HOW LONG MEMORY IN VOLATILITY AFFECTS TRUE DEPENDENCE STRUCTURE

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Long memory in volatility is a *stylized fact* found in most financial return series. This paper empirically investigates the extent to which interdependence in emerging markets may be driven by conditional short and long range dependence in volatility. We fit copulas to pairs of raw and filtered returns, analyse the observed changes in the dependence structure may be driven by volatility, and discuss whether or not asymmetries on propagation of crisis may be interpreted as intrinsic characteristics of the markets. We also use the findings to construct portfolios possessing desirable expected behavior such as dependence at extreme positive levels.

Keywords: Long Memory, FIGARCH models, Copulas, Tail dependence, Emerging markets.

Acknowledgements: We thank the grants received from CNPq (The National Scientific and Technological Development Council of Brazil), and FAPERJ (The Carlos Chagas Filho Rio de Janeiro State Foundation for Research Support), and the Coppead Graduate School of Business of the Federal University of Rio de Janeiro.

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1 Introduction

Recent literature have shown empirical evidence of an increasing degree of integration among stock markets, facilitated probably by the fast transmission of technology. Understanding and measuring these interdependencies is important for portfolio selection, hedging, and accurate assessment of risk in general. In particular, crisis seem to increase the frequency and magnitude of co-movements (joint high gains or joint extreme losses) among stock indexes, risky assets, and economic indicators. Risk managers and the insurance industry in general, have great interest on the accurate computation of probabilities of joint catastrophes.

International equity markets interdependencies have been widely studied through correlations (Longin and Solnik (2001), Ang and Chen (2000), etc.). However, their pitfalls are well known (Embrechts, McNeil and Straumann (2001), Forbes and Rigobon (2000)). A better picture of interdependence, including the measuring of linear and non-linear types of, may be attained by modeling the dependence structure using copulas. Examples include Ané and Kharoubi (2003), Breymann, Dias, and Embrechts (2003), Fermanian and Scaillet (2004), among others.

Copulas are particularly well suited for modeling interdependence at extreme levels, for which many copula families are avaliable, see Joe (1999). In this paper we model markets behavior during crisis fitting copulas to excesses over high thresholds. This alternative approach combines modeling the univariate data using extreme value distributions, and modeling the transformed data using selected copula families.

Let R_1 and R_2 represent the daily log-returns of two stock markets indexes and $H(\cdot,\cdot)$ their bivariate distribution with continuous margins F_1 and F_2. In this paper, the extremes are defined as joint exceedances of high thresholds. More specifically, we take any bivariate high quantile (q_1, q_2) of H as threshold values, and define the joint excesses (X_1, X_2) over the thresholds as $(X_1, X_2) = (R_1 - q_1 \ , \ R_2 - q_2)\mathbf{1}_{[(R_1>q_1)\text{and}(R_2>q_2)]}$, where $\mathbf{1}_{[A]}$ is the indicator function of event A.

Let $G_i(\cdot)$ represent the conditional distribution of $X_i|R_i > q_i$, $i = 1, 2$, and let $G(\cdot,\cdot)$ be the joint conditional distribution of (X_1, X_2). Univariate distributional results are well established, and asymptotic arguments lead to the generalized Pareto distribution (GPD) for modeling the conditional distribution of excesses over high thresholds G_i, see Leadbetter et al. (1983). However, given a data set, finding the distribution G is a very difficult task (see Tawn (1988), Straetmans (1999), Balkema and Embretchs (2004)). This problem may be successfully approached by means of copulas (see Breymann, Dias, and Embretchs (2003), Kolev, Mendes, and dos Anjos (2005)). Having obtained the copula C of (X_1, X_2), the distribution G is easily recovered from $G(x_1, x_2) = C(G_1(x_1), G_2(x_2))$.

When collecting joint bivariate data over a high pair of thresholds, the temporal dependence possibly existing in the univariate data and perhaps in the bivariate data is lost.

Thus, temporal dependence may not be an issue for our data type. However, how much of the observed interdependence is due to conditional short and long range dependence in volatility? In which ways high volatility affects the dependence structure? Being aware of these effects is crucial for fund managers, central banks directors, regulators.

