Relatórios Coppead é uma publicação do Instituto COPPEAD de Administração da Universidade Federal do Rio de Janeiro (UFRJ)

Comissão de Pesquisa
Angela Rocha
Paulo Fernando Fleury
Ricardo Leal

Gerência de Publicações
Simone da Rocha Weitzel

Editoração Eletrônica
Adriana Baptista Pereira

Referenciação e Ficha catalográfica
Ana Rita Mendonça de Moura

Silva, André Carvalhal da.

Measuring risk based on stable distributions: an examination of Latin American stock indexes / André Carvalhal da Siva; Eduardo Facó Lemgruber. – Rio de Janeiro : UFRJ/COPPEAD, 2001.

19 p. ; 27 cm. – (Relatórios Coppead; 344).

ISBN 85-7508-026-1
ISSN 1518-3335

1. Finanças I. Lemgruber, Eduardo Facó. II. Título. III. Série.

CDD – 332

Pedidos para Biblioteca
Caixa Postal 68514 – Ilha do Fundão
21941-970 – Rio de Janeiro – RJ
Telefone: 21-2598-9837
Fax: 21-2598-9835
e-mail: biblioteca@coppead.ufrj.br
Home-page: http://www.coppead.ufrj.br

Measuring Risk Based on Stable Distributions: an Examination of Latin American Stock Indexes

André Carvalhal da Silva [1]

Eduardo Facó Lemgruber

Accurate forecasting of risk is the key to sucessful risk management techniques. Given the fat-tailed characterisitic of financial returns, the assumptions of modeling these returns with the thin-tailed Gaussian distribution is inappropriate. In this paper a more accurate VaR estimate is tested using the "stable" or "α-stable" distribution, which allows for varying degrees of tail heaviness and varying degrees of skewness. Stable VaR measures are estimated and forecasted using the main Latin American stock market indexes. The results show that the stable modeling provides conservative 99% VaR estimates, while the normal VaR modeling significantly underestimates 99% VaR. The 95% VaR stable and normal estimates, using a window length of 50 observations, are satisfactory. However, increasing the window length to 125 and 250 observations worsens the stable and the normal VaR measurements.

1 INTRODUCTION

Value at Risk (VaR) has established itself as one of the standard measures of market risk employed in academic literature and by financial institutions and regulators. VaR can be defined as the maximum loss over a certain time horizon (usually one day or ten days) with a given confidence level. Despite its conceptual simplicity, one of the major concerns about VaR calculations is the lack of consistency between different VaR implementations.

Accurate forecasting of market risk is the key to successful risk management techniques. Given the fat-tailed characteristic of financial returns, the assumptions of modeling these returns with the thin-tailed Gaussian distribution is inappropriate. A vast literature on financial returns[2] has recognized the existence of fat-tailed characteristics. Risk measures are underestimated under these conditions.

---

[1] Doctoral Student and Professor at Instituto COPPEAD de Administração, respectively.

[2] See Cotter (1998), Danielsson and De Vries (1997), Kearns and Pagan (1997), Koedijk and Kool (1992), and Cotter and McKillop (2000).

Alternative approaches to measuring risk include historical simulation, Monte Carlo simulation, stress testing, and extreme value methods[3]. Historical simulation estimates VaR using observed historical portfolio returns, not imposing distributional assumptions, but it does not produce reliable VaR estimates when there are a small number of observations in the tails. Monte Carlo simulation is similar to the historical simulation method, except that the hypothetical changes in prices are created by random draws from a stochastic process. One potential weakness of this method is that it relies on a specific model for underlying risk factors; therefore, it is subject to the risk that the models are wrong. Stress testing, sometimes called "scenario analysis", consists of specifying scenarios of interest to assess possible changes in the value of the portfolio. The biggest drawback of this method is that stress testing is completely subjective. Implausible scenarios will lead to wrong estimates of VaR. Extreme value theory allows for fat-tailed densities, modeling variable's extreme values at the distributional tails. Previous applications of extreme value theory in risk management include Longin (2000), Danielsson and De Vries (1997), among others.

In this paper a more accurate VaR estimate is tested using the "stable" or "α-stable" distribution, which allows for varying degrees of tail heaviness and varying degrees of skewness. Since the seminal works of Mandelbrot (1963) and Fama (1965), stable distributions have been proposed as a model for many types of processes in economics and finance[4]. In VaR estimations it is important to analyze the behavior of the distributions in the tails. The tails of the non-Gaussian stable distributions are much fatter, which will be an important issue in estimating VaR.

This paper's methodology is inspired by Khindanova, Rachev, and Schwartz (2000) in the sense that both papers pursue the same strategy, of developing more precise VaR estimates using stable distributions. Khindanova, Rachev, and Schwartz show that stable VaR modeling outperforms the normal modeling for high values of the VaR confidence level. This paper employs the same methodology in order to assess the stable VaR estimates using the main Latin American stock market indexes.

The paper proceeds in the following section with an outline of stable distributions. Section 3 details the data and the method applied to estimate the

---

[3] For a full explanation of different VaR methods, see Jorion (1997).

