitudinal conduction terms both in the fluid and along the plate are neglected for the

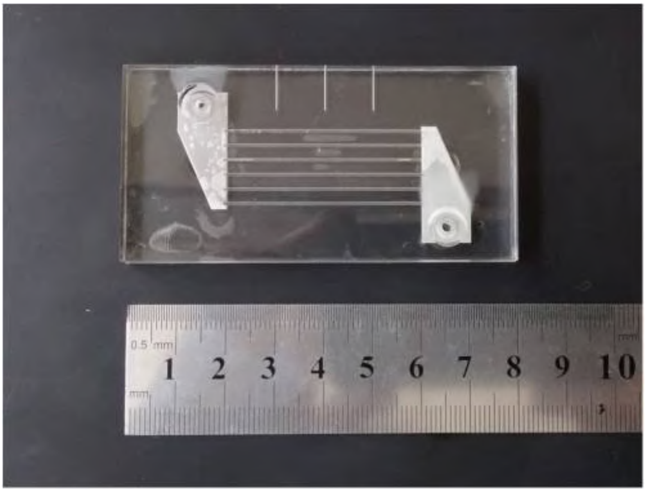


Figure 1. PMMA model of the micro-heat exchanger made by laser ablation

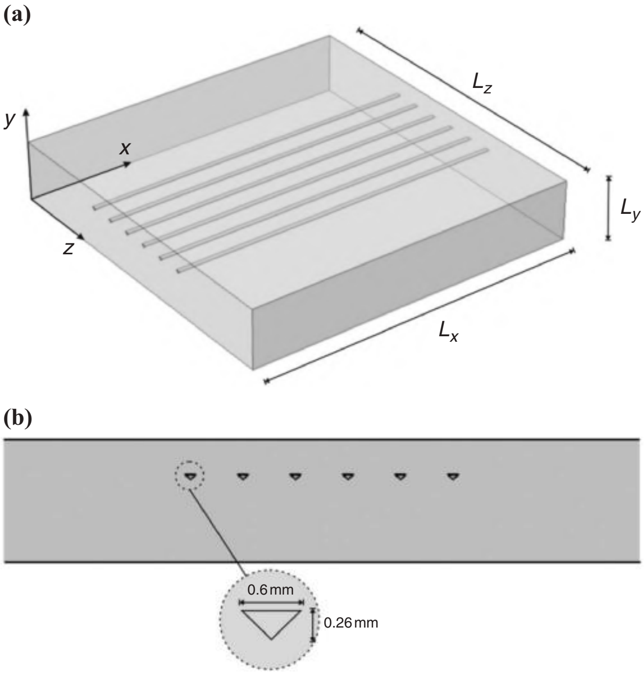


Figure 2. (a) Representation of the micro-heat exchanger for the single domain formulation; (b) details of the micro-channel geometry

sake of simplicity. Therefore, the formulation of the steady state conjugated problem as a single region model is achieved by making use of space variable coefficients represented as functions with abrupt transitions at the fluid-wall interfaces, as follows:

$$w(y,z)\frac{\partial T(x,y,z)}{\partial x} = \frac{\partial}{\partial y}\left(k(y,z)\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k(y,z)\frac{\partial T}{\partial z}\right),$$
$$0 < y < L_y, 0 < z < L_z, 0 < x < L_x \quad (1a)$$

$$T(0, y, z) = T_{in}(y, z) \tag{1b}$$

$$-k_s \frac{\partial T}{\partial y}\bigg|_{y=0} + h_{eff} T(x, 0, z) = h_{eff} T_\infty, \quad k_s \frac{\partial T}{\partial y}\bigg|_{y=L_y} + h_{eff} T(x, L_y, z) = h_{eff} T_\infty \tag{1c,d}$$

$$-k_s \frac{\partial T}{\partial z}\bigg|_{z=0} + h_{eff} T(x, y, 0) = h_{eff} T_\infty, \quad k_s \frac{\partial T}{\partial z}\bigg|_{z=L_z} + h_{eff} T(x, y, L_z) = h_{eff} T_\infty \tag{1e,f}$$

with:

$$w(y, z) = \begin{cases} w_f \overline{u}_f, \text{ fluid} \\ 0, \text{ solid} \end{cases} \quad \text{and} \quad k(y, z) = \begin{cases} k_f, \text{ fluid} \\ k_s, \text{ solid} \end{cases} \tag{1g,h}$$

where $\overline{u}_f$ is the average flow velocity in the micro-channel, w_f is the fluid thermal capacitance, k_f and k_s are the thermal conductivities of the fluid (water) and the solid (PMMA), respectively. The inlet condition given by Equation (1b), refers to both the temperatures at the channel inlets and at the substrate surface.

To improve the computational performance of the formal solution to be derived below using the integral transforms method, the boundary conditions are filtered to become homogeneous, and in this work if suffices to propose the room temperature, T_∞ as the filter:

$$T(x, y, z) = T_\infty + T^*(x, y, z) \tag{2}$$

resulting in the following homogeneous problem:

$$w(y, z)\frac{\partial T^*(x,y,z)}{\partial x} = \frac{\partial}{\partial y}\left(k(y, z)\frac{\partial T^*}{\partial y}\right) + \frac{\partial}{\partial z}\left(k(y, z)\frac{\partial T^*}{\partial z}\right), \quad 0 < y < L_y, 0 < z < L_z, 0 < x < L_x \tag{3a}$$

$$T^*(0, y, z) = T^*_{in}(y, z) = T_{in}(y, z) - T_\infty \tag{3b}$$

$$-k_s \frac{\partial T^*}{\partial y}\bigg|_{y=0} + h_{eff} T^*(x, 0, z) = 0, \quad k_s \frac{\partial T^*}{\partial y}\bigg|_{y=L_y} + h_{eff} T^*(x, L_y, z) = 0 \tag{3c,d}$$

$$-k_s \frac{\partial T^*}{\partial z}\bigg|_{z=0} + h_{eff} T^*(x, y, 0) = 0, \quad k_s \frac{\partial T^*}{\partial z}\bigg|_{z=L_z} + h_{eff} T^*(x, y, L_z) = 0 \tag{3e,f}$$

