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# The State of the Art in Turbulence Modelling in Brazil

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## Abstract

*The present work discusses at length the current status of turbulent research in Brazil. After eight introductory sections on the subject, where some general aspects of the problem are presented, and a brief review of some scientific and engineering approaches is given, the paper strolls over four specific sections, analyzing all work carried out in Brazil in the past twenty five years on turbulence. In fact, the present compilation is restricted to the main events sponsored by the Brazilian Society of Mechanical Sciences. The present review quotes 284 references, presents 6 tables and 16 figures. The paper contents is: Paper Outline, Some Insights, The Traditional Approaches, Some Basic Working Rules, Turbulence Models, One-Point Turbulence Closure Models, Some Other Aproaches to Turbulence Modelling, Some Major Achievements, A Bit of History, Statistics, A Personal View, Gallery, Final Remarks, Cited Bibliography and Compiled Bibliography.*

***Keywords:****Turbulence Flow, Turbulence Modelling*

## Paper Outline

The present review was committioned by the Brazilian Society of Mechanical Sciences, ABCM, in order to give a clear picture of the present status of turbulence modelling in Brazil. As a guiding line, the paper was supposed to concentrate on work that appeared in the last five years in the two major Conferences sponsored by the Association: the Brazilian Congress of Mechanical Engineering - COBEM, and the Brazilian National Meeting on Thermal Sciences, ENCIT. The review, if possible, should not be a mere collection of annotated bibliographies, but offer a critical evaluation of the published literature.

All these aspects were in the minds of the present writers when preparing this manuscript. The broad nature of the subject, however, together with its importance and general interest for the public, made the writing a difficult task. Big issues, such as the inclusion of indexed publications, and of publications appearing in vehicles other than those commonly looked up by the mechanical engineering community, had to be tackled in a very positive and quick way. In the end, it was decided to adhere quite stringently to the main guide lines. The only exception had to do with the period of time covered by this review. In order to give a good historic perspective of the subject, it was decided not to impose any bound on the number, and date, of works considered for review here. In adopting this procedure we particularly regret the exclusion of works published by the physical and mathematical societies.

## Some Insights

Since we are going to discuss at quite some length the phenomenon of turbulence, perhaps it would be appropriate at this stage to define turbulence. It can be said, in a general manner, that a turbulent flow is a flow which is disordered in time and space. Of course, this definition is vague, lacking a precise mathematical formulation. This, however, is the kind of definition we commonly find in treatises that deal with the subject of turbulence. The flows that are present in nature and technology and which are termed turbulent are very complex, exhibiting fairly different dynamics from one occurrence to another. In applications, the flows may be three-dimensional, or quasi-two-dimensional, or may even have some sort of large scale organized structures. All these features, coupled with the random behaviour of the measured properties of the flow, seem to deem the flow mathematically undefinable. The important property which is required of them is that they should be capable of mixing all transportable quantities much faster than if only molecular processes where involved. To cut short an endless discussion, and after Lesieur (1990), we propose here the following simplified definition of turbulence:

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- A turbulent flow must be unpredictable, in the sense that a small uncertainty as to its knowledge at a given initial time will amplify so as to render impossible a precise deterministic prediction of its evolution;
- It has to satisfy the increased mixing property defined above;
- It must involve a wide range of spatial wave lengths.

Despite, the already mentioned vagueness of the above definition, for fluid dynamicists the word turbulence has a precise meaning. Mathematically, in studying turbulent flows we are trying to solve a problem which is based on the assumption that the instantaneous flow variables satisfy the Navier-Stokes equations. These are a deterministic set of equations which apparently yield solutions with a random nature. Thus, the philosophical issues raised on how statistical theories could be used to describe a phenomenon which is deterministic could never be squarely faced until recently. New ideas and mathematical tools advanced by Ruelle and Takens (1971) and by Feigenbaum (1979) have shown that there are non-linear dynamical systems whose solutions, because of sensitive dependence on initial conditions, exhibit a weak causality and hence apparently random nature. Even the discrete algebraic equation $Z_{n+1} = A\ Z_n\ (1 - Z_n)$ demonstrates random solutions whose long time solutions have a critical dependence on vanishingly small changes in the value of parameter A. Because of the dependence of the long term solution on the vanishingly small changes in A, the solutions are non-deterministic. The conclusion is that fluid turbulence is a deterministic problem that evolves with time and space in a very complex fashion due to non-linear interactions.

Although turbulence is a random phenomenon, it is accepted that the flow is completely governed by the Navier-Stokes equations. Considering that a host of numerical schemes exist to deal with systems of non-linear partial differential equations, one should have, in principle, no problem in computing turbulent flows. Actually, this is not the case. The richness of scales that characterizes a turbulent flow imply that, if the flow is to be correctly numerically simulated, all lengths from the large-scale energy containing eddies, to the dissipative scales under which the motion is damped through viscosity, must be resolved.

The dissipation of mechanical energy into heat, however, may be shown through dimensional arguments to occur at very small scales. The dissipative scales are, in fact, given by $\eta = \mathrm{ord}\ (\nu^3/\varepsilon)^{1/4}$, where $\varepsilon$ is the overall viscous dissipation per unit mass, and $\nu$ is the kinematic viscosity. The energy dissipation rate is given by $\varepsilon = \mathrm{ord}\ (u^3/l)$ where u and l are respectively the characteristic velocity and length scales of the largest eddies of a turbulent flow, being, therefore, determined by the geometry of the flow domain. The minimum number of points required in a three dimensional simulation of the flow should then be $N_{length} = \mathrm{ord}\ (l^3/\eta^3) = \mathrm{ord}\ (R_l^{9/4})$, where $R_l$ represents the Reynolds number associated with the large scale eddies. Here, $N_{length}$ represents the number of grid points that the computation or measurement would have to have in order to describe the entire range of scales. In nature and in technological applications, the Reynolds number assumes typically values between $10^6$ and $10^8$, which will result in approximately $10^{14}$ to $10^{18}$ required grid points.

The minimum temporal resolution is found by dividing the smallest spatial scale by the convective velocity of the largest eddies. Likewise, the ratio between the largest scales and the convective velocity gives the large scale temporal resolution. The result is that the number of required time steps is $N_{time} = R_l^{3/4}$.

The number of points required to resolve the spectral properties in turbulence is then $N_{total} = (R_l^{9/4})(R_l^{3/4}) = R_l^3$. This expression shows that an order of magnitude increase in the Reynolds number of the flow will result in a three order of magnitude increase in the number of required computational grid points.

These numbers preclude any direct simulation of flows of practical interest. In fact, even if powerful enough computers were available, the performance of direct simulation of flows at high Reynolds number would be an unfeasible exercise. We realize that firstly by recognizing that the specification of proper initial and boundary conditions on the level of detail required by the smallest dissipative scales is simply not possible. This, unfortunately, in dealing with the Navier-Stokes equations, is a very serious drawback. The non-linear character of the advection terms may give rise to spatial and temporal instabilities in the flow that will excite and amplify the small scales in the motion. The lack of uniqueness of the solution, together with the amplification of the small perturbations on the boundaries of the flow, result in a motion which is apparently random in nature.

## The Traditional Approaches

The decomposition of flow field variables into mean and fluctuating components has led to what is now known as the Reynolds-averaged equations. In many engineering and geophysical problems, the fine details of the flow are really not required, so that often it suffices to have data conveyed by long-time averages. Although the averages were first defined in terms of time averages, further developments have introduced the concepts of space averages and of ensemble averages. The averaged Reynolds equations can simply be derived from the Navier-Stokes equation to which they are very similar, but with additional terms involving the turbulent fluctuations. These additional terms result from the advection terms and may be seen, from a statistical point of view, as second order correlations or moments. Since the practical interest lies usually in the mean properties of the turbulent flow field, the statistical theories try to establish functional relationships between the mean variables of the flow and the fluctuations. This is normally referred to as the "closure" problem in turbulence. The attempt at establishing these functional relationships has been one of the dominant themes of research in turbulence.

Exact equations for the terms involving fluctuations can the derived directly from the Navier-Stokes equations. However, the determination of transport equations for high order correlations always results in the appearance of higher order correlations or moments. We are then faced with the problem of always having more unknowns than equations available for their solution. One way out would be to neglect correlations of some order. Unfortunately, this proves to be unsatisfactory. The reason for this is that despite its apparent random nature, turbulent flows exhibit some degree of "order", which must be captured by the mathematical model. In order to render the system of transport equations closed, we need, therefore, some "closure hypotheses" independent of the physical conservation laws.

Perhaps, we should then now to consider the conditions, or some general principles, to which the closure models should obey.

## Some Basic Working Rules

The sheer complexity of the turbulence phenomenon has prevented researchers from achieving a rational closure of the Reynolds-averaged equations. This has motivated the invention of a number of mathematical models based on different principles and rules, and the evolution of certain heuristics which are supposed to help us to conjecture the broad features of turbulent flows without having to appeal to any of those models of doubtful validity. Of course, as these heuristics are deduced from common knowledge and experience, they will always be open to criticism. However, they must be seen as useful rules that have been subjected to a large scrutiny, and that, for this reason, constitute a serviceable body of knowledge for the engineer.

Some years ago, Narasimha (1989) suggested that the accumulated experience with the various types of heuristic rules could be summarized in the form of the five working rules listed in Table 1.

**Table 1 The five basic working rules**

1. As the Reynolds number of any turbulent flow tends to infinity, the fraction of energy contained in the length and time scales directly affected by the viscosity of the fluid becomes vanishingly small; so do the scales themselves, compared to those accounting for the energy.
2. Any turbulent flow subjected to constant boundary conditions evolves asymptotically to a state independent of all details of its generation save those demanded by overall mass, momentum and energy conservation.
3. If the equations and boundary conditions governing a turbulent flow admit a self preserving (or equilibrium) solution, the flow asymptotically tends towards that solution.
4. A turbulent flow may, before reaching equilibrium, attain a mature state in which the different energetic parameters characterizing the flow obey internal relationships, irrespective of the detailed initial conditions as in Rule 2.
5. Between the viscous and the energetic scales in any turbulent flow exists an overlap domain over which the solutions characterizing the flow in the two corresponding limits must match as the Reynolds number tends to infinity.

These rules have been used by engineers for over a long period without any special acknowledgment of them. Rule 4, in particular, is the basis of all handbook charts and correlations, such as, e.g., the Moody diagram for pipe losses. They are, thus, according to Narasimha (1989), "what comes naturally to workers in turbulence whenever they encounter a totally unfamiliar flow". The rules establish some positions upon which any turbulence model should conform.

At this point we just remind the reader that the working rules have not been deduced from the basic laws of fluid motion and that for this reason they will always remain open to doubt. As a final assessment, we quote Narasimha's very own words when the rules were first presented: "They (the rules) are obviously very useful, being close to reality; but they cannot still be elevated to the status of scientific laws, because the small departures noted from them cannot be dismissed as experimental error, and seem to indicate that the principles are strictly valid only under certain as-yet unstated conditions which would not always be easily obtained."

The most frequently challenged working rules concern the postulated independence from initial conditions, Rules 2 and 4.

## Turbulence Models

Turbulence modelling of a certain type has become a widespread subject in academia and industry. The pressing demand for more efficient models, capable of dealing with evermore complex flows, has resulted in the proliferation of different schools of thought, many of them remotely resembling each other. The implication is a continuous growth in the number, complexity and sophistication of models. The presumption that any monograph on the subject should be able to cover all aspects of turbulence modelling is, therefore, not shared by the present authors. Here, we will try to show the failures and the successes of some of the most popular approaches.

The turbulence models can be ensembled, in general, in four classes (Narasimha (1989)).

**The Impressionistic Models**, that strive to gain insight into the structure and the solution of the problem without any claim to quantitative accuracy in prediction. Examples are the Burgers equation for turbulence, and the Lorenz equation for weather forecast.