[4] See Walter (1990), Zajdenweber (1994), Walter (1994), Cheng and Rachev (1995), McCulloch (1996), Belkacem (1996), Embrechts, Klüppelberg, and Mikosch (1997), Corazza, Malliaris and Nardeli (1997), and Groslambert and Kassibrakis (1999).

stable parameters and the VaR measures. The results for the in-sample evaluation are presented in section 4. In section 5, the results for the out-of-sample forecast evaluation are discussed. Finally, concluding comments are documented in section 6.

2 STABLE DISTRIBUTIONS

Stable distributions[5] are a rich class of distributions, characterized by Paul Lévy (1924), that allow skewness and heavy tails. A random variable X is said to be "stable" or "α-stable" if for X_1 and X_2 and any positive constants a and b,

$$aX_1 + bX_2 \overset{d}{=} cX + d$$

for some positive c and some $d \in R$. In general stable distributions do not have closed form expressions for density and distribution functions. There are three cases in which there is a closed form expression for the stable density: the Gaussian, the Cauchy and the Lévy distributions. While the Gaussian and the Cauchy distributions are symmetric, bell-shaped curves, the Lévy distribution is highly skewed. General stable distributions allow for varying degrees of tail heaviness and varying degrees of skewness. Stable random variables are commonly described by their characteristic functions. There are multiple parameterizations for stable processes. As Zolotarev (1986) shows, there are good reasons to use different parameterizations in different situations. The parameterization most often used now (see Samorodnitsky and Taqqu (1994)) is the following:

$$X \sim S_1(\alpha, \beta, \gamma, \delta) \Leftrightarrow$$

$$E\exp(itX) = \begin{cases} \exp\left\{-\gamma^{\alpha}|t|^{\alpha}\left[1 - i\beta\left(\tan\frac{\pi\alpha}{2}\right)(sign(t))\right] + i\delta t\right\} & \alpha \neq 1 \\ \exp\left\{-\gamma|t|\left[1 - i\beta\frac{\pi}{2}(sign(t))\ln|t|\right] + i\delta t\right\} & \alpha = 1 \end{cases}$$

$$\alpha \in (0,2],\ \beta \in [-1,1],\ \gamma \geq 0 \text{ e } \delta \in R$$

---

5 The purpose of this section is to present the basic characteristics of stable distributions. For additional references, see Nolan (1999a), Nolan, Panorska, and McCulloch (1997), Samorodnitsky and Taqqu (1994), and Khindanova, Rachev, and Schwartz (2000).

where α is an index of stability, β is a skewness parameter, γ is a scale parameter, and δ is a location parameter. When $\alpha=2$, the stable distribution is the Gaussian distribution. As α decreases, the peak gets higher and the tails get heavier. So, the index of stability can be interpreted as a measure of kurtosis. If the skewness parameter $\beta = 0$, the distribution of X is symmetric. If $\beta > 0$, the distribution is skewed to the right, and if $\beta < 0$, the distribution is skewed to the left. When $\beta = 1$, the stable distribution is totally skewed to the right, and when $\beta = -1$, the distribution is totally skewed to the left. The parameters δ and γ play the role of the location and the scale usually played by the mean and variance. For the Gaussian distribution, the first and the second moment completely specify the distribution; for most distributions, including stable models, they do not.

One consequence of heavy tails is that not all moments exist. In most statistical problems, the first moment and the second moment are typically used to describe a distribution. However, these are not generally useful for heavy-tailed distributions. When $\alpha<2$, stable distributions do not have finite second moments, which is one of the arguments against using stable models for real data that have bounded range. However, as Nolan (1999a) points out, the variance is but one measure of spread for a distribution and it is not appropriate for all problems. Furthermore, bounded data are routinely modeled by the normal distribution which has unbounded support.

Distributions with heavy tails are regularly seen in applications in finance. Stable distributions have been proposed as a model for many types of processes in economics and finance[6]. There are several reliable approaches for estimating stable parameters from data. Nolan (1999b) points out that unpublished simulation results suggest that there are three best general methods: the quantile approach, characteristic functions techniques, and maximum likelihood methods. The fastest but the least accurate method is the quantile/fractile method of Fama and Roll (1971) and McCulloch (1986). It estimates stable parameters by matching certain data quantiles with those of stable distributions. Characteristic function methods estimate stable parameters fitting the empirical characteristic function to the theoretical characteristic function. Maximum likelihood methods are the most

[6] See Mandelbrot (1963), Fama (1965), Walter (1990), Zajdenweber (1994), Walter (1994), Cheng and Rachev (1995), McCulloch (1996), Belkacem (1996), Embrechts, Klüppelberg, and Mikosch (1997), Corazza, Malliaris and Nardeli (1997), and Groslambert and Kassibrakis (1999).

accurate but the slowest method for estimating stable parameters. The computer program STABLE (see Nolan (1997))[7] is used to do the maximum likelihood estimates for all four stable parameters. This method is used to ensure accuracy in the results.

3 DATA AND METHODOLOGY

The sample consists of the largest capitalization markets in Latin America: Argentina, Brazil, Chile, Colombia, Mexico, Peru and Venezuela. This study is comprehensive for the region in that it considers 97% of the market capitalization of the Caribbean and Latin American markets at the end of 1998 according to the IFC (1999). The total market capitalization of the Caribbean and Latin American markets was 1.4% of the world's capitalization and 20.6% of the emerging markets capitalization (IFC, 1999).