Following the GITT formalism, it is desirable that the eigenvalue problem be chosen in order to contain as much information as possible about the original problem. The eigenvalue problem is formulated by directly applying separation of variables to problem (3) so that all the information concerning the transitions of the original subdomains are represented within the eigenvalue problem, by means of the space variable coefficients $k(y,z)$ and $w(y,z)$:

$$\nabla \cdot [k(y, z) \nabla \psi(y, z)] = \mu^2 w(y, z) \psi(y, z), 0 < y < L_y, 0 < z < L_z \tag{4a}$$

$$-k_s\frac{\partial\psi}{\partial y}\bigg|_{y=0}+h_{eff}\psi(0,z)=0, k_s\frac{\partial\psi}{\partial y}\bigg|_{y=L_y}+h_{eff}\psi\left(L_y,z\right)=0 \quad (4b,c)$$

$$-k_s\frac{\partial\psi}{\partial z}\bigg|_{z=0}+h_{eff}\psi(y,0)=0, k_s\frac{\partial\psi}{\partial z}\bigg|_{z=L_z}+h_{eff}\psi(y,L_z)=0 \quad (4d,e)$$

Once problem (4) is solved, it yields the eigenvalues, μ_i, and corresponding normalized eigenfunctions, $\tilde{\psi}_i(y,z)$, with:

$$\tilde{\psi}_i(y,z)=\frac{\psi_i(y,z)}{\sqrt{N_i}},\quad N_i=\int_0^{L_z}\int_0^{L_y}w(y,z)\psi_i^2(y,z)dydz \quad (5a,b)$$

Problem (3) can be exactly transformed by operating Equation (3a) with $\int_0^{L_z}\int_0^{L_y}\tilde{\psi}_i(y,z)(\cdot)dydz$ and making use of the boundary conditions, Equations (3(e)-(f)); and (4(b)-(e)), providing the decoupled transformed ODE system:

$$\frac{d\overline{T}_i^*(x)}{dx}+\mu_i^2\overline{T}_i^*(x)=0,\ i=1,2,\ \dots \quad (6a)$$

The initial condition given by Equation (3b) is transformed through the operator $\int_0^{L_z}\int_0^{L_y}w(y,z)\tilde{\psi}_i(y,z)(\cdot)dydz$:

$$\overline{T}_i^*(0)=\overline{T}_{in,i}^*\equiv\int_0^{L_z}\int_0^{L_y}w(y,z)\tilde{\psi}_i(y,z)T_{in}^*(y,z)dydz \quad (6b)$$

Problem (6) can be analytically solved to provide:

$$\overline{T}_i^*(x)=\overline{T}_{in,i}^*\exp\left(-\mu_i^2x\right) \quad (7)$$

and the solution of problem (1) can be analytically expressed by making use of the inversion formula:

$$T(x,y,z)=T_\infty+T^*(x,y,z)=T_\infty+\sum_{i=1}^{\infty}\tilde{\psi}_i(y,z)\overline{T_i}^*(x) \quad (8)$$

Therefore, the main task in this solution path is associated with the solution of the two-dimensional eigenvalue problem with space variable coefficients, Equations (4(a)-(e)), which can also be achieved by means of integral transforms, as described in details in Cotta (1993). The solution of the particular problem given by Equation (4(a)-(e)) is undertaken trough the proposition of the following simple auxiliary eigenvalue problem:

$$\nabla^2\Omega(y,z)+\lambda^2\Omega(y,z)=0, 0<y<L_y, 0<z<L_z \quad (9a)$$

$$-k_s\frac{\partial\Omega}{\partial y}\bigg|_{y=0}+h_{eff}\Omega(0,z)=0,\ k_s\frac{\partial\Omega}{\partial y}\bigg|_{y=L_y}+h_{eff}\Omega\left(L_y,z\right)=0 \quad (9b,c)$$

$$-k_s\frac{\partial\Omega}{\partial z}\bigg|_{z=0}+h_{eff}\Omega(x,y,0)=0,\; k_s\frac{\partial\Omega}{\partial z}\bigg|_{z=L_z}+h_{eff}\Omega(x,y,L_z)=0 \qquad \text{(9d,e)}$$

Problem (9) allows for exact analytical solution after the application of separation of variables.

The proposed expansion of the original eigenfunctions is then given by:

$$transform: \overline{\psi}_i=\int_0^{L_z}\int_0^{L_y}\tilde{\Omega}_i(y,z)\psi(y,z)dydz \qquad \text{(10a)}$$

$$inverse: \psi(y,z)=\sum_{i=1}^{\infty}\tilde{\Omega}_i(y,z)\overline{\psi}_i \qquad \text{(10b)}$$

where the normalized eigenfunctions $\tilde{\Omega}_i$ are given by:

$$\tilde{\Omega}_i(y,z)=\frac{\Omega_i(y,z)}{\sqrt{N_{\Omega_i}}} \qquad \text{(10c)}$$

with:

$$N_{\Omega_i}=\int_0^{L_z}\int_0^{L_y}\Omega_i^2(y,z)dydz \qquad \text{(10d)}$$

Then the integral transformation of the original eigenvalue problem with space variable coefficients given by Equations (4(a)-(e)) is performed by operating on Equations (4a) with $\int_V\tilde{\Omega}_i(y,z)(\cdot)dV$, to yield the following algebraic problem in matrix form after truncation of the expansion to a sufficiently large order M:

$$(\mathbf{A}+\mathbf{C})\{\overline{\boldsymbol{\psi}}\}=\mu^2\mathbf{B}\{\overline{\boldsymbol{\psi}}\} \qquad \text{(11a)}$$

with the matrices' elements given by:

$$A_{ij}=-\int_S(k(y,z)-1)\tilde{\Omega}_i(y,z)\frac{\partial\tilde{\Omega}_j(y,z)}{\partial\mathbf{n}}dS+ \\ +\int_V(k(y,z)-1)\nabla\tilde{\Omega}_i(y,z)\cdot\nabla\tilde{\Omega}_j(y,z)dV \qquad \text{(11b)}$$

$$C_{ij}=\lambda_i^2\delta_{ij} \qquad \text{(11c)}$$

$$B_{ij}=\int_V u(y,z)w(y,z)\tilde{\Omega}_i(y,z)\tilde{\Omega}_j(y,z)dV \qquad \text{(11d)}$$

where V is the domain defined by the coordinates y and z, S is the contour of region V and $\mathbf{n}$ denotes the outward-drawn normal to the surface S.

The algebraic eigenvalue problem (11) can be readily solved numerically to provide results for the eigenvalues μ and corresponding eigenvectors $\{\overline{\boldsymbol{\psi}}\}$ which are then combined by the inverse formula given in Equation (10b) to provide an explicit representation of the desired original eigenfunctions, $\psi(y,z)$. Then, the transformed system solution, given by Equation (7), can be readily substituted into Equations (8), together with the calculated eigenfunctions, yielding the temperature field T representation at any desired position (x, y, z).