This topic is investigated in Poon, Rockinger and Tawn (2002). They used nonparametric measures of tail dependence and found that there is strong evidence in favor of asymptotically independent models for the tail structure of stock market returns. They also found that most of the extremal interdependence is due to heteroskedasticity in stock returns processes, which is removed by applying bivariate GARCH models. However, they neither use joint excesses nor copulas, drawing their conclusions just based on a logistic dependence structure. On the other hand, Longin and Solnik (2001) modeled extreme tails of monthly returns using extreme value theory and found that high volatility *per se* does not seem to lead to an increase in correlation during stressful times. They conclude that the most important factor is market trend.

In efficient markets, the statistical dependence between very distant observations of a price series should be negligible. Thus, existence of long memory in mean of returns is directly related to market inefficiency. Long memory in a return series increases dependence at extreme levels and thus volatility clustering. Derivative markets in stock markets possessing long range dependence would be very profitable, as the value of an option increases with the volatility of the underlying stock price process. Risk management should take this into account. Also, forecasts based on models that take into account the long memory in returns are more likely to provide better medium or long-term predictions.

The concept of long memory was introduced in econometrics by Granger (1980) and Hosking (1981). There does not exist a unique definition of long range dependence. For a stationary sequence (X_t), one may say that it exists if $\sum_h |\rho_X(h)| = \infty$, where $\rho_X(\cdot)$ denotes the autocorrelation function of the sequence (X_t). This also makes the periodogram of the data to show large values for small frequencies. An alternative definition is via the requirement that the spectral density of the sequence (X_t) to be asymptotically of the order $L(\lambda)\lambda^{-d}$ for some $d > 0$ and for a slowly varying function L, as $\lambda \to \infty$.

Many empirical studies have used long memory, or *fractionally integrated* time series models to capture long range dependence in mean and in volatility of financial returns. Several studies found evidence of long memory in returns, for example, Crato (1994), Sadique and Silvapulle (2001), Lobato and Savin (1998), among others.

However, a comprehensive study investigating effects of long and short range memory on dependence structures, and thus on dependence measures during crisis, is still missing. In the present paper we address these issues. We first fit copulas to joint excess returns and compute measures of tail dependence. Then, we filter the data using Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic (FIGARCH) processes, designed to model short and long range dependence in volatility, and fit the same selected copulas to the joint excess residuals. Filtering the data through GARCH type models is not a monotonic transformation. Thus, it is expected that raw log daily returns and their residuals not to possess the same copula. However, how the copula family changes, and how a measure of their asymptotic dependence changes, may provide valuable information on how volatility dynamics affects interdependencies, providing some new insights on markets joint behavior.

Another important issue when assessing interdependencies is asymmetric propagation of shocks. For a given pair of financial returns, their joint (raw) excesses typically are not identically distributed (*i.d.*). In this case they are not exchangeable, that is, $G(x, y) \neq G(y, x)$ for some $x, y \in \Re$. Non-exchangeability in the joint excesses implies that crisis dissipation or transmission is not symmetric. However, identically distributed margins do not guarantee exchangeability. It may happens that independent and identically distributed (*i.i.d.*) residuals from properly filtered returns result in *i.d.* excess data possessing asymmetric dependence structure. That is, their copula C would be such that $C(u, v) \neq C(v, u)$ for some $u, v \in [0, 1]$. In this case, we may interpret the observed asymmetry in the markets joint behavior as an intrinsic characteristic not just due to volatility. We provide examples of such situations.

The empirical investigation uses daily log-returns of the twelve most important emerging markets stock indexes (from Argentina, Brazil, Chile, Mexico, India, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan, and Thailand), from 1rst January 1994 to 31rst January 2005. We find that left-tail dependence is usually stronger than right-tail dependence for both raw and filtered data. We find that most of the asymptotic dependence is due to high volatility: approximately 50% in the case of positive comovements (75% in the case of negative comovements) of the pairs found to be asymptotically dependent, after filtering were best fitted by the product copula. Estimates of the long memory fractional parameter were most of them higher than 0.50 indicating strong long memory dependence in volatility. The Latin American indexes also indicated presence of long memory in mean. Most pairs were best fitted by the symmetric AKS and the asymmetric ALM copulas.