The analysis for the Latin American stock markets is performed for the available data from January 1994 through December 1999, which provide 1545 daily observations for each country. Specifically, the data consist of the closing daily levels of the GENERAL index (Argentina), the IBOVESPA index (Brazil), the IGPA index (Chile), the IBB index (Colombia), the IPC index (Mexico), the IGBVL index (Peru), and the IBC index (Venezuela). All indices but the IBOVESPA are value weighted. The IBOVESPA is trade volume weighted. Daily returns were computed in dollars according to the equation below. All data are obtained from the Datastream database.

$$rd_{i,t} = 100 \times \ln\left[\left(\frac{I_{i,t}}{I_{i,t-1}} \div \frac{X_{i,t}}{X_{i,t-1}}\right)\right]$$

Where $rd_{i,t}$ is the return in US$. $I_{i,t}$ is the closing index level on day *t* in country *i*. $X_{i,t}$ is the day's dollar exchange rate for country *i* on day *t*.

The data are summarized in Table 1. Latin American indexes have high daily volatility in terms of standard deviations. Brazilian IBOVESPA has the highest dollar return volatility, more than 3 times higher than the Chilean daily return standard deviation. The coefficients of kurtosis and skewness indicate deviations from a normal distribution for Latin American countries. The Kolmogorov-Smirnov (KS) test indicates that the data cannot be approximated by a normal distribution.

---

[7] The program STABLE is available on the Web at http://www.cas.american.edu/jpnolan

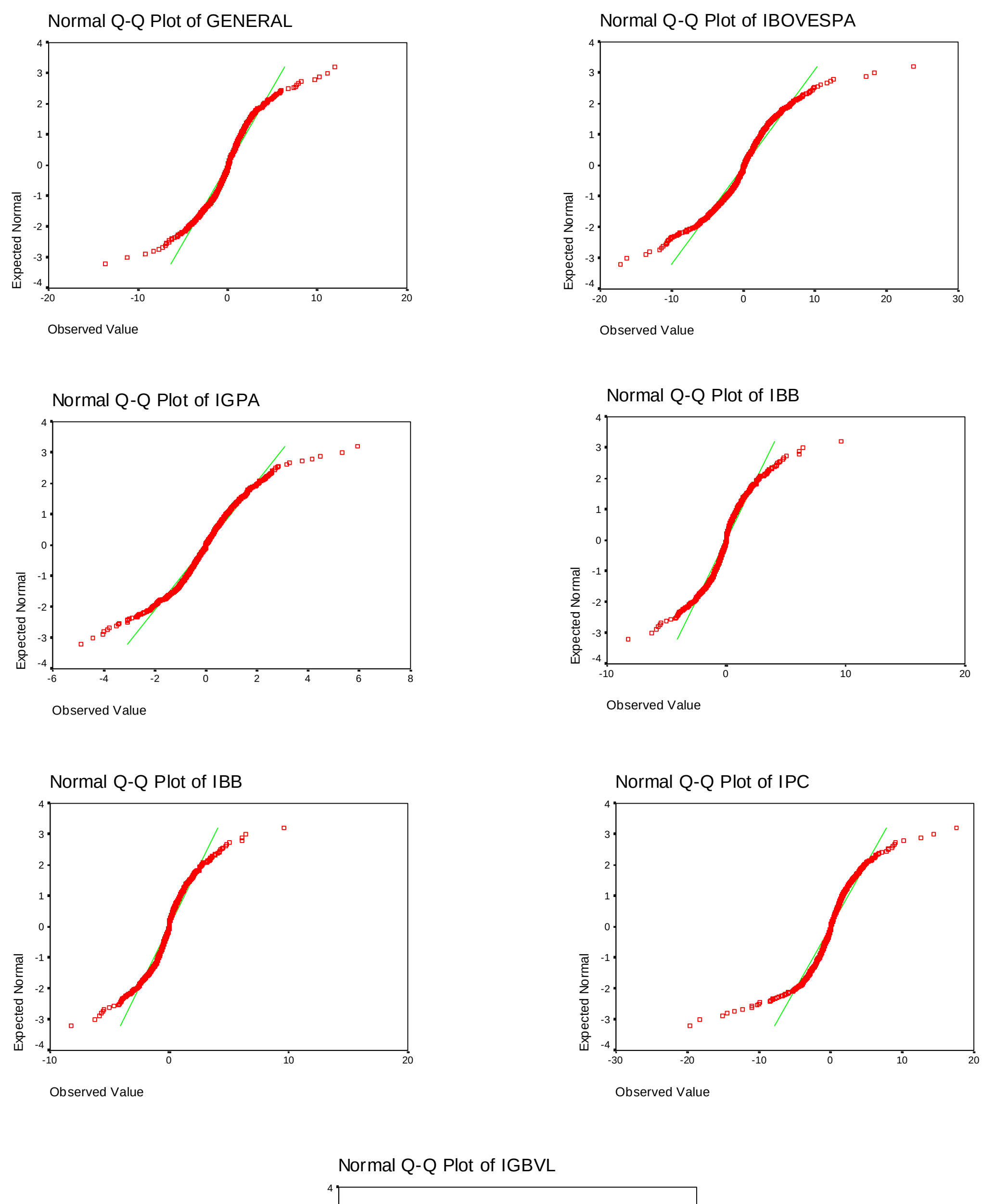
Table 1 - Descriptive Statistics for the Daily Returns in US$ from January 1994 to December 1999