3. Experimental Apparatus and Procedure

3.1 IRT surface temperature measurements

For temperature measurements at the micro-heat exchanger external surface we have employed the experimental apparatus for infrared thermography analysis presented in Figure 3, assembled in the Laboratory of Nano- and Microfluidics and Microsystems, LabMEMS, at the Federal University of Rio de Janeiro, Brazil. The setup employs temperature measurements obtained from a high performance infrared camera FLIR645sc, with 640×480 pixel resolution. For more accurate temperature evaluations, the surfaces are uniformly covered with a graphite ink of known emissivity of 0.97. The main components of the setup are shown in Figure 3 as: infrared camera (FLIR645sc); camera stand; tweezers grabbing the micro-heat exchanger; thermocouples measuring the inlet fluid temperature and room temperature, which are connected to the data acquisition system (Agilent 34970-A); and syringe infusion pump. The experimental procedure is initiated by feeding the syringe with water, which is heated with the syringe heating pad (New Era Pump Systems, Inc.). Then, the data acquisition system, including thermocouples and infrared camera, is started and, next, the syringe infusion pump is turned on with a prescribed volumetric flow rate and the experiment is carried on, with the temperature field at the heat exchanger external surface being monitored and registered through the computer, until steady state is achieved. In Knupp *et al.* (2014a) the variance inference of the infrared thermography temperature measurements has been carried out for a similar experiment, and the standard deviation associated with the experimental error was calculated as $\sigma_{e,IR} = 0.407$ °C. Figures 4(a) and (b) illustrate the infrared thermography images at steady state for the front and back faces of the acrylic substrate, respectively.

3.2 μ-LIF fluid temperature measurements

For fluid temperature measurements we have employed a μ-LIF system, whose experimental apparatus is depicted in Figure 5(a), installed at the Laboratory of Thermodynamics in Emerging Technologies, LTNT, at the Swiss Federal Institute of Technology (ETH Zürich). The main components appearing in the setup are marked in Figure 5(a) as: (a) syringe pump; (b) high frequency acquisition camera of the PIV/LIF system (LaVision); (c) microscope; (d) reservoir with rhodamine B solution for

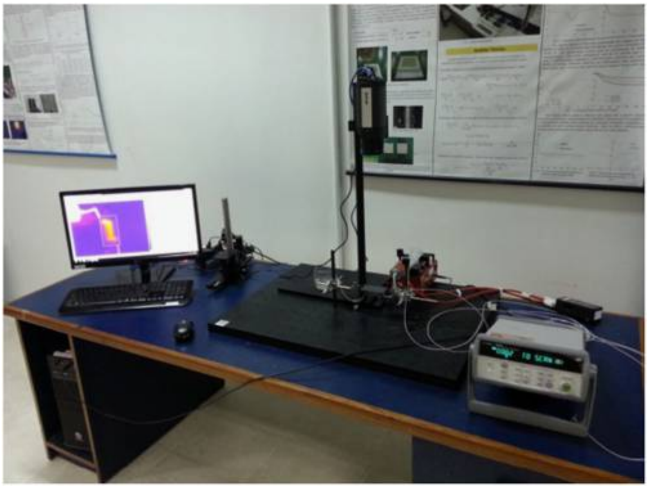
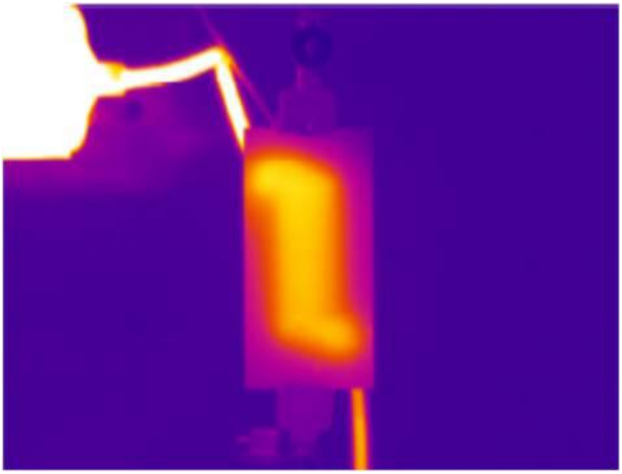
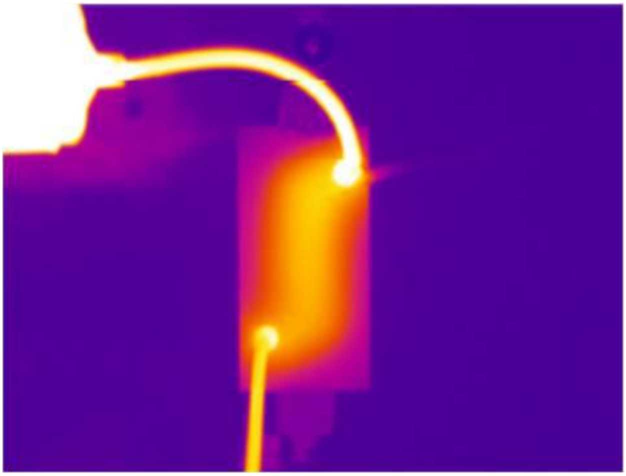


Figure 3. Experimental apparatus for infrared thermography analysis of the micro-heat exchanger

(a)



(b)



Notes: (a) Front face; (b) back face with coolant inlet-outlet

Figure 4.
Infrared thermography images of micro-heat exchanger external surfaces

filling the injection syringes; (e) heating micro-channels; (f) micro-heat exchanger under investigation; (g) cup for collection of the rhodamine B solution at the outlet of the heat exchanger; (h) PID controller system for the heating micro-channels; (i) computer interface for the PID controller system; (j) computer for the PIV/LIF system (LaVision) acquisition; and (l) laser control (PIV/LIF system). Figure 5(b) depicts in details the micro-heat exchanger positioned on the microscope for the μ-LIF fluid temperature measurements acquisition. The procedure is first initiated with the heating micro-channels filled with the rhodamine B solution, but no flow, positioned at the microscope. This heating micro-channels system (Escher *et al.*, 2010) is equipped with an RTD array for temperature monitoring and a PID controller system, in such a way that it is possible to prescribe a desired constant temperature to the rhodamine B solution inside the channels.