**The Physical Models**, that aim at predicting quantities of interest based on assumptions not inconsistent with the observed or understood physics of the problem, appealing to experimental data for model parameters when necessary. The models of Emmons for boundary-layer transition, of Kutta-Joukowski for the lift of an aerofoil, and of Kolmogorov for the spectrum of turbulence, all fall within this category.

**The Rational Models**, that investigate the nature of problem and solutions through simpler models derived from more complete systems by some limiting process. The Burgers model for weak shocks, the Newtonian model for hypersonic flows and the rapid distortion of turbulence are all rational models.

**The Ad Hoc Models**, that provide estimates of quantities of interest without insistence that all assumptions be physically or mathematically justified in detail. The Boussinesq eddy viscosity model, the mixing length model, the $\kappa$ - $\varepsilon$ differential model, these are all Ad Hoc models.

The models that serve industry are all Ad Hoc Models. In particular, gradient transport models have become very popular over the years. The appeal of an attractive blend of simplicity and acceptable predictive ability has stimulated a continued development of these models. In the next section, we discuss these models in detail.

## One-Point Turbulence Closure Models

One-point closure models were developed in order to model inhomogeneous flows in practical applications. Starting from the Reynolds-averaged equations, they aim at establishing functional relationships between the turbulent fluctuations, through their second-order moments at the same point in space, and the mean flow variables. This can be made in several ways. The simplest of the possibilities is to introduce an algebraic relationship.

Using the notion that the collective interaction of eddies can be represented by an increase in the coefficient of viscosity, Boussinesq postulated the second-order moments to be proportional to the local mean rate of strain through an "eddy viscosity", $\nu_t$. This notion is a direct translation of Newton's viscosity law to turbulent flows. Since turbulence is a property of the flow, the eddy viscosity should be a function of the mean flow parameters. For three-dimensional flows, it should even be a vectorial quantity.

Despite the boldness of this assumption, clear even to the earliest workers in turbulence, the eddy viscosity hypothesis has received critical attention over the last 120 years. Turbulence models have become more and more sophisticated but the usefulness of the eddy viscosity concept is still large.

However, as pointed out by Bradbury (1997), the idea that turbulent stresses are strongly dependent on local strain rates (whatever model for $\nu_t$ is used) implies that the length scale of the turbulence is smaller than the scale of the velocity local gradient - this scale is typically of the order of the flow width. However, even casual observation of flow visualization shows that the main turbulent structures are of the same order as the fluid width. It is, therefore, amazing that proportionality relations involving only stresses and strain rate work so well.

Of course, the concept of eddy viscosity is phenomenological, having no mathematical basis. The result is that a constitutive relationship for $\nu_t$ still needs to be constructed.

The eddy viscosity can be expressed algebraically in terms of quantities derived from algebraic or differential equations. Depending on the nature and on the number of equations used, a classification for the eddy viscosity models follows.

**Zero-equation or algebraic models.** These models use an algebraic expression for the definition of $\nu_t$. Typical examples are the mixing-length concept of Prantdl, the Cebeci Model and the Baldwin-Lomax Model.

**One-equation models.** These models employ a single transport differential equation which has to be solved for the fluctuating field. An extra algebraic expression is normally also required for the complete specification of the flow. Examples of one-equation models are the models of Bradshaw, of Nee and Kovasnay, of Johnson and King and of Goldstein.

**Two-equation models.** These models use two transport equations for the description of the fluctuating field. Examples are the $\kappa/\varepsilon$ model, the $\kappa/\omega$ model, the $\kappa/\Omega$ model, the $\kappa/\tau$ model, and many others.

So far, the simplest prescription for the second-moments has been achieved through the algebraic models. These models are built on an analogy between the randomizing effects of the turbulent eddies on the mean flow, and the corresponding random molecular motion in a gas. The implication is that the "turbulent viscosity" can be seen as formed from a product of the density of the fluid, of a local characteristic length, $l_c$, and of a local characteristic velocity, $u_c$. The analogy with the kinetic theory of gases is clear; however, in a turbulent flow, both the characteristic parameters must be expected to vary locally with the flow.

The mixing length theory assumes the fluctuations in both directions to have the same order of magnitude, and these to be proportional to the distance from which discrete "lumps of fluid" are fetched, times the local mean velocity gradient. This hypothesis reduces our closure problem to the determination of the characteristic length of the flow, that is, of the mixing length. For simple boundary layer flows, $l_c$ can be assumed to be proportional to the distance from the wall. For free turbulent flows, $l_c$ can be assumed to be proportional to the width of the turbulent region. The list is not exhaustive. Here, it suffices to say that every class of flow has its particular formulation of the mixing length.

The mixing length concept works quite well for a class of problems, but suffers from all the problems inherent to the eddy viscosity hypothesis. In particular, in complex flows, it may be impossible to specify any form for the mixing-length. In general, the model has shown to be appropriate for two-dimensional flows with mild pressure gradients and mild curvature. No flow separation or rotation effects are allowed in these models. In addition, the algebraic models are useful for two-dimensional shock-separated boundary layers with weak shocks, and in computing three dimensional boundary layers with small cross flow. In fact, several authors have sought extensions of the algebraic eddy viscosity model to three dimensional flows (see, for example, Rotta(1979)). The success, however, has been very limited.

We draw our curtains on the mixing length model by pointing out to the reader that this model assumes the flow to be isotropic, and takes no account of processes of convection or diffusion of turbulence.

In the one-equation models, a transport equation is used to compute the second-order moments. This can be made in two ways. Through the specification of a single characteristic parameter that can be used to represent the fluctuating field, or, alternatively, relating the characteristic velocity in the eddy viscosity model to a turbulent property. In the latter approach, the characteristic length still has to be determined through an algebraic equation.

Almost all one-equation models use the turbulent kinetic energy, $\kappa$, as the reference parameter. An exact equation can be derived for $\kappa$ through some simple algebraic manipulation of the Navier-Stokes equations. This equation, however, presents some turbulent correlations which have to modelled. The diffusion and the production terms are modelled as gradient transport terms. The dissipation term, obtained through a local isotropy assumption, is defined in terms of an algebraic equation that involves the fluid density, the turbulent kinetic energy and the length scale.

In practice, very little advance of this type of models over the zero-equation models exists. The fact that the characteristic length scale still has to be fixed by empirical arguments imports to these models many of the drawbacks of the algebraic models. The material found in literature, however, clearly supports the idea that $\kappa^{1/2}$ is a better velocity scale than those employed in the algebraic models.

A recent attempt at reviving the one equation models has been made by Johnson and King. They begin with a beforehand assumed eddy viscosity algebraic behaviour which has as its velocity scale the maximum Reynolds stress. Next, the maximum turbulent kinetic energy, $\kappa_m$, is supposed to vary very little with the transversal direction so that an ordinary differential equation can be constructed to its prediction. The closure is concluded considering that the ratio of the local kinetic energy to the shear stress is constant. This model is aimed at a very limited class of flows, namely, at two-dimensional separated flows without curvature or rotation. The general results of the model are very good.

The next level of sophistication is to introduce a second transport equation, from which the characteristic length can be calculated. This gives rise to the two-equation models. These models are considered to embody more physics than the previous ones. A list of the various available two-equation models can be found in Launder and Spalding (1972) and more recently in a number of review articles (see, e.g., Lakshminarayana (1986), Wilcox (1988)).

The variables adopted by researchers to determine the length scale vary from one work to another. One may choose as the second variable a parameter associated with the mean frequency of the most energetic motions. That was exactly the original proposal of Kolmogorov (1942). The turbulent energy dissipation rate per unit mass, $\varepsilon$, has been preferred by a number of authors. Another very popular choice has been the specific dissipation rate, $\omega$. A transport equation for the length scale itself was derived by Rotta (1951).

The standard two-equation models fail to capture many of the features associated with complex flows. The fact that they are still imprisioned to the eddy viscosity concept makes them very vulnerable when dealing with flows that present curvature, separation, rotation, and three-dimensionally among other effects. The advantages and simplicity of the two-equation models, however, should not be overlooked. These models are much superior to the algebraic and one-equation models in mildly complex flows. Typical successful application of these models are the flows in jets, channels, diffusers, and annulus wall boundary layers without separation.

The weaknesses and shortcomings of these models have been listed by Hanjalic (1994); they are:

- Linear stress-strain relationship through the eddy viscosity hypothesis;
- Scalar character of eddy viscosity;
- Scalar character of turbulence characteristic scales - insensitivity to eddy anisotropy;
- Limitations to define only one time- or length scale of turbulence for characterizing all turbulent interactions;
- Failure to account for all viscous processes governing the behaviour of $\varepsilon$ or other scale-determining quantities by virtue of the simplistic form of the basic equation for that variable;
- Inadequate incorporation of viscosity damping effects on turbulence structure (low Reynolds number models);
- Inability to mimic the preferentially oriented and geometry-dependent effects of pressure reflection and the eddy-flattening and squeezing mechanisms due to the proximity of solid- or interface surface;
- Frequently inadequate treatment of boundary conditions, in particular at the solid wall.

A logical way of deriving more sophisticated models, of a certain universality, consists in deriving full transport equations for each of the second-order moments directly from the Reynolds-averaged equations. This procedure, inevitably, results that every derived equation involves various correlations among fluctuating quantities which are not exactly determined. These, of course, must be modelled in terms of the mean flow variables, the second-moments themselves, and at least one characteristic time scale. Models of this nature are usually termed Reynolds-stress transport models. According to some authors, these models represent the highest degree of complexity that has been exercised in conventional turbulence modeling.

Additionally, to a more universal description of the turbulence, these models are considered to adhere more rigorously to some modelling principles which are judged by some to be crucial. The first of the principles has to do with the mathematical formalism of the closure problem. Consistency conditions such as dimensional coherence of all terms, tensorial consistency, coordinate-frame and material frame independence, satisfaction of realizability conditions, limiting properties of turbulence, all these are satisfied by the Reynolds-stress models. The second principle is related to the physics of the turbulence. For example, the decreasing influence of higher-order moments upon the mean flow properties is a feature that must be observed in turbulent models.

The Reynolds stress models are reported (Hanjalic (1994)) to provide a superior representation of two-dimensional non-equilibrium flows. They are also supposed to perform better in unsteady and periodic flows and in all situations where the turbulent field exhibits hysteresis as compared with the mean flow field. The models account for the effects of anisotropy, curvature, rotation, three-dimensionality, and hence normally performs better than the two-equation models under these conditions. A shortcoming feature of the models is the need for a scale-determining equation, which grows in importance in pressure dominated flows, or in flows with unproperly treated boundary conditions.

For three dimensional flows, the Reynolds-stress models will result in 6 transport equations which must be simultaneously solved together with the equations of motion. This results in 10-12 equations to be solved for the mean flow and turbulence quantities. This is certainly a Herculean task even for present day modern computers. Hence, the order is to simplify the Reynolds stress transport equations. One try is to reduce these equations to algebraic equations. Such models are termed algebraic Reynolds stress models.

The essential idea of these models is that only the convection and diffusion terms contain stress gradients. Thus, if through some alternative procedure, these processes could be modelled by equations not containing stress gradients, then the differential transport equations for the Reynolds stresses would be reduced to algebraic equations. The need for solving turbulent transport equations is not eliminated here; however, the calculated flow variables are now scalars.

A still simpler approach considers the stress transport processes to be completely negligible. This local-equilibrium approximations are used mainly for understanding the physical nature of complex flows. In may cases, they often lead to useful overall models of the stresses.

In fact, many of the ideas that we have been considering in the last paragraphs were advanced quite some time ago. As early as 1940, Chou introduced, in an absolutely original paper, the equations for the second and third moments of the turbulent fluctuations. At the time, he proposed to "close" the equation for the triple velocity correlation by assuming the fourth-order moments to be proportional to the sum of the three products of two double velocity correlations. In addition, further hypotheses on the correlations involving turbulent pressure gradients and velocity fluctuations were needed. These correlations were derived by Chou from a Poisson equation. The equations for vorticity decay were also found and the terms of energy decay were improved.