Market	Index	Average Return	Standard Deviation	Minimu Return	Maximum Return	Kurtosis	Skewness	Normality (KS)	# Obs
Argentina	GENERAL	0.00%	1.97%	-13.66%	11.97%	6.06*	-0.11*	0.09*	1545
Brazil	IBOVESPA	0.07%	3.16%	-17.25%	23.72%	5.77*	0.05	0.09*	1545
Chile	IGPA	0.01%	0.96%	-4.90%	5.93%	3.83*	0.03	0.06*	1545
Colombia	IBB	-0.04%	1.28%	-8.24%	9.63%	6.60*	0.14*	0.10*	1545
Mexico	IPC	-0.01%	2.42%	-19.69%	17.54%	11.97*	-0.75*	0.10*	1545
Peru	IGBVL	0.01%	1.46%	-9.55%	7.62%	6.00*	-0.15*	0.08*	1545
Venezuela	IBC	-0.01%	2.42%	-31.50%	20.72%	26.13*	-0.73*	0.10*	1545

Note: * indicates significant at the 5% level
KS indicates the Kolmogorov-Smirnov test

Figure 1 shows Quantile-Quantile (Q-Q) plots for the seven Latin American indexes. The heavy tails in the data cause the sample variance to be large, and the normal fit poorly describes both the center and the tails of the distribution. In figure I, both lower and upper percentile values diverge substantially from the corresponding normal values.

Figure 1 - Quantile-Quantile (Q-Q) Plots for Latin American Indexes



Value-at-Risk (VaR) can be defined as the maximal loss on a given, fixed portfolio, which can be observed in a given period of time at a prespecified confidence level:

$$P(\Delta X \leq -VaR) = 1 - \alpha$$

where ΔX is the relative change in the portfolio value over the time horizon t. Typically the confidence level α is chosen to be 95% or 99%, and the time horizon to be one day or two weeks. For the purpose of testing VaR models in this work, α is chosen to be 95% and 99%, and, for computation purposes, the time horizon to be one day.

The methodology is inspired by Khindanova, Rachev, and Schwartz (2000). For each of the 7 Latin American time series of returns, stable and normal VaR models are analyzed applying in-sample (entire distribution) and out-of-sample forecast evaluations. Stable VaR parameters were derived using the computer program STABLE to do the maximum likelihood estimates for all four stable parameters (α, β, γ, and δ). This method is used to ensure accuracy in the results. In-sample evaluation is performed for stable and normal VaR estimates. Stable VaR estimates are computed by calculating the negative of the $(1-\alpha)^{th}$ quantile of the fitted stable distribution, and compared with VaR estimates based on the normal distribution. Biases of stable and normal VaR estimates are computed by subtracting the empirical VaR from the model (stable and normal) measurements.

Out-of-sample forecast evaluation is conducted for both the stable and the normal VaR models by comparing predicted VaR with observed returns. At each time t, a VaR_t measure is obtained using wl (window length) recent observations of returns R_{t-1}, R_{t-2}, ..., R_{t-wl}. The following window lengths are considered: 50, 125 and 250 trading days. For the purpose of forecast evaluation, two testing intervals (T) are considered: 250 and 500 days. The accuracy of the model is verified using the failure rate model proposed by Kupiec (1995), which gives the proportion of times VaR is exceeded in a given sample. Kupiec developed confidence regions for the number of times the actual loss exceeds the previous day's VaR. Table II shows the confidence regions for the parameters considered in this study: α = 95% ($p=1-\alpha=5\%$), α = 99% ($p=1-\alpha=1\%$), and T=250 and 500 days. In the next section the results for the VaR in-sample evaluation of each Latin American series of return are presented.

Table 2 - Admissible VaR Exceedings for Different Confidence Levels and Testing Intervals

VaR confidence level α	Testing Interval T	Admissible VaR Exceedings	
		Significance level	
		5%	1%
99%	250	[0,5]	[0,7]
	500	[2, 9]	[1,11]
95%	250	[7, 19]	[5, 22]
	500	[16, 35]	[14, 38]

Note: calculated based on Kupiec (1995)

4 RESULTS FOR VAR IN-SAMPLE EVALUATION

For each Latin American time series of returns, stable and normal VaR models are analyzed applying in-sample evaluations. Stable VaR parameters are derived using the computer program STABLE to do the maximum likelihood estimates for all four stable parameters (α, β, γ, and δ). In-sample evaluation is performed for stable and normal VaR estimates. Table III shows the estimated parameters of stable and normal densities for each series of returns.

Table 3 - Parameters of Stable and Normal Densities for the Latin American Stock Market Indexes

Series	Normal		Stable			
	Mean	Std Dev	α	β	γ	δ (multiplied by 10^3)
Argentina (GENERAL)	0.0000	0.0197	1.4585	-0.1455	0.0093	0.5989
Brazil (IBOVESPA)	0.0007	0.0316	1.4139	-0.0549	0.0147	1.0309
Chile (IGPA)	0.0001	0.0096	1.5647	0.0481	0.0052	-0.0593
Colombia (IBB)	-0.0004	0.0128	1.3490	-0.0048	0.0056	0.0052
Mexico (IPC)	-0.0001	0.0242	1.4749	0.0561	0.0109	-0.1654
Peru (IGBVL)	0.0001	0.0146	1.4926	0.0835	0.0069	-0.1536
Venezuela (IBC)	-0.0001	0.0242	1.3616	0.0003	0.0010	-0.0009

All data series can be modeled by stable distributions having $1<\alpha<2$, which is consistent with empirical studies for modeling financial return data. Figure II displays the adequacy of the stable and normal distributions for each Latin

American stock index. The graphical evidence supports that stable distributions explain and model daily returns better than normal distributions.