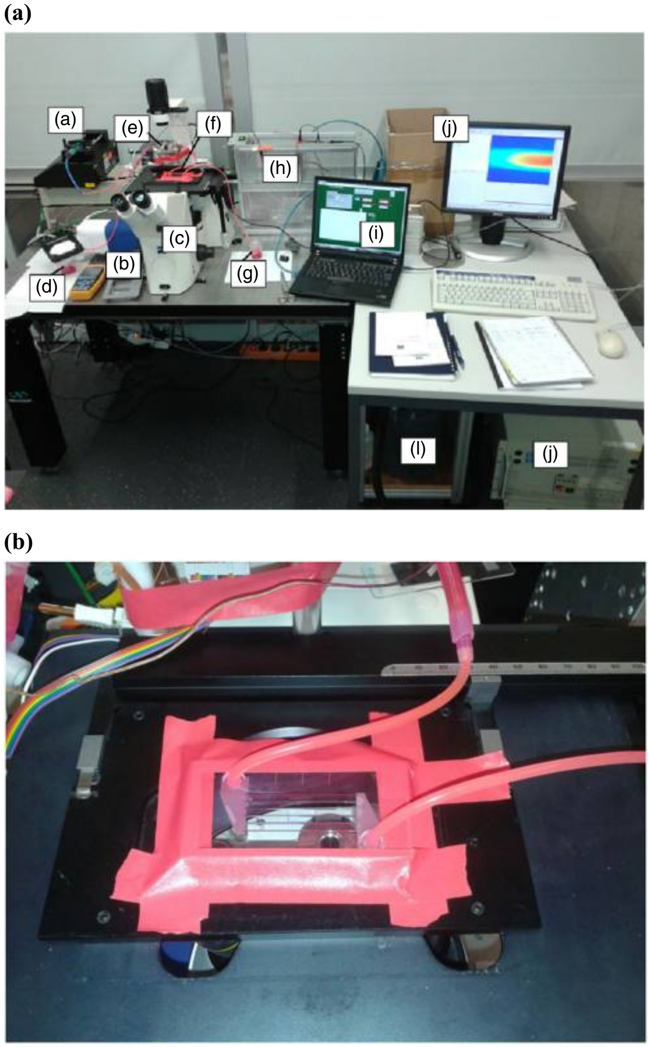


Figure 5.
(a) Experimental apparatus for fluid temperature measurements through μ-LIF; (b) micro-heat exchanger positioned at the microscope

This procedure is employed for the calibration of the LIF system regarding the correlation between the rhodamine B solution fluorescence intensities and corresponding temperatures. A detailed description of the μ-LIF procedure for fluid temperature measurements concerning this experimental setup can be found in Renfer *et al.* (2013).

Having finished the calibration step, the experimental procedure for obtaining fluid temperature measurements inside the heat exchanger under investigation is initiated by feeding the syringe with the rhodamine B solution used in the calibration step. Then the data acquisition of the μ-LIF system is started, and next the syringe infusion pump is turned on with a prescribed volumetric flow rate and the experiment is carried on, until steady state is achieved. Once the acquired images are stored (for posterior treatment), the syringe pump is turned off, and the experimental analysis can be carried on for another selected position in the heat exchanger. In all experiments it was used the same rhodamine B solution, prepared with distilled water and a 0.2 mM concentration of rhodamine B. This concentration was chosen based on the investigation of Renfer *et al.* (2013) in which the 0.2 mM concentration was found to yield satisfactory optimal signal to noise ratio.

Concerning the experimental error related to the LIF measurements, Figure 6(a) illustrates the temperature distribution along the center of seven micro-channels employed during the calibration step, in all cases the temperatures are supposedly constant and equal to each other, which does not occur in the measurements due to experimental errors. In this case, the data presented in Figure 6(a) were used in order

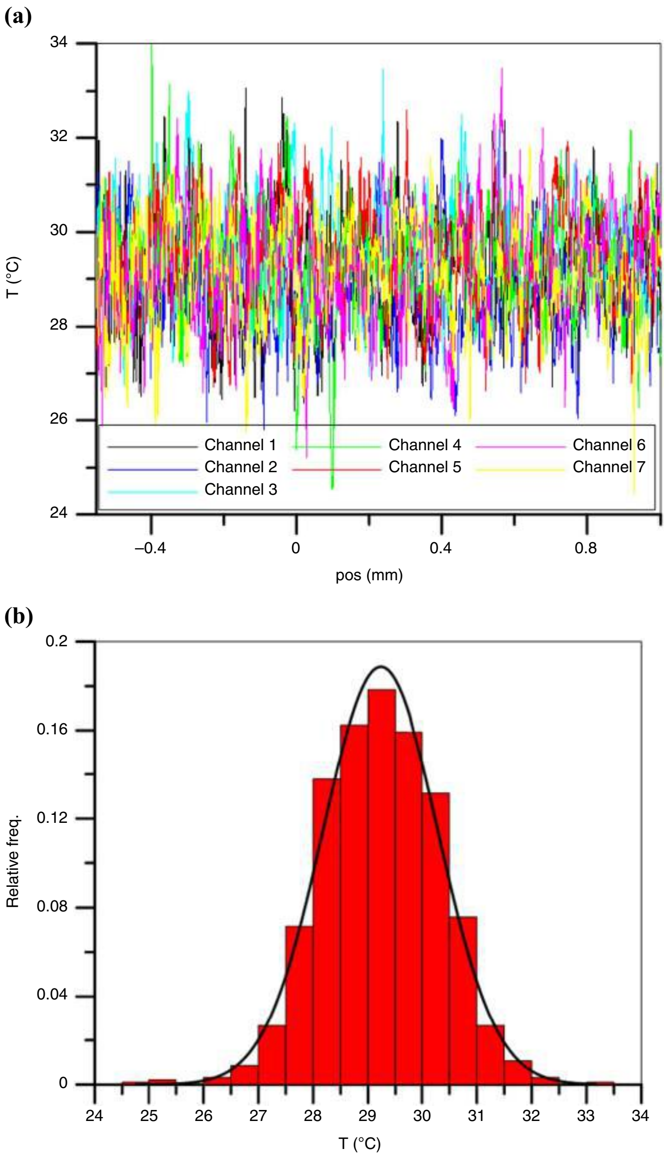


Figure 6.
(a) Temperature distribution along the center of seven microchannels employed during the μ-LIF calibration step; (b) histogram plotted from the data in Figure 6(a)

to infer the standard deviation associated with the experimental errors, yielding the value of $\sigma_{e,LIF} = 0.955$ °C, resulting in an uncertainty of +/−1.91 °C for 95 percent confidence level. This value is not far from the observations of Coolen *et al.* (1999) which stated an uncertainty of +/−1.7 °C with 95 percent confidence level for their LIF temperature measurements. Figure 6(b) depicts the histogram plotted from the data in Figure 6(a), also illustrating that the experimental error closely follows a normal distribution.