Essentially, Chou argued that because the Navier-stokes equations are the basic dynamical equations of fluid motion, it does not suffice to consider only the mean turbulent motion. The turbulent fluctuations should be as important as the mean motion and, for this reason, the equations for them should also be considered.

What Chou really achieved was a systematic way of building up differential equations for the velocity correlations for each successive order from the equations of turbulent fluctuations. The problem now lies on the difficulties to be found in solving simultaneously the Reynolds-averaged equations and the equations for the high order moments. The three major difficulties found at the time were:

- The non-linear character of the equations;
- The correlation functions are slowly varying functions of space and time, whereas the fluctuations are all rapidly varying functions of them;
- To mathematically solve the set of non-linear differential equations, an extra physical condition is needed (similarity hypothesis).

These difficulties were finally overcome by Chou (1987) himself by developing the method of successive substitution, to solve the mean motion and velocity fluctuation equations simultaneously.

## Other Approaches to Turbulence Modelling

Another method of describing the turbulent fluctuations is the spectral method, first introduced by Taylor (1935, 1938). The introduction of Fourier analysis into the problem leads to some benefits. The differential operators are converted into multipliers, the physics of turbulence is given a relatively simple picture, and the degrees of liberty of the turbulent system are better defined. The use of Fourier analysis is particularly useful when the turbulence is homogeneous, that is, when it is statistically invariant under translation. Unfortunately, the spectral approach does not solve the problem of closure, it just appears in a different form.

Using the spectral approach, several theories were built for the description of turbulent flows. Some are:

- The direct-interaction approximation;
- The quasi-normal theory.

The first approach is analytical and has only given useful results for isotropic, homogeneous, turbulence. The quasi-normal theories, which assume all cumulants above some given order greater than two to vanish, lead to a negative energy spectra. Attempts at fixing this problem have led to considerations that require an extra assumption about time scales that is questionable in the energy containing range. These methods are of narrow application, make a variety of hypotheses whose validity is difficult to assess in physical terms, and, for this reason, will not be considered further here.

Other methods that heavily rely on numerical simulations have been developed more recently. One of them, direct simulation, has received a lot of attention.

The remarkable recent advances in computing power have opened an era of simulation. The application of paralleled codes on computers with efficient networking and large memory capacity has become a very powerful way of computing turbulence. Ideally, we would like to perform computations by solving the complete Navier-Stokes equations. However, due to the resolution restrictions observed in section 2, the calculations are presently restricted to flows with a Reynolds number of the order of $10^4$. Despite the severe limitations imposed by the resolution requirements, direct simulation has proved to be a powerful tool for understanding the physics of turbulence and furnishing data for the development and the improvement of turbulence models. In fact, it is capable of providing data on turbulence that is virtually unobtainable from experiments such as pressure-velocity correlations.

So far, our models for the simulation of complex turbulent flows, have relied on the use of statistical averages for the variables of interest. Another possibility is the use of filters on the same variables. This approach, normally termed large eddy simulation, presents the advantage of naturally introducing the scales of the resolved variables. The result is that, i) in principle, it is simple to prepare the input data for numerical models that are consistent with those scales; ii) the interaction that occurs between numerics and physics in the solution of the equations of motion is very strong, and the use of this approach makes it easier to have a better understanding of this process. The term large eddy simulation is frequently associated in literature with a space filtering operation. The use of time filtering, however, also has been tested; this results in the elimination of highly fluctuating components in time, allowing the use of large time steps in the numerical integration of the motion equations.

Let us now look in more detail at the philosophy of large eddy simulation. The method requires that a filtering operation is applied to the Navier-Stokes equations. Next, the velocity field is decomposed into a large scale velocity and a sub-grid velocity scale. This produces a new problem. The non-linearity of the advective term will now result in four different terms. Indeed, for a general space filtering operation, the classical averaging rules do not apply. Only one of these resulting terms is analogous to the Reynolds stresses. The other terms arise from the fact that, as we have just said, the filtering operation in not idempotent. The subgrid scale modelling problem can then be defined as finding expressions for the subgrid terms as a function of the large scale variables. Actually, the subgrid scale modelling problem is not a well posed problem. In the physical problem, the propagation to the large scales of the uncertainties contained in the subgrid scales will contaminate the former yielding a flow with an unpredictable nature. Now, consider we have constructed a subgrid scale model, and that, therefore, we have at our disposal a closed set of motion equations where everything is expressed in terms of the large scales. Then the simulations conducted with these set of equations will not be able to propagate any disturbances which would generate different flow fields. This implies that a large eddy simulation of turbulence will not faithfully reproduce the large scale evolution from a deterministic point of view, at least for times greater than the predictability time.

In large eddy simulation, the calculated flow must then be interpreted as a different realization of the actual flow. What one would hope to happen is that realization to have the same statistical properties of the real flow and the same spatially organized structures (though, at a different position from the reality).

Based on these concepts, Lesieur (1990) defines what a good large eddy simulation of turbulence should be.

**Low grade definition** the simulation must predict correctly the statistical properties of turbulence (spectral distributions, turbulent exchange coefficients, et cetera).

**High grade definition** moreover, the simulation must be able to predict the shape and topology (but not the phase) of the organized vortex structures existing in the flow at the scales of the simulation.

An important question to be asked now is: How small the scale of the resolved motion has to be? We begin to answer by reminding the reader that in high Reynolds number turbulent flow, the smaller the scale of the motion, the more isotropic it becomes, and that, in fact, an "inertial" subrange exists. Thus, if the scale of the large scale motion lies on the inertial subrange, the behaviour of the unresolved scales could then be assessed by invoking their near isotropic properties. The implication is that the proper master scale to be used in large eddy simulation of turbulence is precisely the grid spacing for isotropic meshes. For anisotropic meshes, a product average or a Euclidean norm can be used.

## Some Major Achievements

The classical theories of turbulence have achieved some results in the past which clearly have had a definitive influence in our perception of the problem. Next, we shall discuss some of these successes.

The second-order two-point correlations play a leading part in turbulence theory. We have discussed in some detail in the previous sections the engineering approaches for the one-point correlations. However, if our interest is the underlying structure of the turbulence, we should consider the velocity correlations at two or more points. These fundamental concepts were advanced by Taylor (1935) in a paper where he also introduced the concepts of statistical homogeneity and isotropy. Subsequently, Taylor (1938) introduced the three-dimensional energy spectrum in wavenumber (i.e., the Fourier transform of the two-point correlation in space), an entity whose calculation has become one of the fundamental objectives of turbulence theory. This function measures how much energy is contained between the wave numbers $\kappa$ and $\kappa + d\kappa$.

The closure problem for isotropic turbulence can be formulated in wave-number space. In this way, when we consider the transport of turbulent energy, this will be in wavenumber rather than configuration space. Large scale structures are associated with small values of $\kappa$, whereas small structures are associated with high values of $\kappa$. Thus, the transfer of energy will occur from one range of eddy scales to another. This process is known as the "cascade of energy".

The energy balance equation is obtained from the equation for the single-time correlations. This is just the mean motion equation, after some manipulation, specialization to homogeneous turbulence and Fourier transformed. A further specialization to the isotropic case, reduces the spectral tensor to its isotropic form. The trace of the tensor then gives the energy spectrum, E. The resulting equation for E, involves then a production term, a dissipation term, and a transport term. The transport term corresponds to the triple velocity correlations coming from non-linear interactions of the Navier-Stokes equations; this term just redistributes energy in wave number space.

The usual interpretation of the energy balance equation is that the energy in the flow stored at small $\kappa$ (that is, at large scales) is transferred by the non-linear transport term to large $\kappa$ (that is, to small scales), where it is dissipated through heat by the action of viscosity. The non-linear term describes conservative processes, namely inertial transfer of energy from one wave number to a neighboring one. Now, from ample experimental evidence, we know that the energy is determined by the lowest wavenumber, that the dissipation rate is determined by the highest wavenumber, and that the two ranges do not overlap even for very low values of the Reynolds number. It follows that the non-linear transport term can be made to dominate over an as large as we like portion of the wavenumber space, by simply increasing the Reynolds number.

The above ideas were formalized by Kolmogorov (1941) in two famous assumptions.

**Kolmogorov's first hypothesis of similarity**. At very high, but not infinite Reynolds numbers, all the small-scale statistical properties are uniquely and universally determined by the scale, $l$, the mean energy dissipation rate, $\varepsilon$, and the kinematic viscosity, $\nu$.

**Kolmogorov's second hypothesis of similarity**. In the limit of infinite Reynolds number, all small-scale statistical properties are uniquely and universally determined by the scale, $l$, and the mean energy dissipation rate, $\varepsilon$.

By a simple dimensional argument, the first hypothesis implies that the energy spectrum can be written as: $E(\kappa) = \nu^{5/4} \varepsilon^{1/4} f(\kappa l)$, where f is a universal function.

The second hypothesis implies that, in the limit as Reynolds number tends to infinity, $E(\kappa)$ should become independent of the viscosity. This amounts to the energy spectrum assuming the form $E(\kappa) \propto \varepsilon^{2/3} \kappa^{-5/3}$, the famous Kolmogorov's $\kappa^{-5/3}$ law. This law is remarkably well verified experimentally, although the fine-scale motion does not necessarily have the desired degree of isotropy postulated by the author, and the proportionality constant cannot be deduced convincingly by theory.

A second major success of turbulence theory is the theory of Taylor (1921) for the turbulent diffusion of fluid particles. A random process where at any instant the future state of the process is entirely determined by its state at that particular instant and independent of its prehistory is called a Markov process. Alternatively, we say we have a Markov process when the future is independent of the past for a known present. For the turbulent diffusion of fluid particles, this is certainly not the case. In the turbulent motion of a fluid, the motion of the particles is continuous - so is the exchange of transferable quantities - and there is a correlation in time between properties of a fluid particle at subsequent times. This memory behaviour of turbulent diffusion implies that this process cannot be considered a Markov process. Taylor extended these notions to the turbulent flow diffusion by considering the path of a marked fluid particle during its motion through the flow field.

Considering the displacement of marked fluid particles, Taylor introduced the Lagrangian autocorrelation and the integral time scale. Then, further considering the flow to be homogeneous in space and time, an exact result for the mean square distance traveled by a diffusing marked fluid particle was derived in terms of the Lagrangian correlation coefficient, $R_L$. The difficulty is that a theoretical solution for $R_L$ cannot be found. Also, experimental measurements of Lagrangian quantities in turbulent flow are difficult to make, so that information about $R_L$ is scarce. Nevertheless, two important results concerning the limiting cases of $t \rightarrow 0$ and of $t \rightarrow \infty$ were enunciated.

**Short diffusion times:** $R_L = 1$. The distance traveled by the marked particle is equal to its velocity multiplied by the time elapsed. A classical result from Newtonian mechanics. In other words, one can say that, at short diffusion times, the motion of the marked particle is predictable, provided we know the initial conditions; that is, the motion is deterministic.

**Long diffusion times.** The root mean square particle displacement is proportional to the square root of the time elapsed. This is the same result one would find for the classical random walk of discontinuous movements, where the particle variance is proportional to the square root of the number of steps.

The latter result characterizes the analogy between the gas kinetic theory and the diffusive motion of eddies. Thus, the r.m.s. distance traveled by a fluid particle can be expressed in terms of a diffusion coefficient defined as a function of single-time means and a Lagrangian integral time-scale. The above results, although simple, gave us a deep insight into the nature of turbulent diffusion. Nothing was really solved by the results, as the closure problem still persisted, but some connections and "transforms" of the problem were established.

Another great achievement of the traditional approaches is the universal log-law for the mean velocity distribution in the near wall region. Using the notion that the near wall turbulent flow could be divided into distinct regions, with distinct dominant physical effects, scaling velocities and lengths, Millikan, (1939) resorted to a "matchability" argument to work out a functional relationship for the velocity profile. The resulting logarithmic expression has become one of the great paradigms of turbulence theory. Validated by a huge number of experimental data, the so-called law of the wall has become a bench mark result to which all simulations of wall flow have to conform to. Evidence in favor of the law is, therefore, strong. Some authors, however, have placed the law under a severe scrutiny, claiming the velocity profile to be power-like, and not logarithmic-like!