Figure 2: Stable and Normal Fitting for Latin American Indexes

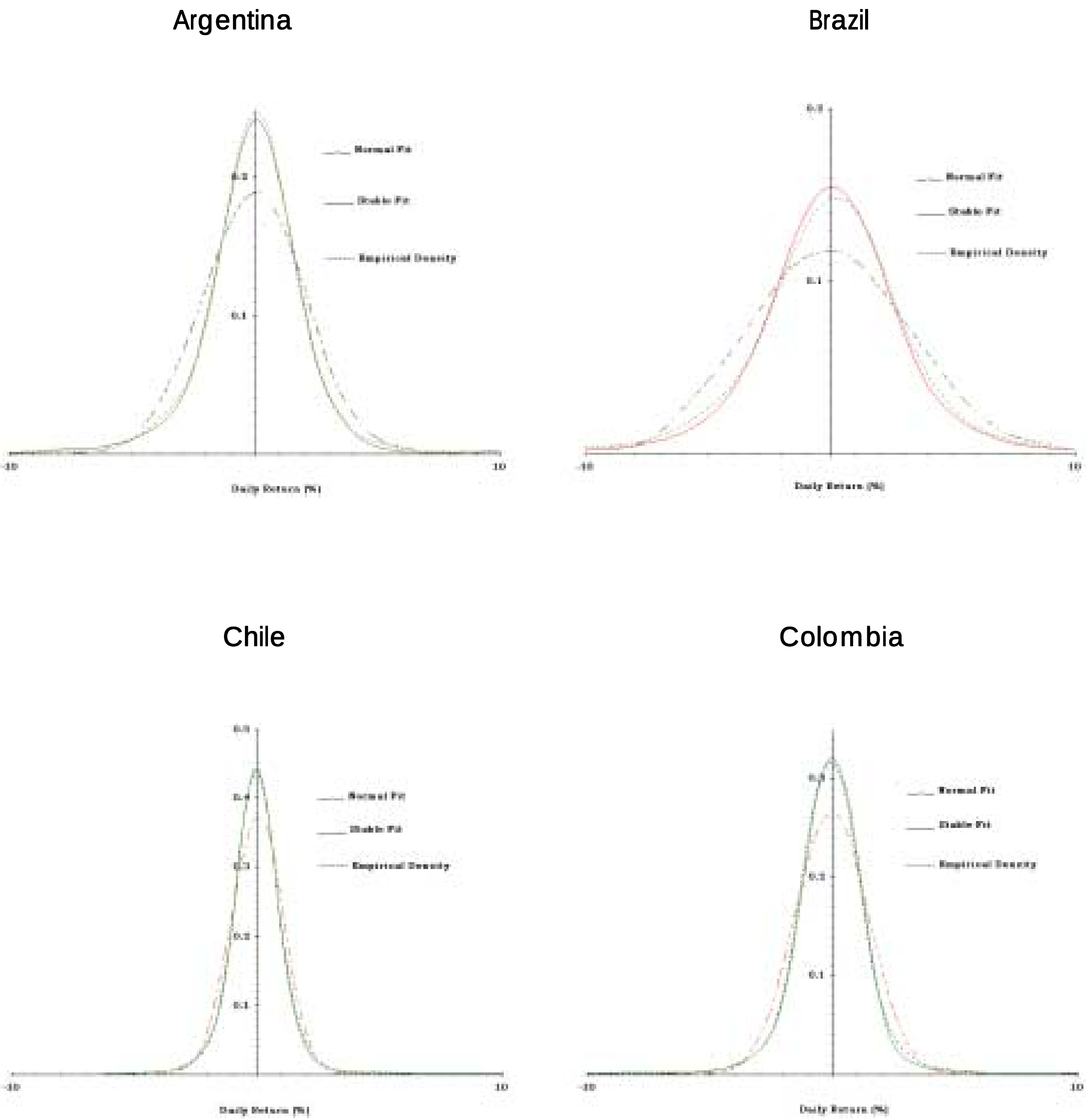
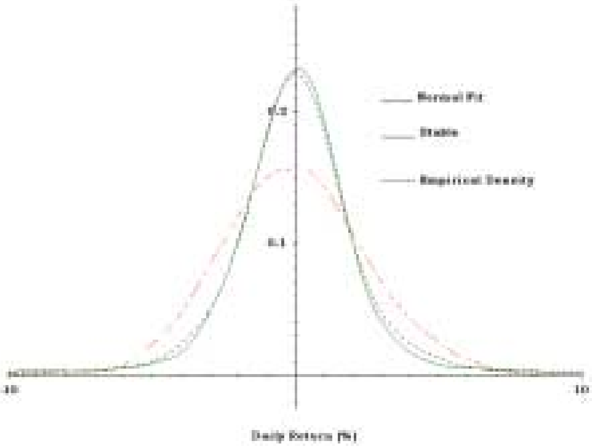
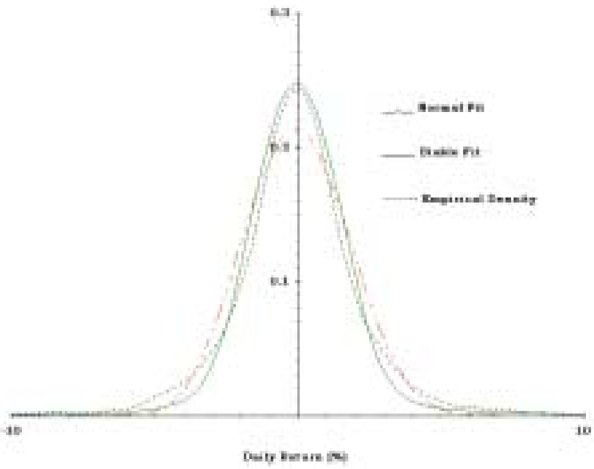