4. Results and discussion

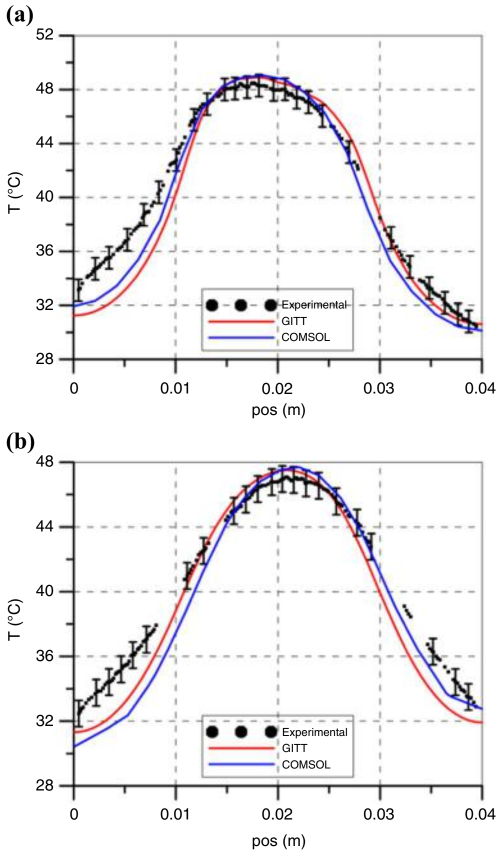
As shown in Figure 2(a), for the problem formulation considered, only the heat exchange section is modeled, i.e. the inlet and outlet distribution manifolds are not taken into account. It was considered that the 2 mL/min volumetric flow rate is equally distributed between all six channels, resulting in a volumetric flow rate of 1/3 mL/min in each channel. We have also considered uniform velocity profiles in each channel, $\overline{u}_f$ in Equation (1g), which was a considered a reasonable approximation from previous works (Knupp *et al.*, 2014a). For the physical properties of the materials involved, we have employed: $k_s = 0.17$ W/mK (PMMA), $k_f = 0.64$ W/mK, and $w_f = 4.186\times10^6$ J/m^{3}K (water as the working fluid) (Lienhard and Lienhard, 2008). The inlet temperature distribution, required by Equation (1b), was obtained from the complete model simulation by the Comsol solver (COMSOL Multiphysics, 2014), in order to set the same channels inlet temperatures for comparison purposes. For the heat transfer coefficient, h_{eff}, appearing in Equations (1(c-f)), we have considered natural convection, with $h_{eff} = 15$ W/m^{2}K.

Table I shows a close inspection of the convergence of the front surface temperatures along the heat exchanger width and length (transversal centerline and longitudinal centerline), in (a) and (b), respectively, with respect to the truncation order (M) of the algebraic eigenvalue problem. One may observe that the results are converged to practically three significant digits at the selected positions for $M < 70$. It should be highlighted that in all performed calculations we have employed only ten terms in the temperature field expansion in terms of the calculated eigenfunctions, which are enough to yield a full five digits convergence throughout the domain, confirming that the variable coefficients eigenvalue problem involves most of the computational effort in this problem solution and convergence.

(a) Transversal centerline			
M	$y = 0.01$ m	$y = 0.02$ m	$y = 0.03$ m
$M = 30$	39.940	47.354	40.843
$M = 40$	38.691	47.690	39.465
$M = 50$	38.787	47.419	39.963
$M = 60$	38.830	47.481	39.914
$M = 70$	38.821	47.457	39.954
(b) Longitudinal centerline			
M	$x = 0.03$ m	$x = 0.04$ m	$x = 0.05$ m
$M = 30$	49.680	47.062	45.688
$M = 40$	48.400	47.882	45.816
$M = 50$	48.329	47.629	46.656
$M = 60$	48.355	47.372	46.599
$M = 70$	48.394	47.457	46.558

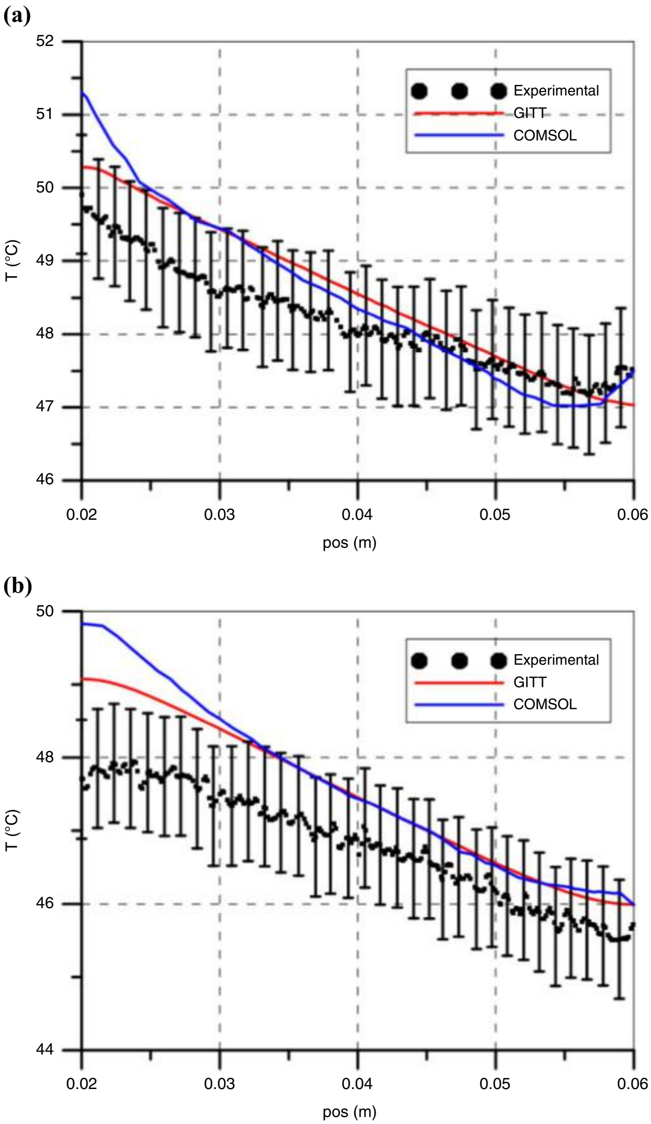
Table I. Convergence of the front surface temperature along the micro-heat exchanger (a) width (transversal centerline) (b) length (longitudinal centerline) with the order of the algebraic eigenvalue problem

Figure 7 depicts the theoretical predictions of the surface temperatures in a case with volumetric flow rate of 2 mL/min, obtained with both the single domain model solved through GITT and the COMSOL Multiphysics finite elements commercial code (COMSOL Multiphysics, 2014), together with the infrared camera experimental measurements obtained at the transversal centerline of the substrate external surfaces. In Figure 7(a) are provided the results corresponding to the micro-heat exchanger front face, whereas in Figure 7(b) the back face is considered. Similar comparative results are presented in Figures 8(a) and (b), which depict the surface temperature profiles in the longitudinal direction at the centerline along the length of the micro-heat exchanger, corresponding to the front face, (a), and the back face, in (b). From these results samples, it is clear that a good adherence between the hybrid theoretical prediction via GITT and the experimental measurements is achieved, indicating the adequacy of the proposed model and solution methodology. It should also be observed the good agreement between the solution provided by GITT employing the single domain formulation and the numerical finite elements solution (COMSOL) of the conjugated problem, with the difference between these predictions remaining within the experimental error.