In fact, the derivation of both laws is equally consistent and rigorous. However, they are based on entirely different assumptions. Both derivations start from similarity and asymptotic considerations. The log-law is obtained from the assumption that for sufficiently large local Reynolds number and sufficiently large flow Reynolds number, the dependence of the velocity gradient on the molecular viscosity disappears completely. In the alternative approach that leads to the power-law, the velocity gradient is assumed to possess a power-type asymptotic behaviour, where the exponent and the multiplying parameter are supposed to depend somehow on the flow Reynolds number. Thus, the velocity gradient dependence on molecular viscosity does not disappear however large the local and the flow Reynolds numbers may be. The form of the power-like scaling law yields a family of curves whose parameter is the Reynods number. The resulting envelop of curves is shown to be very close to the universal log-law.

The original result, however, has remained unshaken. The fact that the log-law can be obtained from formal asymptotic methods and that it has been shown to apply to a variety of flows, has firmly silenced its critics. In fact, the large body of useful services covered by the law has made it to transcend in importance. The universal laws of friction for smooth and rough pipes are just one of the pungent examples of application of the log-law.

Typical examples of application of the law include cases with wall transpiration, roughness, transfer of heat, compressibility, three-dimensionality, and even separation of flow. Of course, for all of these situations, some modifications need to be made in the original formulation to comply with specific flow requirements. However, the essence of the law and its general form are the same for all cases.

For boundary layer flows, a second universal law was added to our collection of great achievements. In the outer region of the boundary layer the log-law may no longer apply, since the conditions in which it is based are no longer valid. However, experimental evidence has shown that, if we consider as our reference quantities the skin-friction velocity and the boundary layer thickness, a velocity defect similarity relation can be constructed which turns out to be valid for the whole turbulent part of the boundary layer. This result was suggested by Von Karman to be considered a postulate, the velocity-defect law.

Impressed by the similar behaviour between the flow in the outer region of a boundary layer and the flow in a wake, for both exhibit large scale mixing processes controlled primarily by inertia effects rather than viscous effects, Coles, (1956) proposed a correction function to the classical log-law. The resulting expression, built purely on experimental grounds, includes in its validity domain both the turbulent part of the wall region and the outer region. The equation for the correction function was called by Coles the law of the wake. The actual function may be approximated by an antisymmetrical sine function multiplied by a profile parameter which does not depend on the transversal flow coordinate.

The above list of successes is, of course, not exhaustive. Some other great achievements such as the stability theory for the flow between two rotating cylinders developed by Taylor, (1923), or the boundary layer approximations due to Prandtl, (1904), could easily have been included here. The main feature of the chosen achievements is that they were advanced by exploiting plausible physical arguments through dimensional and similarity analyses.

The reader will be keen in identifying some of the above results with some of the basic working rules advanced in the section *Some Basic Working Rules*.

## A Bit of History

The progress in the art of turbulence modelling in Brazil must, in some way or another, be related to the degree of development of its graduate courses in physics, applied mathematics and engineering. This is equivalent to say that turbulence is a contemporary subject in Brazil, still in its infancy and, therefore, in a very incipient state. In fact, the very first graduate engineering course in Brazil was only inaugurated at the Federal University of Rio de Janeiro (UFRJ) in the sixties. In a historical deed, the Chemical Engineering Division of the Institute of Chemistry founded in March of 1963 the Chemical Engineering Graduate Program. This act was a direct result from a previous official visit to the United States of America by Prof. Alberto Luiz Coimbra to study in detail all the aspects of the engineering graduate courses taught at the Universities of Houston, Rice, California (Los Angeles and Berkeley), Stanford, Cal. Tech., Minnesota, M.I.T. and Michigan. The visit, which took place in December of 1960, was sponsored by the Organization of American States, (OAS), and by the Congregation of the Chemical School of the Federal University of Rio de Janeiro. As a follow up to this visit, in August of 1961 the Deans of the Engineering Schools of the Universities of Houston and of Texas came to Rio de Janeiro.

Together with their Brazilian pears, the visiting deans established a preliminary plan for the implementation of a graduate engineering course at the Federal University of Rio de Janeiro that was finally presented in October, 1961, to the brazilian coordinator of the Alliance for Progress. This plan was made accessible to the community in December of 1961 at a seminar on university reform and on the teaching of engineering organized by the Engineering Club of Rio de Janeiro. As an exercise to a complete implementation of the graduate program, it was decided that short and intensive courses on the subjects of Boundary Layer Theory and Turbulence, Flow in a Porous Medium and Computer Programming would be offered in the months of July and August of 1962. The courses would be jointly sponsored by the OAS, the Brazilian National Research Council (CNPq), the Chemical Institute of UFRJ and the University of Houston (UH), and would be given by the UH lecturers Drs. Abraham E. Dukler and Elliot I. Organick.

This initiative, in fact, was determinant in the definition of the areas of excellence that the graduate courses to be established were this have throughout the sixties. The special care dedicated to fluid mechanics was clear, and that is where this review wants to concentrate on.

The formal inauguration of the Chemical Engineering Program occurred in March, 1963. Still benefiting from financial help from the OAS, and with further aid from the Fulbright Commission and the Rockfeller Foundation, the coordinators of the graduate program invited four American lecturers to, together with six other Brazilian colleagues, provide the initially required teaching and supervision. The four U.S. lectures were Profs. Donald Katz, Louis Brand, Cornelius John Pings Jr. and Frank M. Tiller. Only two years later, and under the guidance of the Chemical Engineering Program, the Mechanical Engineering Program started functioning. In the years that followed, nine other graduate programs were created giving origin to the Post-Graduate Engineering School of the Federal University of Rio de Janeiro (COPPE/UFRJ).

The role of those three initial courses on the development of the subjects of fluid mechanics and of heat transfer in Brazil was seminal. Levered by the presence of great scientists on those subjects, the chemical and mechanical graduate programs became an important center for fluid mechanics and heat transfer studies. Indeed, all twenty four M.Sc.(Magister Scientia) theses that were presented at COPPE/UFRJ from 1964 to 1966 had to do in some way or another with fluid mechanics or heat transfer processes. Only in 1968, and only with the entrance of the graduate programs in Metallurgical, Electrical, Civil, Nuclear, Naval and Manufacturing Engineering, the tide of power started to change.

In a broader sense, as we shall see, the above mentioned facts unfolded in a national level. The cast of great scientists temporarily associated to COPPE/UFRJ was to become a key factor in the development of the sciences of fluid mechanics and of heat transfer in the whole of Brazil. Several examples immediately leap to the mind. The close relation, almost intimate, between COPPE/UFRJ and Houston University through Profs. Coimbra and Tiller prompted several students to go abroad to study with Prof. A. E. Dukler and other Houston lecturers. These students were late to come back to Brazil and, on moving to different institutions, they were ultimately the responsible for the establishment of many other graduate fluid mechanics programs in Brazil. Another notable example was the passage of Prof. Ephraim M. Sparrow, currently an Emeritus Professor at Minnesota University, through COPPE/UFRJ. After spending two years in Brazil, he went on to supervise a large number of Brazilian students over a span of more than twenty years. His impression on the development of fluid mechanics science in Brazil is therefore unquestionable. Indeed, when the Mechanical Engineering Program started, its staff consisted only of four lecturers: Profs. Sparrow and Jacques Louis Mercier, and the assistant lecturers Théo F. C. Silva and Franscisco N. de Farias. As Prof. Mercier left after a span of one year, the role of turning the graduate program in mechanical engineering into a success was left almost entirely to Prof. Sparrow.

In the next sections we will try to establish a firm connection between all the above facts and the present state of turbulence research in Brazil. Here, just for the sake of future reference, we will quote a number of selected theses. They are:

- Gileno A. Barreto, "Hot-wire Anemometer - Construction and Calibration", M.Sc. Thesis, 1964.
- Ralf Gielow, "The Graetz-Nusselt Problem for a Turbulent Flow", M.Sc. Thesis, 1965.
- João S. D'Avila, "Solutions of the Boundary Layer Equations for a Turbulent Flow", M.Sc. Thesis, 1968.
- Arno Bollmann, "Study of a Turbulent Jet in a Highly Turbulent Environment", M.Sc. Thesis, 1969.
- José Augusto F. Gouvêa, "Heat Transfer in Turbulent Flows", M.Sc. Thesis, 1969.
- Carlos E. Lopes, "Turbulence in the Vicinity of Fluid Interfaces", M.Sc. Thesis, 1970.

These examples were chosen to illustrate the state of turbulence research in Brazil by the end of the sixties. They, by no means, represent the totality of work done in turbulence at that time. Note, however, that any of the above titles could be perfectly valid titles for theses to be presented in the past few years.

The growth in the engineering sciences in Brazil provided by the consolidation of several graduate courses quickly started pressing for the creation of an appropriate forum where discussions of scientific interest were to take place. As a result, several symposia and conferences were to have birth in the late sixties, early seventies.

For the mechanical engineering community, the landmark was the creation of the Brazilian Congress of Mechanical Engineering. Aiming at discussing the problems - in a broad sense - related to mechanical engineering in Brazil, a small group of twelve researchers assembled in Florianópolis, Santa Catarina, for the First Brazilian Symposium on Mechanical Engineering. The Symposium was held in 1971 and produced twelve technical papers. Since then, and at every two years, the Brazilian Congress of Mechanical Engineering (COBEM) has been staged on a regular basis; it is, unquestionably, the most important conference in Brazil on Mechanical Engineering. In its latest edition, the astonishing number of 631 papers were presented.

By 1986 the area of thermosciences had achieved such a degree of development in Brazil that the creation of an event uniquely devoted to it was a must. A new conference was then organized which was to be termed the Brazilian National Meeting on Thermal Sciences (ENCIT). The conference, which is held every two years, became an immediate success. In the last event, held in Florianópolis, Santa Catarina, 331 papers were presented.

## Statistics

Before we make any critical comment on the existing literature on turbulent flow, let us first work out the statistics of the past COBEM's and ENCIT's. On compiling all relevant work to this review we have tried as much as possible to include all material of interest. Hence, the orientation was to consider every possible contribution to the subject. As such, all papers that dealt with turbulence in one way or another were considered.

The key to abbreviations is shown in Table 2. Table 3 shows the details of all the events covered by the present review. A comparison between the total number of published work and the number of papers on turbulence can the drawn from this table and Table 4. Note that the papers on turbulence account for roughly 10% of the total contribution to the ENCIT's. Concerning the COBEM's, about 5% of the contributed papers were related to turbulence.

**Table 2 Key to abbreviations**

Aero. = Aeronautics
Comb. = Combustion
TurbMach. = Turbomachines
Int.Flow = Internal Flow
Heat Exch. = Heat Exchangers
E.F.M. = Environmental Fluid Mechanics
Ind.Proc. = Industrial Processes
Instr. = Instrumentation
M.-Phase = Multi-Phase Flow
Hot-W. = Hot-Wire Anemometry
Visual. = Flow Visualization

**Table 3 ABCM conferences**

| Conference | Date, Organizing Committee | No. Papers |
|---|---|---|
| COBEM-1971 | 19-24/Nov, Prof. C.E.Stemmer/SC | 12 |
| COBEM-1973 | 5-7/Nov, COPPE/UFRJ | 85 |
| COBEM-1975 | 9-11/Dec, COPPE/UFRJ | 106 |
| COBEM-1977 | 12-14/Dec, CT/UFSC | 133 |
| COBEM-1979 | 12-15/Dec, UNICAMP | 169 |
| COBEM-1981 | 15-18/Dec, PUC/Rio | 162 |
| COBEM-1983 | 13-16/Dec, UFU/MG | 197 |
| COBEM-1985 | 10-13/Dec, ITA/SP | 239 |
| ENCIT-1986 | 10-12/Dec, PUC/Rio | 61 |
| COBEM-1987 | 7-11/Dec, DEM/UFSC | 443 |
| ENCIT-1988 | 6-8/Dec, INPE,CTA | 94 |
| COBEM-1989 | 5-8/Dec, PEM/COPPE/UFRJ | 350 |
| ENCIT-1990 | 10-12/Dec, DEM/UFSC | 216 |
| COBEM-1991 | 11-13/Dec, IPT,UNESP,UNICAMP,USP | 401 |
| ENCIT-1992 | 1-4/Dec, PUC/Rio | 189 |
| COBEM-1993 | 7-10/Dec, DEM/UnB/DF | 521 |
| ENCIT-1994 | 7-9/Dec, EPUSP,IPT/SP | 137 |
| COBEM-1995 | 12-15/Dec, UFMG,PUC/MG,CEFET/MG | 631 |
| ENCIT-1996 | 11-14/Nov, DEM/UFSC | 331 |

Table 4 classifies all gathered literature according to the area of application. A classification of all theoretical work according to the type of adopted turbulence model or problem methodology is presented in Table 5. The experimental works are classified in Table 6.