Figure 2: Stable and Normal Fitting for Latin American Indexes (Cont.)

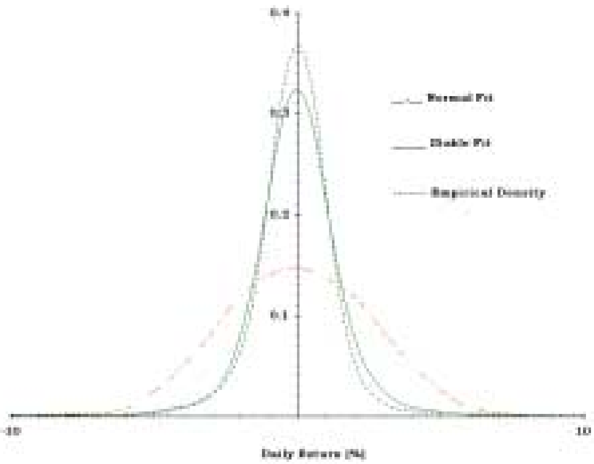
Mexico



Peru



Venezuela



VaR estimates are computed at confidence levels $\alpha=95\%$ and $\alpha=99\%$ by calculating the negative of the 5% and 1% quantile, respectively. The easiest way to find these values is to use the STABLE program. Table IV shows the 95% and 99% VaR estimates for each series of returns. The empirical, normal and stable VaR measures are reported in Table IV.

Table 4 - Empirical, Normal and Stable VaR Measures for the Latin American Stock Market Indexes

Series	99% VaR			95% VaR		
	Empirical	Normal	Stable	Empirical	Normal	Stable
Argentina (GENERAL)	5.6%	4.6%	8.6%	3.2%	3.3%	3.2%
Brazil (IBOVESPA)	10.0%	7.4%	14.2%	5.0%	5.2%	5.0%
Chile (IGPA)	2.7%	2.2%	3.4%	1.5%	1.6%	1.5%
Colombia (IBB)	3.9%	3.0%	6.0%	2.0%	2.1%	2.0%
Mexico (IPC)	7.7%	5.7%	8.5%	3.3%	4.0%	3.3%
Peru (IGBVL)	4.2%	3.4%	5.2%	2.1%	2.4%	2.1%
Venezuela (IBC)	6.9%	5.6%	10.6%	3.5%	4.0%	3.5%

Note: VaR numbers are the negative values of the VaR estimates

Table 5 reports the biases of stable and normal VaR estimates, computed by subtracting the empirical VaR from the model (stable and normal) measurements. All 99% stable VaR estimates are higher than the empirical 99% VaR, while for all data sets, the normal modeling underestimates the empirical 99% VaR. At the 95% confidence level, the stable VaR estimates are practically identical to the empirical 95% VaR, while the normal modeling overestimates the empirical VaR. The mean biases of the stable and normal VaR models are also shown in Table V. At the 99% confidence level, the mean bias under the stable method is higher in absolute terms than the normal method (2.21% and -1.30%, respectively), but the normal method clearly underestimates the empirical VaR. At the 95% confidence level, the mean bias under the normal method is higher than the stable method (0.29% and 0.00%, respectively). The results for the in-sample evaluation of stable and normal VaR show that the stable modeling provides conservative 99% VaR estimates, and provides very accurate 95% VaR estimates. The normal modeling underestimates the empirical 99% VaR and overestimates a little the empirical 95% VaR.

Table 5 - Biases of Normal and Stable VaR Measures for the Latin American Stock Market Indexes

Series	99% VaR^*_m - 99% $VaR_{Empirical}$		95% VaR^*_m - 95% $VaR_{Empirical}$	
	Normal	Stable	Normal	Stable
Argentina (GENERAL)	-1.00%	3.00%	0.10%	0.00%
Brazil (IBOVESPA)	-2.60%	4.20%	0.20%	0.00%
Chile (IGPA)	-0.50%	0.70%	0.10%	0.00%
Colombia (IBB)	-0.90%	2.10%	0.10%	0.00%
Mexico (IPC)	-2.00%	0.80%	0.70%	0.00%
Peru (IGBVL)	-0.80%	1.00%	0.30%	0.00%
Venezuela (IBC)	-1.30%	3.70%	0.50%	0.00%

Mean Bias	-1.30%	2.21%	0.29%	0.00%

* denotes normal, stable methods.