Notes: (a) Front face; (b) back face with coolant inlet-outlet

Figure 7. Comparison of infrared thermography temperature measurements at the external surfaces (transversal centerline) against the GITT (red) and COMSOL (blue) solutions



Notes: (a) Front face; (b) back face with coolant inlet-outlet

Figure 8. Comparison of infrared thermography temperature measurements at the external surfaces (longitudinal centerline) against the GITT (red) and COMSOL (blue) solutions

Results are now also reported for fluid temperature measurements employing the μ-LIF system described in Section 3.1. Figure 9 indicates the three positions investigated. In this experimental setup we have no control on the depth position (in the channel) at which the fluid temperatures are monitored, since the channels depth is within the depth of field of the microscope. In order to understand the differences in temperatures at varying depths, the theoretical temperatures are presented at three different depths within the micro-channel, as illustrated in Figure 10. Figure 11(a)-(c) then depict the theoretical temperature profiles in the fluid, in comparison with the measurements

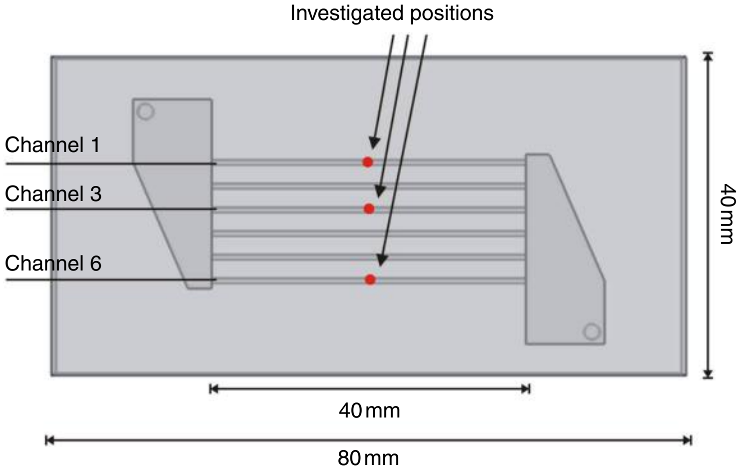


Figure 9.
Micro-heat exchanger representation with investigated positions in μ-LIF measurements

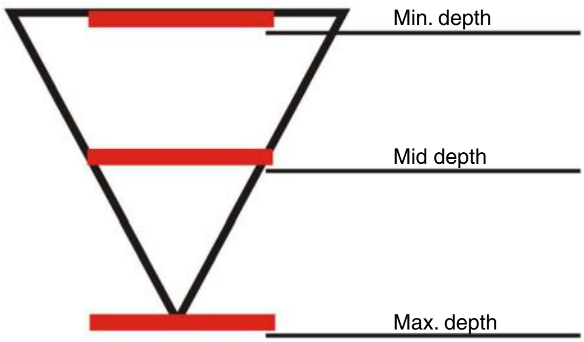


Figure 10.
Depth positions within the micro-channel for the theoretical evaluations

obtained through the μ-LIF system, transversally to channels 1, 3, and 6, respectively, as marked on Figure 9, at the heat exchanger longitudinal central plane. The experimental results show larger temperature variations than predicted along the transversal direction within the channel for channels 1 and 6, in Figure 11(a) and (c), where increasing and decreasing temperatures are observed, respectively. One may notice that in both cases the cold side refers to the border, suggesting that the micro-heat exchanger is losing more heat than predicted by the model through the borders. This fact may be explained by the difficulty in insulating the contact between the PMMA substrate and the metallic holder of the microscope, which ends up working as a fin attached to the PMMA substrate. On the other hand, for the central channel in Figure 11 (b), which is less sensitive to these borders effects, a much better adherence is observed between the predictions via GITT and the experimental measurements.

5. Concluding remarks

The methodology that combines the GITT and a single domain reformulation strategy is applied in the hybrid numerical-analytical treatment of a multi-stream micro-heat exchanger application, involving a three-dimensional configuration with triangular

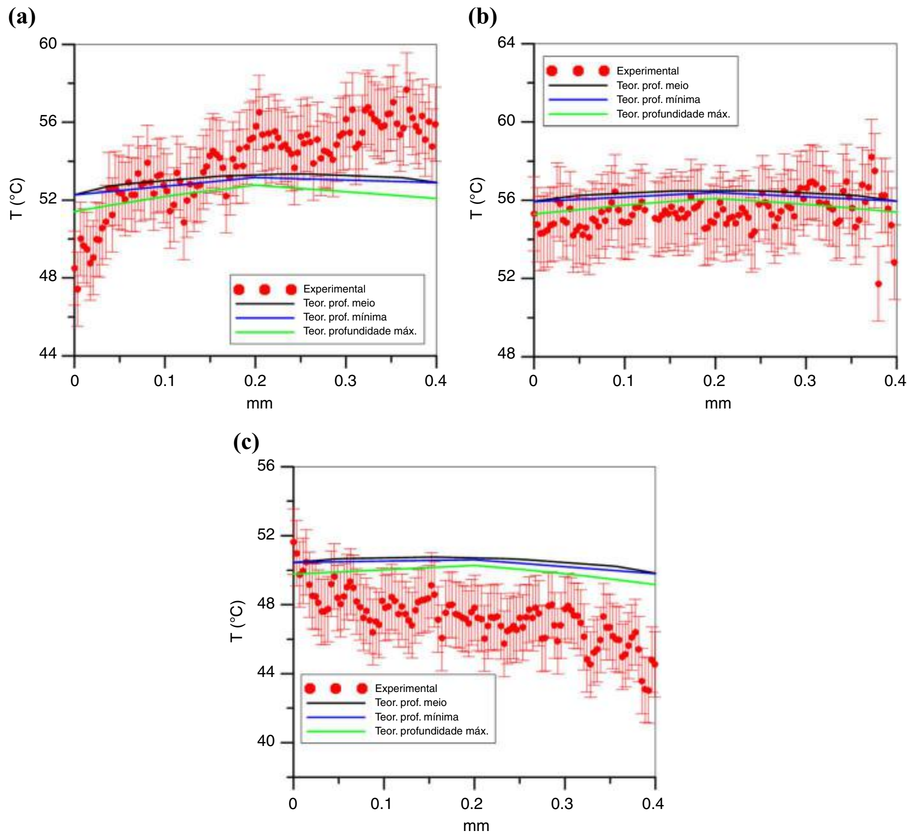


Figure 11.
Calculated and measured temperature profiles in the fluid transversally to channels (a) 1, (b) 3 and (c) 6, at the heat exchanger central position (marked on Figure 9)