**Table 4 Work motivation**

| | Aero. | Comb. | TurbMach. | Int.Flow | Heat Exch. | E.F.M. |
|---|---|---|---|---|---|---|
| 1973 | | | | | | 2 |
| 1975 | | | | | 1 | |
| 1977 | | | 1 | 1 | | |
| 1979 | 1 | | | 2 | | |
| 1981 | | | | 2 | | |
| 1983 | | | | | 1 | |
| 1985 | | | | 2 | | |
| 1986 | | | | 3 | 1 | |
| 1987 | 3 | | | 2 | 2 | |

(continued)

| | | | | | | |
|---|---|---|---|---|---|---|
| 1988 | 2 | | 1 | | | 1 |
| 1989 | 4 | | 1 | | | 2 |
| 1990 | 5 | 1 | 1 | 2 | 3 | 1 |
| 1991 | 6 | 3 | | 4 | 1 | 4 |
| 1992 | 5 | 1 | | 2 | | 2 |
| 1993 | 3 | 3 | 2 | 1 | | 1 |
| 1994 | 2 | 2 | 1 | 1 | | 2 |
| 1995 | | 1 | | 1 | 2 | 1 |
| 1996 | 8 | 6 | 2 | 5 | 2 | 4 |

| | Fundamental | Ind.Proc. | Instr. | M.-Phase | Total |
|---|---|---|---|---|---|
| 1973 | 1 | | | 1 | 4 |
| 1975 | 4 | | | | 5 |
| 1977 | 7 | 1 | | | 10 |
| 1979 | 4 | | | | 7 |
| 1981 | | | 1 | | 3 |
| 1983 | 3 | 3 | | | 7 |
| 1985 | 5 | | | | 7 |
| 1986 | 5 | | | | 9 |
| 1987 | 3 | | | | 10 |
| 1988 | 2 | | | 2 | 8 |
| 1989 | 9 | 1 | | 5 | 22 |
| 1990 | 6 | | | 2 | 21 |
| 1991 | 2 | | 1 | 1 | 18 |
| 1992 | 6 | | 2 | | 18 |
| 1993 | 8 | 1 | | 1 | 20 |
| 1994 | 5 | 1 | | | 14 |
| 1995 | 10 | 1 | | | 16 |
| 1996 | 5 | 12 | | | 44 |

Table 5 Turbulence Models

| | Eddy Viscosity Models | | | Algebraic Stress Models | | |
|---|---|---|---|---|---|---|
| | 0 Eq. | 1. Eq. | 2. Eq. | Classic | RNG | Generic |
| 1973 | | | | | | |
| 1975 | | 1 | 1 | | | |
| 1977 | 1 | | 3 | | | |
| 1979 | 1 | 1 | 1 | | | |
| 1981 | 1 | | 1 | | | |
| 1983 | | | | | | |
| 1985 | 1 | | 1 | | | |
| 1986 | 1 | | 1 | | | |
| 1987 | 1 | | 3 | | | |
| 1988 | 1 | | 1 | | | |
| 1989 | | | 4 | | | |
| 1990 | 1 | | 6 | 1 | | |
| 1991 | 5 | | 4 | | | |
| 1992 | 2 | | 3 | 2 | | |
| 1993 | 2 | | 5 | | | |
| 1994 | 1 | 1 | 6 | | | |
| 1995 | 1 | | 4 | | | |
| 1996 | 1 | | 14 | 2 | 1 | |

| | Transition | Asymp. Tech. | Vortex Meth. | Other | Total |
|---|---|---|---|---|---|
| 1973 | | | | 1 | 1 |
| 1975 | | 1 | | | 3 |
| 1977 | | 2 | | | 6 |
| 1979 | | 2 | | | 5 |
| 1981 | | | | | 2 |
| 1983 | | 1 | | | 1 |

(continued)

| | | | | | |
|---|---|---|---|---|---|
| 1985 | | | 1 | 2 | 5 |
| 1986 | | 1 | 1 | | 4 |
| 1987 | | | | | 4 |
| 1988 | | 1 | | 1 | 4 |
| 1989 | | 4 | | 3 | 11 |
| 1990 | 3 | 7 | | | 18 |
| 1991 | 1 | 6 | | | 16 |
| 1992 | 1 | 2 | 2 | | 12 |
| 1993 | 4 | 4 | | 1 | 16 |
| 1994 | 1 | | 1 | 2 | 12 |
| 1995 | | 6 | | 2 | 13 |
| 1996 | 3 | 6 | 1 | 4 | 32 |

**Table 6 Experimental works**

| | Hot-W. | Optical | Pitot Tube | Visual | Other | Total |
|---|---|---|---|---|---|---|
| 1973 | 1 | | 1 | 1 | | 3 |
| 1975 | 1 | | 1 | | | 2 |
| 1977 | 3 | 1 | | | | 4 |
| 1979 | 1 | | 1 | | | 2 |
| 1981 | 1 | | | | | 1 |
| 1983 | 2 | 1 | 3 | | 1 | 7 |
| 1985 | 2 | | | | | 2 |
| 1986 | 4 | | 1 | | 2 | 7 |
| 1987 | 1 | | 4 | 2 | | 7 |
| 1988 | 1 | | 2 | | 1 | 4 |
| 1989 | 2 | 4 | 4 | | 1 | 11 |
| 1990 | | | 2 | | 1 | 3 |
| 1991 | 1 | | | 1 | 2 | 4 |
| 1992 | 2 | | 3 | | 1 | 6 |
| 1993 | | 1 | 1 | | 2 | 4 |
| 1994 | | | | | 2 | 2 |
| 1995 | 1 | 1 | 1 | | | 3 |
| 1996 | 3 | | 4 | 1 | 5 | 13 |

## A Personal View

Let us now make a critical evaluation of all the work listed in the previous section.

The number of produced works that touch on turbulent flow is, as fully demonstrated by Table 2, not negligible. In fact, we are sure that those numbers will surprise most people. In particular, in the last ENCIT forty five papers on turbulence were presented. This figure would justify in its own right the realization of a conference only devoted to turbulence. Here, we remind the reader that a large, worldwide, conference on turbulence would attract about one hundred papers. Of course, one may argue that the number corresponding to ENCIT'96 cannot be taken as a general trend for the number of published works. However, even the average number of published works over the last eight years, twenty two, is not bad at all.

The distribution of the works according to their area of interest, Table 3, shows that Aeronautics and Fundamental Studies are still the leading scorers. Examples of works on Internal Flows, Heat Exchangers, Combustion or Turbomachinery, are very few. The areas of Instrumentation and Multiphase Flows are almost barely touched.

The true picture of the turbulence research carried out in Brazil by the mechanical engineers and scientists, however, is provided by Table 4. The striking feature there is the overwhelming majority of works on the two-equation modelling of turbulence and on similarity and asymptotic methods. The number of works in any of these two classes is even larger than the number of works that employ algebraic turbulence models. No work was found on the direct numerical simulation of flows. Only one work was found on large eddy simulation, and two on Reynolds stress modelling.

The compilation of all experimental work on turbulence is shown in Table 5. The numbers are extremely timid. They show that no tradition exists in Brazil for the performance of experimental studies on turbulent flows. In particular, the hot-wire anemometry technique, so important for the measurement of turbulent flow properties, is shown to be very incipient in Brazil.

The performance of good, reliable and accurate experiments is however crucial for the development of the science of turbulence. As we saw before, the models that serve industry are all Ad Hoc models. We have also seen that most of the work carried out in Brazil makes use of two-equation differential models and that these models are vulnerable to a series of weaknesses and shortcomings. Unfortunately, these difficulties must always be overcome on an individual basis through new modelling procedures resulting from new experimental observations.

In fact, as observed by Prof. L. J. S. Bradbury in a personal communication to the present authors, the bound between the available data and the available turbulence models is very tight. Indeed, as vastly demonstrated by common experience, the progress in turbulence modelling is directly fastened to the progress in turbulence measurement techniques. Thus, according to Bradbury: "When only mean velocity measurements were available, only integral theories satisfying overall momentum conservation could be formulated. A small number of disposable constants were needed but agreement with experiments was poor outside the data set. With turbulent measurements, differential models could be developed which satisfied local momentum conservation. As better measurements became available, models satisfying an enlarging hierarchy of conservation relationships were developed with an enlarging hierarchy of disposable constants. In no case are the models rational models - they are all, to a greater or lesser extent, empirical, and modelling is really a sophisticated way of "curve fitting" to the experimental data. The number of disposable constants in Reynolds stress models is large and they are adjusted to give the best fit to the largest set of experimental data".

In relation to the large number of theoretical work on two-equation differential models and, particularly, on numerical techniques, we must acknowledge this to be related to type of training that the Brazilian scientists received in their undergraduate and graduate courses. In fact, the increasing lack of training on experimental techniques and on the fundamental aspects of turbulence is a worldwide trend that is only being repeated in Brazil. The pressing demand for publications, the absolute shortage of money for research, the computer revolution, all these are factors that conspire against the establishment of experimental programs and time consuming fundamental studies. Researchers these days have become more practical, choosing as their research lines topics which can be developed at a certain pace and with the minimum of trouble; this will assure them a profitable carrier.

The aggravating circumstance for the Brazilian case is that we have never had any tradition on these issues and, as a result, nothing really was built in the past in those lines. The competence of Brazilian researchers to implement turbulence models for a variety of flow situations is undoubtedly high. In the literature reviewing process several examples of very complex problem geometries were found. Despite a clear dominance of numerical methods that involve finite differences and finite volumes, quite a number of works were found on finite elements, vortex dynamics, integral methods, the generalized integral transform technique, integral methods, and many others.

The number of works which resort to perturbation techniques is also significative. These techniques have a long history of service to fluid mechanics, dating back to Prandtl (1904) and the derivation of the boundary layer theory. The law of the wall, is perhaps the soundest case of application of singular perturbation methods to turbulent flow. In recent times, in the past editions of COBEM's and ENCIT's, extensions of the law for more complex flow situations have been sought with success. Such diverse effects as flow transpiration, flow compressibility, transfer of heat at the wall, surface roughness, shock wave interaction, flow separation, or a combination of all these factors, have been incorporated to the classical formulation yielding a set of useful expressions for the local velocity and temperature profiles, and for the skin-friction coefficient and the local Stanton number.

The tradition on theoretical fluid mechanics in Brazil has certainly its roots on the early graduate courses set out in the sixties. At that time, and by virtue of the strong mathematical character of the first fluid dynamicists, in particular Prof. Coimbra, the emphasis had always been on the development of the fundamental aspects of problems. Experimental investigations were always rare. It is, therefore, only natural to expect this trend to be reflected in the past publications appearing in COBEM and in ENCIT. In fact, since the early editions of COBEM some very interesting theoretical works on turbulent flow were present. The years of 1975 and of 1977 were clearly vintage years for works on turbulence. All five works presented in 1975 and all ten works presented in 1977 were top quality. In the years to follow some great works were continuously been added to ABCM's archive.