5 OUT-OF-SAMPLE PERFORMANCE EVALUATION OF VAR ESTIMATES

For each Latin American time series of returns, an out-of-sample forecast evaluation is conducted for both the stable and the normal VaR models by comparing predicted VaR with observed returns. At each time *t*, an estimate VaR_t is obtained using wl (window length) recent observations of returns R_{t-1}, R_{t-2}, ..., R_{t-wl}. The following window lengths are considered: 50, 125 and 250 trading days. For the purpose of forecast evaluation, two testing intervals (T) are considered: 250 and 500 trading days. The accuracy of the model is verified using the failure rate model proposed by Kupiec (1995), which gives the proportion of times VaR is exceeded in a given sample (see Table II). Table VI reports the results of the 99% VaR exceedings for the stable and normal modeling.

Table 6 - Out-of-Sample Evaluation of 99% VaR Exceedings for the Latin American Stock Market Indexes

Series	Testing Interval T	99% VaR Exceedings					
		Window length = 50		Window length = 125		Window length = 250	
		Normal	Stable	Normal	Stable	Normal	Stable
Argentina (GENERAL)	250 500	2 11**	4 10**	2 12*	0 4	1 7	0 0
Brazil (IBOVESPA)	250 500	3 13*	1 8	1 10**	0 1	3 9	0 1
Chile (IGPA)	250 500	6** 15*	4 6	2 10**	0 2	1 10**	0 0
Colombia (IBB)	250 500	6** 13*	5 7	5 13*	0 0	4 13*	0 0
Mexico (IPC)	250 500	2 11**	2 8	1 6	1 5	2 7	1 2
Peru (IGBVL)	250 500	1 11**	1 5	1 9	0 6	1 11**	0 5
Venezuela (IBC)	250 500	2 10**	2 7	1 14*	1 4	1 15*	1 1

* significant at the 1% level
** significant at the 5% level

The results indicate that normal models for the 99% VaR estimates commonly produce numbers of exceedings above the acceptable range, which implies that normal VaR modeling significantly underestimates VaR at the 99% confidence level. On the other hand, stable VaR estimates are within the permissible range.

Table 7 reports the results of the 95% VaR exceedings. The 95% VaR normal and stable estimates, using a window length of 50 observations, are satisfactory. However, increasing the window length to 125 and 250 observations worsens the normal and the stable VaR measurements.

Table 7 - Out-of-Sample Evaluation of 95% VaR Exceedings for the Latin American Stock Market Indexes

Series	Testing Interval T	95% VaR Exceedings					
		Window length = 50		Window length = 125		Window length = 250	
		Normal	Stable	Normal	Stable	Normal	Stable
Argentina (GENERAL)	250	9	12	3*	6**	3*	3*
	500	29	30	20	21	17	16
Brazil (IBOVESPA)	250	6**	8	4*	4*	4*	4*
	500	25	24	22	22	19	19
Chile (IGPA)	250	12	9	6**	6**	6**	5**
	500	37	28	21	18	29	24
Colombia (IBB)	250	15	13	14	10	13	10
	500	28	30	27	22	32	29
Mexico (IPC)	250	8	10	5**	8	2*	3*
	500	29	25	27	28	18	23
Peru (IGBVL)	250	8	10	2*	6**	1*	1*
	500	26	27	22	22	20	21
Venezuela (IBC)	250	8	9	3*	5**	1*	2*
	500	29	31	23	20	27	23

* significant at the 1% level
** significant at the 5% level

We can conclude by Table VI and VII that the stable method results in satisfactory 99% VaR estimates, while the normal VaR modeling significantly underestimates 99% VaR. Both the stable and the normal 95% VaR measurements are in the admissible range for the window of 50 observations, but are outside of the admissible interval at the window lengths of 125 and 250 days.

6 CONCLUSION

Value at Risk (VaR) has established itself as one of the standard measures of market risk employed in academic literature and by financial institutions and regulators. Accurate forecasting of VaR is the key to successful risk management techniques. A vast literature on financial returns has recognized the existence of fat-tailed characteristics. Risk measures based on the thin-tailed Gaussian distribution are underestimated under these conditions.

In this paper a more accurate VaR estimate is tested using the “stable” or “α-stable” distribution, which allows for varying degrees of tail heaviness and varying degrees of skewness. Stable distributions have been proposed as a model for many types of processes in economics and finance. The tails of the non-Gaussian stable distributions are much fatter, which is an important issue in estimating VaR.

This paper is inspired by Khindanova, Rachev, and Schwartz (2000) in the sense that both papers pursue the same strategy, of developing more precise VaR estimates using stable distributions. The same methodology is employed in order to assess the stable VaR estimates using the main Latin American stock market indexes.

The results for the in-sample evaluation of stable and normal VaR show that the stable modeling provides conservative 99% VaR estimates, and provides very accurate 95% VaR estimates. The normal modeling underestimates the empirical 99% VaR and overestimates a little the empirical 95% VaR. The results for the out-of-sample forecast evaluation indicates that the stable method produces satisfactory 99% VaR estimates, while the normal VaR modeling significantly underestimates 99% VaR. Both the stable and the normal 95% VaR measurements are in the admissible range for the window of 50 observations. However, increasing the window length to 125 and 250 observations worsens the stable and the normal 95% VaR measurements.