cross-section micro-channels. The methodology here advanced is carefully validated against experimental results from non-intrusive techniques, namely infrared thermography measurements of the substrate external surface temperatures and fluid temperature measurements obtained through Laser Induced Fluorescence. A very good agreement among the proposed hybrid methodology predictions, a finite elements solution from the COMSOL code, and the experimental findings has been achieved. The proposed methodology has been demonstrated to be quite flexible, robust, and accurate. Most important, the hybrid nature of the approach, providing analytical expressions in all but one independent variable, and requiring numerical treatment at most in one single independent variable, makes it particularly well suited for computationally intensive tasks such as in optimization, inverse problem analysis, and simulation under uncertainty.

References

Aparecido, J.B., Cotta, R.M. and Ozisik, M.N. (1989), "Analytical solutions to two-dimensional diffusion type problems in irregular geometries", *J. of the Franklin Institute*, Vol. 326 No. 3, pp. 421-434.

COMSOL Multiphysics (2014), "COMSOL Multiphysics version 4.3", available at: www.comsol.com (accessed August 17, 2014).

Coolen, M.C.J., Kieft, R.N., Rindt, C.C.M. and van Steenhoven, A.A. (1999), "Application of 2-D LIF temperature measurements in water using a Nd: YAG laser", *Experiments in Fluids*, Vol. 27 No. 5, pp. 420-426.

Cotta, R.M. (1990), "Hybrid numerical-analytical approach to nonlinear diffusion problems", *Num. Heat Transfer, Part B*, Vol. 17 No. 2, pp. 217-226.

Cotta, R.M. (1993), *Integral Transforms in Computational Heat and Fluid Flow*, CRC Press, Boca Raton, FL.

Cotta, R.M. (1994a), "Benchmark results in computational heat and fluid flow: the integral transform method", *Int. J. Heat Mass Transfer*, Vol. 37 No. S1, pp. 381-394.

Cotta, R.M. (1994b), "The integral transform method in computational heat and fluid flow", *Special keynote lecture Proc. of the 10th Int. Heat Transfer Conf., Vol. 1*, *Brighton*, August, pp. 43-60.

Cotta, R.M. (Ed.) (1998), *The Integral Transform Method in Thermal and Fluids Sciences and Engineering*, Begell House, New York, NY.

Cotta, R.M. and Ozisik, M.N. (1987), "Diffusion problems with general time-dependent coefficients", *Rev. Bras. Ciências Mecânicas*, Vol. 9 No. 4, pp. 269-292.

Cotta, R.M. and Mikhailov, M.D. (1997), "Heat conduction: lumped analysis", *Integral Transforms, Symbolic Computation*, Wiley-Interscience, Chichester.

Cotta, R.M. and Mikhailov, M.D. (2006), "Hybrid methods and symbolic computations", in Minkowycz, W.J., Sparrow, E.M. and Murthy, J.Y. (Eds), *Handbook of Numerical Heat Transfer*, 2nd ed., John Wiley, New York, NY.

Cotta, R.M. and Ozisik, M.N. (1986), "Laminar forced convection in ducts with periodic variation of inlet temperature", *Int. J. Heat Mass & Transfer*, Vol. 29 No. 10, pp. 1495-1501.

Cotta, R.M., Knupp, D.C. and Naveira-Cotta, C.P. (2014), "Unified integral transforms algorithm for convection-diffusion in irregular geometries and complex configurations", *Invited Keynote Lecture, ICCHMT International Symposium on Convective Heat and Mass Transfer in Sustainable Energy, CONV-14,* Kusadasi, June.

Diniz, A.J., Aparecido, J.B. and Cotta, R.M. (1990), "Heat conduction with ablation in a finite slab", *Int. J. Heat & Technology*, Vol. 8 Nos 3-4, pp. 30-43.

Escher, W., Brunschwiler, T., Michel, B., and Poulikakos, D. (2010), "Experimental investigation of an ultrathin manifold microchannel heat sink for liquid-cooled chips", *Journal of Heat Transfer*, Vol. 132 No. 8, pp. 081402-1.

Hetsroni, G., Mosyak, A., Pogrebnyak, E. and Yarin, L.P. (2005), "Heat transfer in microchannels: comparison of experiments with theory and numerical results", *Int. J. Heat Mass Transfer*, Vol. 48 Nos 25-26, pp. 5580-5601.

Horvat, A. and Catton, I. (2003), "Numerical technique for modeling conjugate heat transfer in an electronic device heat sink", *Int. J. Heat Mass Transfer*, Vol. 46 No. 12, pp. 2155-2168.

Knupp, D.C., Cotta, R.M. and Naveira-Cotta, C.P. (2013a), "Heat transfer in microchannels with upstream-downstream regions coupling and wall conjugation effects", *Num. Heat Transfer, Part B- Fundamentals*, Vol. 64 No. 5, pp. 365-387.

Knupp, D.C., Naveira-Cotta, C.P. and Cotta, R.M. (2012), "Theoretical analysis of conjugated heat transfer with a single domain formulation and integral transforms", *Int. Comm. Heat & Mass Transfer*, Vol. 39 No. 3, pp. 355-362.

Knupp, D.C., Naveira-Cotta, C.P. and Cotta, R.M. (2013b), "Conjugated convection-conduction analysis in micro-channels with axial diffusion effects and a single domain formulation", *ASME J. Heat Transfer, Special Issue on Micro/Nanoscale Heat and Mass Transfer*, Vol. 135 No. 9, pp. 091401-091411.

Knupp, D.C., Naveira-Cotta, C.P. and Cotta, R.M. (2014a), "Theoretical – experimental analysis of conjugated heat transfer in nanocomposite heat spreaders with multiple microchannels", *Int. J. Heat Mass Transfer*, Vol. 74, July, pp. 306-318.