This brings us to a paradox. Despite the superior quality of work that was being generated, few people managed to continue in the system producing quality work. In reality, many vanished after a few years. In more recent times, this has become a worrying pattern. Researchers with a very good academic record, who performed very good research work in their Ph.D. courses, simply do not seem capable of following it up in their carriers. The consequence is that, despite a continued entrance of new people into the fluid dynamics community, the number of published works remains nearly constant in the last eight years. The reasons for this phenomenon are unknown to the present authors. Maybe researchers are carrying out more applied studies, which take them away from their original specialization field; maybe their academic loads become too heavy or the struggle for financial aid is tough and unrewarding. The number of feasible explanations is high and academic at this stage. The important point to notice is that a lot of well educated scientists are not surviving in the present system and that something must be done.

## Gallery

This section was thought as to provide the reader with a brief but faithful view of the type of studies carried out on turbulence in Brazil in the last twenty five years, illustrating the past achievements through pictures and graphs taken directly from the original works. The expectation is that the selection will be representative enough to cover all aspects discussed in previous sections. In this sense, the main body of work is concentrated on asymptotic techniques and on two-equation differential models.

The first ABCM Conference with full proceedings was the 1973 COBEM. In that occasion, only a single work was presented on the subject of turbulence. The work, due to Gaspareto and Giorgetti, was a very interesting attempt at developing hot-wire probes for the measurement of water flow properties. The wires were coated with epoxy, yielding an alternative method for the use of hot films. The description of the main aspects of the problem, including the whole probe production process and the calibration process was, unfortunately, very short and incomplete, what makes it virtually impossible to reproduce the author's results. In taking the measurements a DISA 55D01 anenometer unit was used. Graphs for the flow mean velocity in the wake behind a cylinder were presented for two different Reynolds numbers. Typical measured velocity profiles are shown in Fig. 1, which is here reproduced in its totality. In fact, in reproducing all figures, we have decided to keep the original captions.

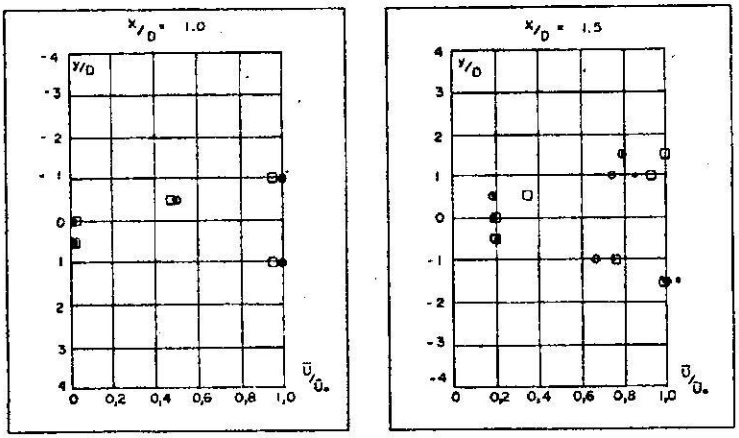


FIGURA 3.7 PERFIL DE VELOCIDADES NA ESTEIRA $Re_D = 1025$
SONDAS: • FILME; o FIO 13μm, □ FIO 5μm

**Fig. 1 Hot-wire results of Gaspareto and Giorgetti.**

The next COBEM presented some high quality work. The return to Brazil of several research students who had gone abroad for their Ph.D. degrees brought back some innovating ideas. All works present in the 1975 proceedings were, in fact, to have far reaching influence in the development of turbulent research in Brazil. In view of the statistics we have seen before, we are obliged to mention here the work of Carajilescov. In his work, an one-equation differential model was used to describe the details of the flow through a triangular array of rods with different aspect ratios and Reynolds number.

The calculations were in reasonable agreement with the experimental data of other authors, allowing the author to make a good assessment of any existing secondary flow. Results for the shear stress distribution are shown in Fig. 2.

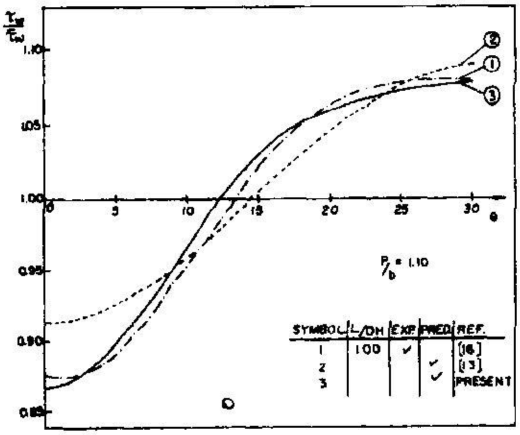


Fig. 3. Wall Shear Stress Distribution

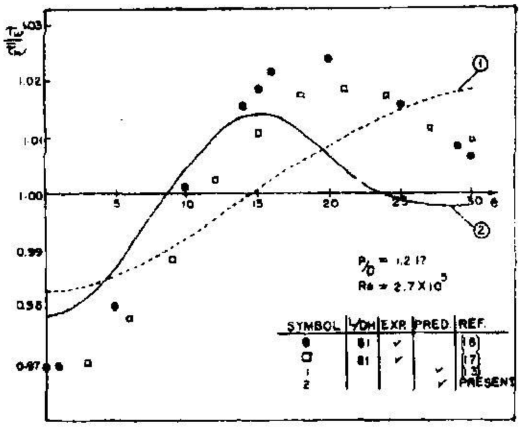


Fig. 4. Wall Shear Stress Distribution

**Fig. 2 Results of carajileskov.**

The year of 1977 was also very good for works on turbulence. Therefore, the task of selecting a representative reference of all works published was really difficult. The works of Militzer, Pimenta, Alves, Pereira Filho, Alvim Filho and of Menon, all resulted from their graduate dissertations. The works of Crabb and of Nickel were top quality, but had no participation, direct or indirect, of brazilians. From a historical perspective, it is the judgment of the present authors that perhaps the article with the most relevant result was the work of Pimenta. His experiments were part of a great collective effort of the Heat and Mass Transfer Group at Stanford University to assess the properties of transpired turbulent boundary layers, and the new data he gathered for flow over a rough an porous wall became reference work. Some of his data for the local skin-friction coefficient and for the Stanton number are shown in Fig. 3.

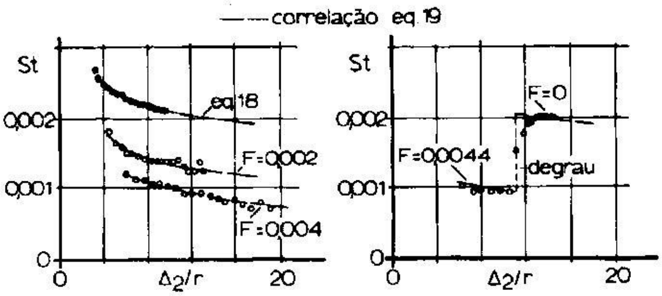


Fig. 4 Números de Stanton com injeção

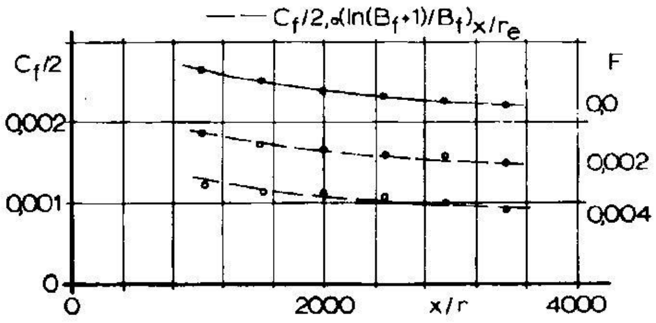


Fig.5 Coeficiente de atrito com injeção

**Fig. 3 The experimental results of Pimenta.**

The next COBEM did not bring any great news. Compared with the previous conferences, the results presented did not show any significative change. Seven works where published in the proceedings. A very interesting one was the experimental work of Ferreira on turbulent thermal convection in a horizontal layer between parallel plates. The work illustrates the various possible configurations for the thermal convection process, comparing the data of several authors, including his own. Fig. 4 shows the flow configurations tackled in this work. Results for the high order moments were presented.

The year of 1981 showed a sharp decline in the number of works on turbulence. Only three works were published that year. From these, the most interesting one was the work of Frota and Moffat. In this work, a triple hot-wire system was developed to measure the instantaneous values of the velocity components. The technique allowed the measurement of the turbulent properties of complex turbulent three-dimensional flows with the use of a single probe. According to the authors, both the mean velocity profiles and the turbulent shear stresses could be measured with an accuracy of 1.4% and 3% respectively, provided the probe axis had a maximum misalignment of 20° with the flow direction. The paper is reasonably detailed but the results are poorly presented. The figures and graphs have a small size and for this reason are difficult to reproduce.

In the next COBEM we had a singular fact; all published works on turbulence were experimental works. Besides, and more importantly, five of the seven presented works were fully developed in Brazil, two of them dealing with the difficult subject of hydrodynamic stability. To illustrate that vintage year for experimental work we show here graphs for the critical Taylor number and the critical Reynolds number obtained, respectively by Purquerio and by Santana et al., for two different flow geometries: the flow of a Newtonian fluid in the interior of two concentric rotating cylinders and the flow of a non-Newtonian fluid in a capillar reometer.

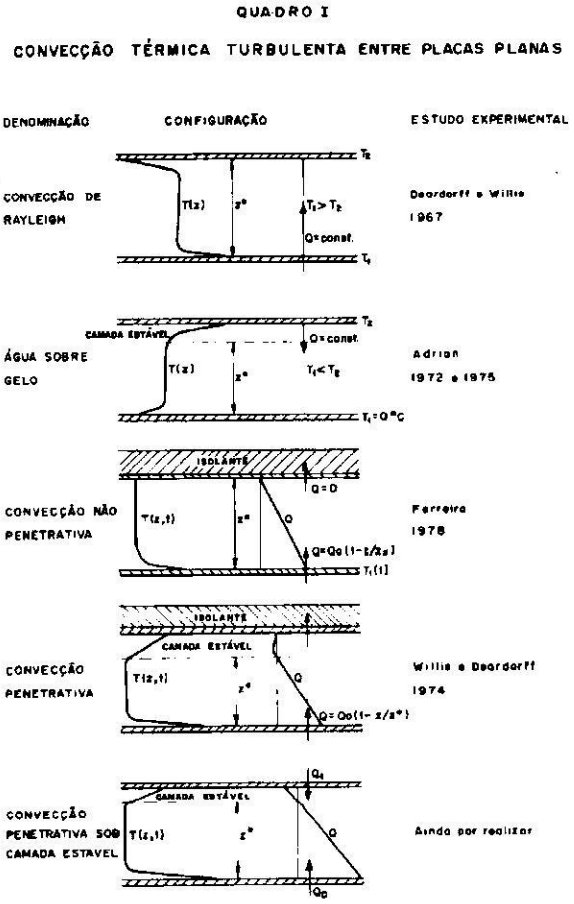


**Fig. 4** Possible flow configurations for turbulent thermal convection in a horizontal layer between parallel plates (after Ferreira).

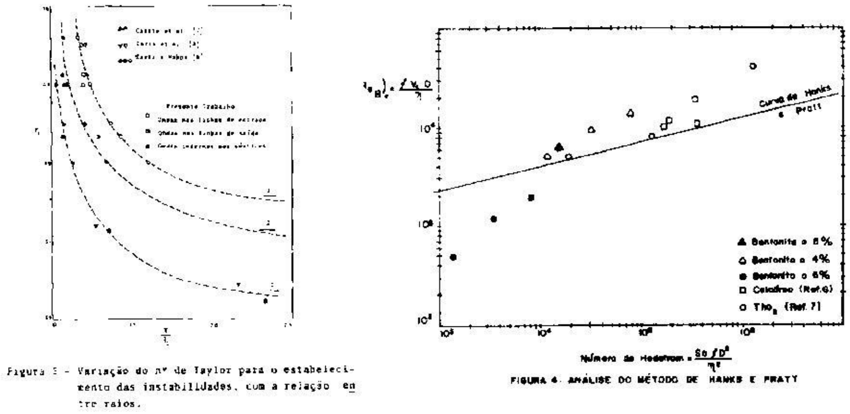


Figura 3 - Variação do nº de Taylor para o estabelecimento das instabilidades, com a relação entre raios.