7 REFERENCES

BELKACEM, L. *Processus stables et applications en finance*. 1996. Thèse (doctorat en Sciences Mathématiques) - Université Paris IX Dauphine.

CHENG, B.; RACHEV, S. Multivariate stable futures prices. *Journal of Mathematical Finance,* v. 5, p.133-153, 1995.

CORAZZA, M.; MALLIARIS, A.; NARDELLI, C. Searching for fractal structures in agricultural futures markets. *Journal of Futures Markets*, New York, v. 17, n. 4, p. 433-473, June 1997.

COTTER, J. Testing distributional models for the Irish equity market. *Economic and Social Review*, v. 29, p. 257-269, 1998.

COTTER, J. ; McKILLOP, D. The distributional characteristics of a selection of contracts traded on the London International Financial Futures Exchange. *Journal of Business Finance and Accounting*, v. 27, n. 3/4, p. 487-512, Apr./May 2000.

DANIELSSON, J.; DE VRIES, C. *Value at risk and extreme returns*. London: London School of Economics, Financial Markets Group, 1997. (Discussion Paper, n. 273).

EMBRECHTS, P.; KLÜPPELBERG, C.; MIKOSCH, T. *Modeling extremale events with emphasis on insurance and finance.* Berli: Springer Verlag, 1997. 655 p.

FAMA, E. F. The behavior of stock market prices. *Journal of Business*, v. 38, n.1, p. 34-105, Jan. 1965.

_______; ROLL, R. Parameter estimates for symmetric stable distributions. *Journal of the American Statistical Association*, v. 66, n.334, p. 331-338, 1971.

GROSLAMBERT, B.; KASSIBRAKIS, S. The alpha-stable hypothesis: an alternative to the distribution of emerging market returns. *Emerging Markets Quarterly*, v. 3, n. 1, p. 21-38, Spring 1999.

INTERNATIONAL FINANCE CORPORATION. *Emerging stock markets fact book.* Washington, D.C.: World Bank/IFC/MIGA Publications, May 1999

JORION, P. *Value at risk:* the new benchmark for controlling market risk. Chicago: Irwin, 1997. 332p.

KEARNS, P.; PAGAN, A. Estimating the density tail index for financial time series. *The Review of Economics and Statistics*, v. 79, n. 2, p. 171-175, May 1997.

KHINDANOVA, I.; RACHEV, S.; SCHWARTZ, E. Stable modeling of value at risk, In: THE STATE OF THE ART OF VALUE AT RISK CONFERENCE, Sept. 2000, Edinburgh. *Proceedings...* Edinburgh: SIRIF, 2000.

KOEDIJK, K.; KOOL, C. Tail estimates of East European exchange rates. *Journal of Business and Economic Statistics*, v. 10, n. 1, p. 83-96, Jan. 1992.

KUPIEC, P. Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives*, v.3, n.2, p. 73-84, Winter 1995.
LÉVY, P. Theorie de l'addition des variables aléatories. *Bulletin de la Société de France*, 52, p. 49-85, 1924.

LONGIN, F. From value at risk to stress testing, the extreme value approach. *Journal of Banking and Finance,* v. 24, n. 7, p. 1097-1130, July 2000.

MANDELBROT, B. B. The variation of certain speculative prices. *Journal of Business*, v. 36, n.4, p. 394-419, 1963.

McCULLOCH, J. H. Simple consistent estimators of stable distribution parameters. *Communications in Statistics, Simulation and Computation,* v. 15, n. 4, p. 1109-1136, 1986.

McCULLOCH, J. H. Financial applications of stable distributions. In: MADDALA, G.; RAO, C. (eds). *Statistical methods in finance:* handbook of statistics. New York: North-Holland, 1996.

NOLAN, J. P.; PANORSKA, A. K.; McCULLOCH, J. H. *Estimation of stable spectral measures.* Washington: American University, Mar. 1997. (Working paper).

_______. Numerical computation of stable densities and distribution functions. *Communications in Statistics – Stochastic Models,* v. 13, n. 4, p. 759-774, 1997.

_______. *Stable distributions*. Washington: American University, Apr. 1999a. (Working Paper).

_______. *Fitting data and assessing goodness-of-fit with stable distributions*, Washington: American University. June 1999b. (Working paper).

SAMORODNITSKY, G.; TAQQU, M. S. *Stable non-Gaussian random processes*, New York: Chapman and Hall, 1994.

WALTER, C. Lévy-Stable distributions and fractal structure on the Paris market. *AFIR International Colloquial*, Paris, v.3, p. 242-259, 1990.

WALTER, C. *Les structures des hasards en economie:* efficience des marchés, lois stables et processus fractals. 1994. Thèse (doctorat ès Science Economiques) - Institut d'Etudes Politiques, Paris.

ZAJDENWEBER, D. Self-Similar properties of the CAC40 index. *Revue d'Economie Politique*, v. 104, p. 407-434, 1994.

ZOLOTAREV, V.M. *One-dimensional stable distributions.* Providence. American Mathematical Society, 1986. (Translation of 1983 Russian original).