Knupp, D.C., Naveira-Cotta, C.P. and Cotta, R.M. (2014b), "Unified integral transforms in single domain formulation for internal flow three-dimensional conjugated problems", *Proceedings of the 15th International Heat Transfer Conference, IHTC-15*, *Kyoto, August 10-15*.

Koshlyakov, N.S. (1936), *Fundamental Differential Equations of Mathematical Physics*, ONTI, Moscow.

Lienhard, J.H. IV and Lienhard, J.H.V (2008), *A Heat Transfer Textbook*, Phlogiston Press, Cambridge, MA.

Luikov, A.V. (1968), *Analytical Heat Diffusion Theory*, Academic Press, New York, NY.

Luikov, A.V. (1980), *Heat and Mass Transfer*, Mir Publishers, Moscow.

Luikov, A.V., Aleksashenko, V.A. and Aleksashenko, A.A. (1971), "Analytical methods of solution of conjugated problems in convective heat transfer", *Int. J. Heat and Mass Transfer*, Vol. 14 No. 8, pp. 1047-1056.

Machado, H.A. and Cotta, R.M. (1995), "Integral transform method for boundary layer equations in simultaneous heat and fluid flow problems", *Int. J. Num. Meth. Heat & Fluid Flow*, Vol. 5 No. 3, pp. 225-237.

Maranzana, G., Perry, I. and Maillet, D. (2004), "Mini and micro-channels: influence of axial conduction in the walls", *Int. J. Heat Mass Transfer*, Vol. 47 Nos 17-18, pp. 3993-4004.

Mikhailov, M.D. and Özisik, M.N. (1994/1984), *Unified Analysis and Solutions of Heat and Mass Diffusion*, Dover Publications John Wiley, New York, NY.

Morini, G.L. (2004), "Single-phase convective heat transfer in micro-channels: a review of experimental results", *Int. J. Thermal Sciences*, Vol. 43 No. 7, pp. 631-651.

Nunes, J.S., Cotta, R.M., Avelino, M. and Kakaç, S. (2010), "Conjugated heat transfer in micro-channels, in microfluidics based microsystems: fundamentals and applications", in Kakaç, S., Kosoy, B., and Pramuanjaroenkij, A. (Eds), *NATO Science for Peace and Security Series A: Chemistry and Biology*, Vol. 1, pp. 61-82.

Ozisik, M.N. (1968), *Boundary Value Problems of Heat Conduction*, Int. Textbooks Co, Scranton, PA.

Ozisik, M.N. (1980), *Heat Conduction*, John Wiley, New York, NY.

Perelman, Y.L. (1961), "On conjugate problems of heat transfer", *Int. J. Heat and Mass Transfer*, Vol. 3, pp. 293-303.

Perez Guerrero, J.S. and Cotta, R.M. (1992), "Integral transform method for navier-stokes equations in stream function-only formulation", *Int. J. Num. Meth. in Fluids*, Vol. 15 No. 4, pp. 399-409.

Renfer, A., Tiwari, M.K., Tiwari, R., Alfieri, F., Brunschwiler, T., Michel, B. and Poulikakos, D. (2013), "Microvortex-enhanced heat transfer in 3D-integrated liquid cooling of electronic chip stacks", *Int. J. Heat and Mass Transfer*, Vol. 65, October, pp. 33-43.

Serfaty, R. and Cotta, R.M. (1990), "Integral transform solutions of diffusion problems with nonlinear equation coefficients", *Int. Comm. Heat & Mass Transfer*, Vol. 17 No. 6, pp. 851-864.

Serfaty, R. and Cotta, R.M. (1992), "Hybrid analysis of transient nonlinear convection-diffusion problems", *Int. J. Num. Meth. Heat & Fluid Flow*, Vol. 2 No. 1, pp. 55-62.

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2. K.H.Jyothiprakash, Jyothiprakash K.H., Y.T.Krishnegowda, Krishnegowda Y.T., VenkataramKrishna, Krishna Venkataram, SeetharamuK.N., K.N. Seetharamu. 2018. Effect of ambient heat-in-leak on the performance of three-fluid cross-flow heat exchanger. *International Journal of Numerical Methods for Heat & Fluid Flow* **28**:9, 2012-2035. [Abstract] [Full Text] [PDF]
3. R. M. Cotta, C. P. Naveira-Cotta, D. C. Knupp, J. L. Z. Zotin, P. C. Pontes, A. P. Almeida. 2018. Recent advances in computational-analytical integral transforms for convection-diffusion problems. *Heat and Mass Transfer* **54**:8, 2475-2496. [Crossref]
4. FuGuangming, Guangming Fu, AnChen, Chen An, SuJian, Jian Su. 2018. Integral transform solution of natural convection in a cylinder cavity with uniform internal heat generation. *International Journal of Numerical Methods for Heat & Fluid Flow* **28**:7, 1556-1578. [Abstract] [Full Text] [PDF]
5. Diego C. Knupp, Fabricio S. Mascouto, Luiz A.S. Abreu, Carolina P. Naveira-Cotta, Renato M. Cotta. 2018. Conjugated heat transfer in circular microchannels with slip flow and axial diffusion effects. *International Communications in Heat and Mass Transfer* **91**, 225-233. [Crossref]
6. Jose Luiz Zanon Zotin, Diego Campos, Renato Machado Cotta. Conjugated heat transfer in complex channel-substrate configurations: Hybrid solution with total integral transformation and single domain formulation 184-191. [Crossref]
7. T. Antonini Alves, R.A.V. Ramos, C.R.M. Maia. 2016. Aplicação da transformada integral generalizada e da transformação conforme na solução de um problema de convecção forçada laminar em dutos de setor de anel circular. *Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería* **32**:4, 261-269. [Crossref]
8. R.M. Cotta, C.P. Naveira-Cotta, D.C. Knupp, J.L.Z. Zotin, P.C. Pontes. 2016. Eigenfunction Expansions for Coupled Nonlinear Convection-Diffusion Problems in Complex Physical Domains. *Journal of Physics: Conference Series* **745**, 022001. [Crossref]
9. Renato M Cotta, Carolina Palma Naveira-Cotta, Diego C. Knupp. 2016. Nonlinear eigenvalue problem in the integral transforms solution of convection-diffusion with nonlinear boundary conditions. *International Journal of Numerical Methods for Heat & Fluid Flow* **26**:3/4, 767-789. [Abstract] [Full Text] [PDF]
10. J Zotin, Diego Knupp, Renato Cotta. Analytical–Numerical Solutions for Conjugated Heat Transfer in Multistream Microsystems 349-367. [Crossref]