FIGURA 4 - ANÁLISE DO MÉTODO DE HANKS E PRATT

**Fig. 5** The transition results of Purquerio and of Santana et al.

The following conference had the same number of published works, seven. From these, the most important were the two experimental works of Leite et al. on plane turbulent jets. In these works, the organized motion in the near field of a turbulent plane jet under periodic controlled acoustic excitations were investigated visually and quantitatively through smoke-wire and hot-wire techniques. The Strouhal number was made to vary from 0.15 to 0.60 and the time-dependent excitation from 0.5% to 49%. The main conclusion was that the fundamental component attains a maximum value for Strouhal = 0.18 at x/H = 4.0 along the centerline. Flow visualization results obtained through the smoke-wire technique are shown in Fig. 6.

**Fig. 6** **Organized motion in the near field of a turbulent plane jet.**

The year of 1986 inaugurated the ENCIT's. In all, nine papers on turbulence were presented in this conference. The works were about average, reflecting mainly the work carried out in Brazil at that time; the meeting had only one foreign entry. In fact, most works were just a small departure of former works that were being studied for many years; some for more than ten years. The work of Pimenta and Alvim Filho studied the mixing flow provoked by the interaction of two confined axisymmetric jets. Several graphs were presented with the axial distribution of static pressures and the transversal profiles of the averaged velocities and turbulent intensities. Figure 7 shows some of the data.

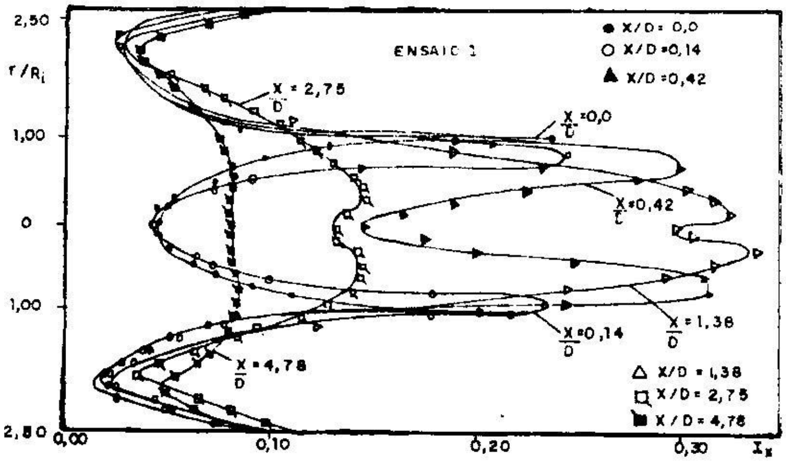


Figura 8 – Perfis de Intensidade de Turbulência

**Fig. 7** **The experimental data of Pimenta and Alvim Filho**

The 1987 COBEM section on turbulent flow had only three articles. From those, the only one worth of a note was the work of Leite and Scofano. By this time, Leite had a very comprehensive oeuvre on plane turbulent jets, certainly one of the most important in the history of events sponsored by ABCM. For this year, no figures will be presented.

The 1988 ENCIT had no particular section on turbulence. Even so, eight works on the subject were presented that year. Embarking on a trend that was to dominate the studies on turbulence in Brazil in the coming years, the turbulent flow in a compressor was investigated by Deschamps, Ferreira and Prata through a two-equation differential model. The limitation in paper length resulted in a very concise article where many aspects of the analysis could not be explained in detail. For example, only graphs of the pressure distribution where presented. These graphs are shown in Fig. 8. In the years to come these authors were to publish many other interesting papers on the subject.

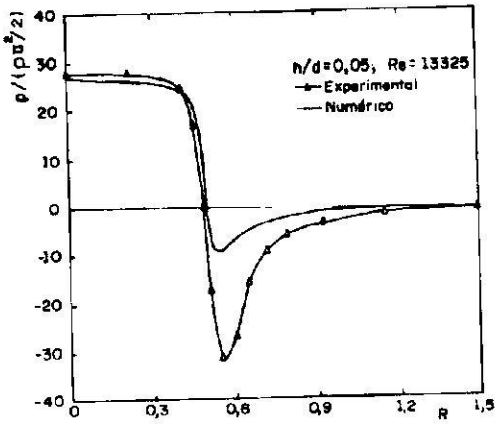


Fig. 6 - Comparação entre resultados numérico e experimental; h/d=0,05, Re=13.325.

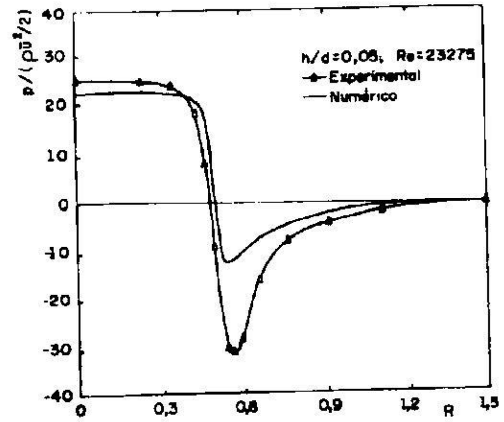


Fig.7 - Comparação entre resultados numérico e experimental; h/d=0,05, Re=23.275.

**Fig. 8 The results of Deschamps et alli.**

The tenth COBEM presented a very good selection of works on turbulence. The highlight here was the large number of works, nine, devoted to the fundamental aspects of the problem. Half of the twenty two articles were on experimental techniques; five on two-equation differential models. The most important contribution to turbulence in this meeting was given by the twin works of Coelho on the modelling of turbulent jets in cross flow. Some of his results are reproduced in Fig. 9.

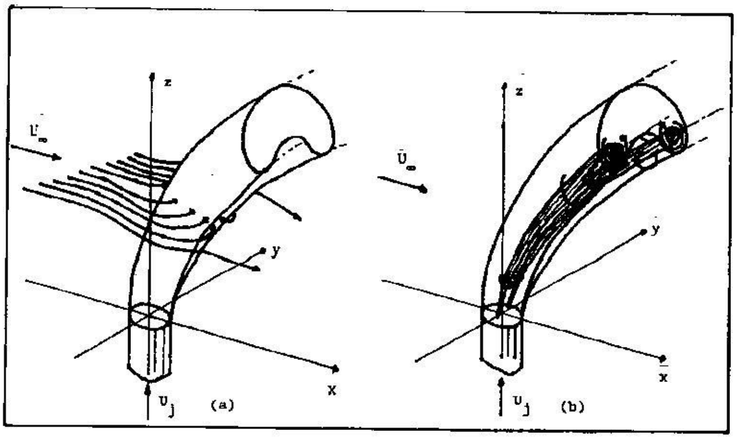


FIGURE 2.2: Models for a jet in a cross-flow: (a) entraining surface; (b) vortex pair.

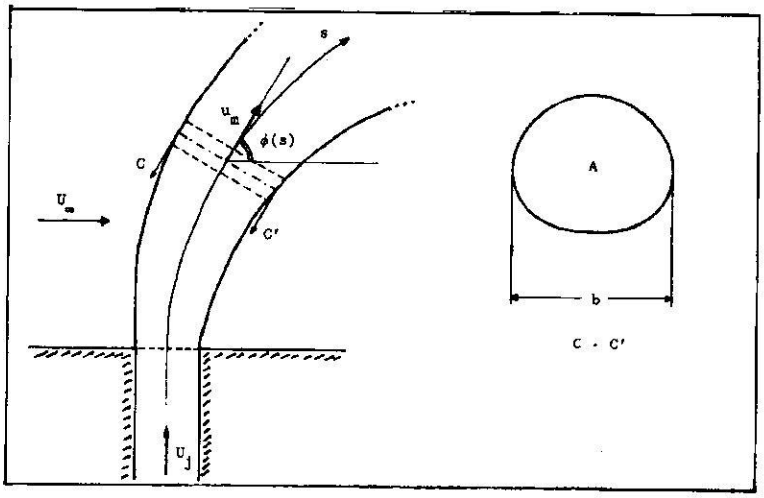


FIGURE 2.3: Nomenclature for an element of the jet.

**Fig. 9 The flow configuration of a turbulent jet in cross flow.**

The major conclusion reached by Coelho was that turbulent entrainment and the transport of the transversal component of vorticity have a strong influence on the dynamics of the mixing layer in the initial region of the jet. Further considerations on the formation of the wake behind the jet led to two main conclusions: 1) The deflection of the jet in the near field of these flows is mainly due to entrainment rather than to pressure drag. 2) The transversal component of vorticity has a strong influence on the formation of the pair of trailing vortices, inducing a rapid transference of transversal vorticity into the pair of vortices which is being formed.

In 1990, the third ENCIT had two specific sections on turbulent flow. The biggest contingent of papers this time was on asymptotic techniques applied to turbulent flow. The emphasis on the implementation aspects of numerical methods to turbulent flow was also high. The sad news here was the small number of experimental works. The simulation of incompressible 2D and 3D turbulent flows by a finite element method performed by Brasil Jr. et al. through the κ-ε model is presented in Fig. 10. Two cases were presented in the paper: turbulent backward facing step and annular turbulent jet. Only the former case is shown here.

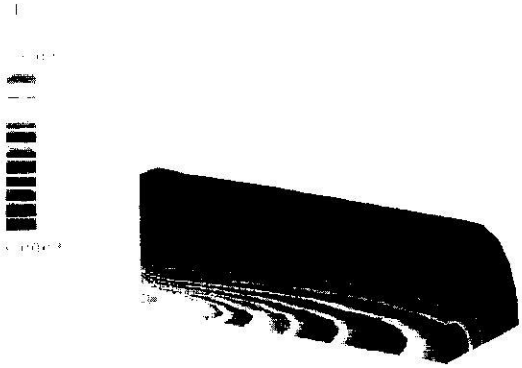


Fig. 10 The results of Brasil Jr for the turbulent kinetic energy levels.

The 1991 COBEM section on boundary layer theory and turbulence had twelve papers. A very interesting work, however, was presented in the section on thermal convection, vaporization and condensation. The work, due to Yanagihara and Torii, studied the influence of an array of longitudinal vortices generated by half-delta wings on the heat transfer of laminar boundary layers. Hot-wire velocity measurements and heat transfer experiments were carried out to evaluate the mechanism of heat transfer increase. The main conclusion was that arrays of counter rotating longitudinal vortices present better heat transfer characteristics than co-rotating arrays. Some of the results found by the authors are shown in Fig. 11.

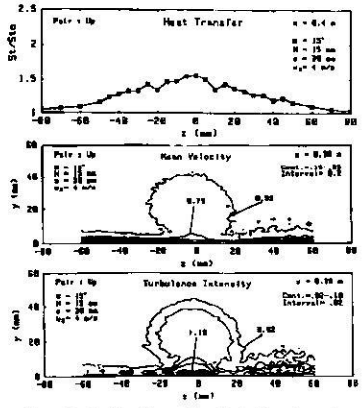


Figure 7 Heat transfer results and velocity contours for a pair with the common flow up at x= 0.4 m

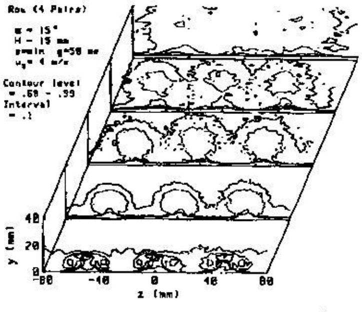
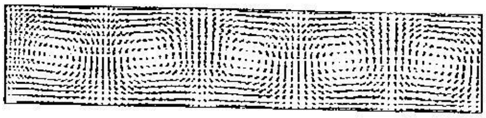


Figure 8 Streamwise velocity contours for an array of counter-rotating vortices (α= 15°, and g= 50 mm)

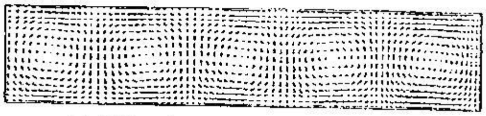
Fig. 11 Experimental results of Yanagihara and Torii.

The IV ENCIT was not a very good meeting for κ-ε modellists; only three works on two-equation differential models were presented from a total of eighteen works. The selected reference for this compilation, however, deals with the κ-ε model applied to a three-dimensional turbulent swirling flow in a rectangular duct of large aspect ratio. The results, obtained by Nogueira and Nieckele, show the effects of the Reynolds number and of the swirl intensity in the flow field, as shown in Fig. 12.

(a) $Z/D_h = 0,0$, $Re = 2 \times 10^4$, $P/D = 4,4$



(b) $Z/D_h = 1,1$, $Re = 2 \times 10^4$, $P/D = 4,4$



(c) $Z/D_h = 2,5$, $Re = 2 \times 10^4$, $P/D = 4,4$

Figura 5. Desenvolvimento das Velocidades Transversais

**Fig. 12 The numerical predictions of Nogueira and Nieckele.**

The 1993 COBEM showed a relative balance among the several entries for the sections on turbulent flow. One of the most interesting works was surely the numerical predictions of Kobayashi and Pereira for the flow over a two-dimensional hill covered with vegetation. The equations of motion were solved with the aid of an extended κ-ε turbulence model which included terms due to the drag caused by the plant canopy. Typical results for the kinetic turbulent energy are shown in Fig. 13.

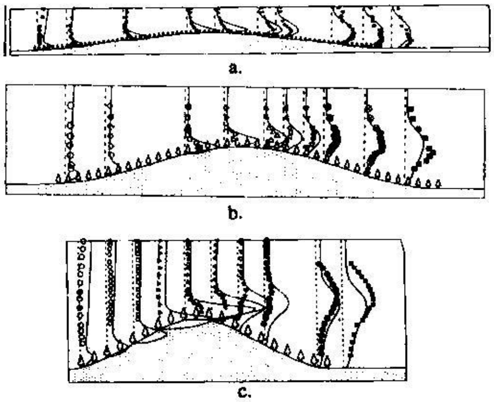


Figura 5. Previsões de Perfis de energia cinética turbulenta k com $C_D$=0,8 e $C_{4\varepsilon}$ = 1,95 e resultados experimentais para as três geometrias.

**Fig. 13 The numerical predictions of Kobayashi and Pereira.**

The year of 1994 was very quiet. The works on turbulence were again dominated by numerically oriented studies, with a clear prevalence of two-equation models. To illustrate this fact, we quote here the article of Vasconcellos and Maliska. Over the years Maliska realized an important work on the development of procedures for the computation of fluid mechanics. The next figure gives the reader a glimpse of his work. The paper performs a numerical study of the turbulent flow in a bifurcating channel using a multidomain procedure.

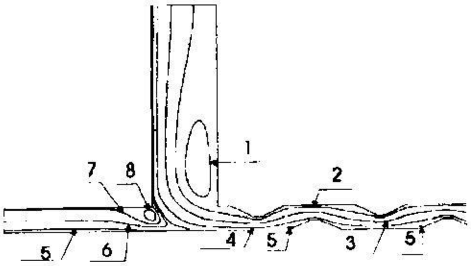
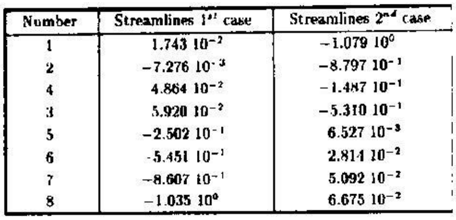


**Figure 9:** Streamlines for the second geometry



| Number | Streamlines $1^{st}$ case | Streamlines $2^{nd}$ case |
|---|---|---|
| 1 | $1.743\ 10^{-2}$ | $-1.079\ 10^{0}$ |
| 2 | $-7.276\ 10^{-3}$ | $-8.797\ 10^{-1}$ |
| 4 | $4.864\ 10^{-2}$ | $-1.487\ 10^{-1}$ |
| 3 | $5.920\ 10^{-2}$ | $-5.310\ 10^{-1}$ |
| 5 | $-2.502\ 10^{-1}$ | $6.527\ 10^{-3}$ |
| 6 | $-5.451\ 10^{-1}$ | $2.814\ 10^{-2}$ |
| 7 | $-8.607\ 10^{-1}$ | $5.092\ 10^{-2}$ |
| 8 | $-1.035\ 10^{0}$ | $6.675\ 10^{-2}$ |

**Table 2:** Streamlines · Fig. 5 and Fig. 9

**Fig. 14 The numerical predictions of Vasconcellos and Maliska.**

The XIII COBEM had 33 entries on turbulent flow. Despite the large number of possible candidates for our representative article, the choice here was obvious. In fact, the only work on large eddy simulation in the history of COBEM's and ENCIT's to date was published in this event. Pinho and Silveira Neto performed the simulation of a turbulent flow in a rectangular cavity using the sub-grid Smagorinsky isotropic model and the MacCormack compressible discretization method. The paper presents pictures of the calculated flow, showing the temporal evolution of the vorticity field. The main results are shown in Fig. 15.

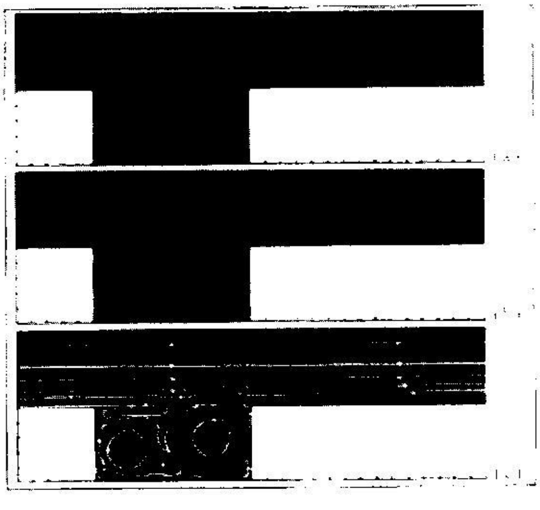


**Fig. 15 The large eddy simulation of Pinho and Silveira Neto.**

In 1996, forty five articles on turbulent flow were published in the ENCIT proceedings. From these, fourteen articles dealt with two-equation differential models; a obvious majority. The pick of a representative paper became then, again, very difficult. One of the articles that really called the attention of the present authors was the article of Queiroz et al. on the dispersion of contaminants released in an environment with a known dispersive capacity. Different formulations for the diffusity tensor were tested which were numerically solved. The results were compared with an exact analytical solution and with solutions provided by classical integral methods. Results for the local concentration profiles are shown in Fig. 16.

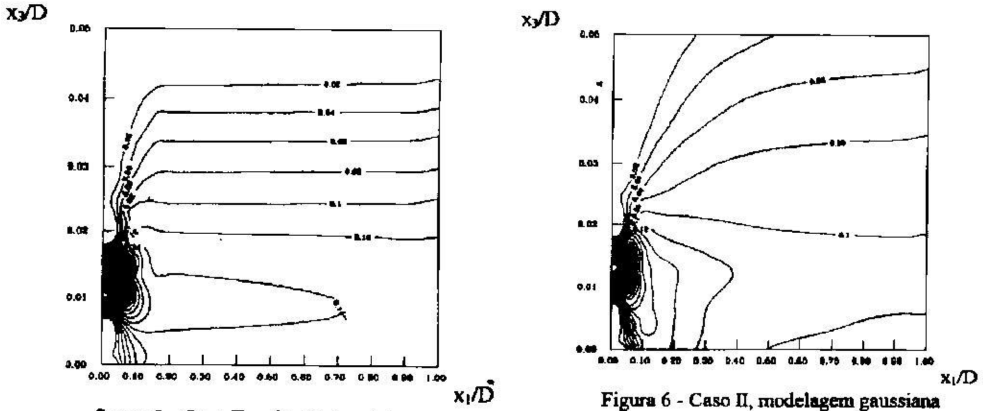


figura 5 - Caso II, solução numérica.

Figura 6 - Caso II, modelagem gaussiana correspondente à classe B.

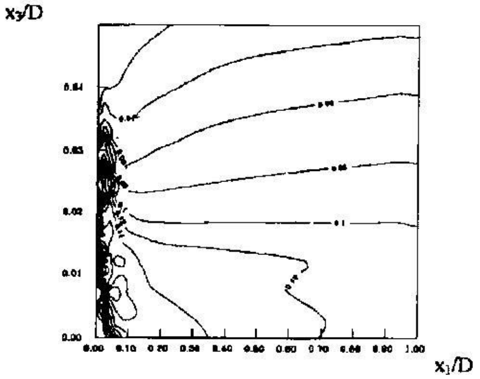


Figura 7 - Caso II, solução exata.

**Fig. 16 The experimental results of Queiroz.**

The feeling of the present authors is that the "turbulence" community has still much to mature. The high quality of the works presented in the first editions of COBEM were in part due to the foreign supervision of most works. As the number of papers increased and the authors were left to carry out their own research, the quality started to suffer. The number of people active in turbulence is still very small and the nuclei of most work generated in Brazil easily identified. It is true that the recent progresses have been remarkable; however, much still remains to be done.

## Final remarks

The purpose of this work was left clear at its outset: to give a picture of the present status of turbulence modelling in Brazil. The strategy for doing this was also clear. We started with a tour on the subject, aiming at giving the reader a view over most approaches to turbulence modelling. Next, after a short historical recollection, we presented a detailed statistics of all past COBEM's and ENCIT's in what concerns turbulence. The critical evaluation of this statistics was left to a separated section. We must emphatically point out that every opinion expressed in this work is the responsibility solely of its authors. The Brazilian Society of Mechanical Sciences holds no responsability for any judgment or conclusion upheld here.

During the collection of the relevant material, the authors tried to be as careful as possible so as to avoid the omission of any related work. The task of reviewing all proceedings, however, was very difficult and time consuming and this may have caused some references not to be spotted, for which we apologize in advance.

The present personal view on the subject of turbulence must not be taken here as conclusive. The authors are surely biased by their own experiences on the field, so that further views on the subject must be sought by the interested reader.

The general conclusion is that experimental and fundamental studies on turbulent flow must be stimulated in the future. Also, the mechanical engineering community must seek closer links with the physics community.

**Bibliography generation.** There are many ways for formatting bibliographies. The present work has made extensive use of the LATEX system and the companion program BIBTEX written by Oren Patashnik. Basically, three .bib files were prepared: class.bib, encit.bib and cobem.bib. These can be obtained directly from the authors.

**Acknowledgments.** In writing this work the authors have been strongly influenced by many ideas of Profs. Leslie Bradbury and Roddam Narasimha; fruitful discussions on the subject have challenged the authors to always re-think every fundamental aspect of the problem, forcing them to carefully consider every single word laid here; this has left an indelible mark on the final format of the present work.

In the references' compilation process we benefited from a valuable help from Mr. Eduardo Nunes.

The work was financially supported by the Brazilian National Research Council (CNPq) through the Research Grant No 350183/93-7.

## Cited Literature

The cited literature constitutes only a small portion of the texts on the subject, relevant to the present review. Due to lack of space, we have decided to include here only those references essential to a complete understanding of the text. They form a very short tour on the world of turbulence, being a small representation of the main lines of thought on the subject.

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## Compiled Literature

In opposition to the previous section, here we tried as much as possible to consider the largest number of eligible references. For this reason, the compiled literature was gathered considering all works that had anything to do with turbulence. This criterion led to a huge amount of work, forcing the authors to look at every paper on an individual basis. We firmly believe that only a very small fraction of all possible entries may have escaped from us. In this sense we are sure the present collection of articles is really representative of the work made in Brazil in turbulence.

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