The remainder of this paper is as follows. In Section 2 we give copula and tail dependence definitions and the expressions of the copulas used. In Section 3 we provide a brief review of fractional integration within the volatility context. Section 4 goes over the empirical analysis and interprets the results. Section 5 concludes.

2 Copulas and dependence

Let X_1, X_2 be continuous random variables with distribution function $G(x_1, x_2)$ and marginal distributions G_1, G_2, correspondingly. For every $(x_1, x_2) \in [-\infty, \infty]^2$ consider the point in $[0, 1]^3$ with coordinates $(G_1(x_1), G_2(x_2), G(x_1, x_2))$. This mapping from $[0, 1]^2$ to $[0, 1]$ is an 2-*dimensional copula*, or a bivariate copula.

The following basic theorem (given in the bivariate case) is the main result in copula theory, e.g. Sklar (1959), and partially explains the importance of copulas, see also Nelsen (1999), p. 41.

Sklar's Theorem. *Let G be a bivariate dimensional distribution function with margins G_1, G_2. Then there exists a 2-dimensional copula C such that for all $(x_1, x_2) \in [-\infty, \infty]^2$,*

$$G(x_1, x_2) = C(G_1(x_1), G_2(x_2)). \tag{1}$$

Conversely, if C is a bivariate copula and G_1, G_2 are distribution functions, the function G defined by (1) is a 2-dimensional distribution function with margins G_1, G_2. Furthermore, if the marginals are all continuous, C is unique. Otherwise, C is uniquely determined on $RanG_1 \times RanG_2$.

Therefore, the copula function is one of the most useful tools for dealing with multivariate distributions with given or known univariate marginals. Additionally, copulas can be employed in probability theory to characterize dependence concepts. In particular, in this paper we compute the upper and lower tail dependence coefficients. The *coefficient of upper tail dependence* is defined by

$$\lambda_U = \lim_{\alpha \to 0^+} \lambda_U(\alpha) = \lim_{\alpha \to 0^+} Pr\{X_1 > G_1^{-1}(1-\alpha) | X_2 > G_2^{-1}(1-\alpha)\} \ ,$$

provided a limit $\lambda_U \in [0, 1]$ exists. If $\lambda_U \in (0, 1]$, then X_1 and X_2 are said to be *asymptotically dependent* in the upper tail. If $\lambda_U = 0$, they are *asymptotically independent*. Similarly, the *lower tail dependence coefficient* is given by

$$\lambda_L = \lim_{\alpha \to 0^+} \lambda_L(\alpha) = \lim_{\alpha \to 0^+} Pr\{X_1 < G_1^{-1}(\alpha) | X_2 < G_2^{-1}(\alpha)\} \ ,$$

provided a limit $\lambda_L \in [0, 1]$ exists.

Let C be the copula of (X_1, X_2). It follows that

$$\lambda_U = \lim_{u \uparrow 1} \frac{\overline{C}(u,u)}{1-u}, \text{ where } \overline{C}(u,v) = Pr\{U > u, V > v\} \text{ and } \lambda_L = \lim_{u \downarrow 0} \frac{C(u,u)}{u} \ .$$

Copulas for modeling the joint exceedances $(X_1, X_2) | [X_1 > q_1, X_2 > q_2]$ were studied in Nelsen (1999), Joe (1999), Frees and Valdez (1998), Juri and Wüthrich (2002), Charpentier (2004), among others. Frees and Valdez (1998) worked out the expression of the copula pertaining to the bivariate Pareto distribution (Clayton copula). Juri and Wüthrich (2002) characterize the limiting dependence structure in the upper-tails of two random variables assuming their dependence structure is Archimedean. All these results lead to the *Clayton* or *Kimeldorf and Sampson* copula.

Let C be the Clayton or Kimeldorf and Sampson copula, that is, $C(u, v; \delta) = (u^{-\delta} + v^{-\delta} - 1)^{-1/\delta}$. Note this is family B4 in Joe (1999), and family (4.2.1) in Nelsen (1999), known as the Pareto family of copulas, or the Clayton family. Since C has lower tail dependence, for fitting purpose we use one its *associated* copulas, that is, the copula $C^{'}(u,v) = u + v - 1 + C(1-u, 1-v)$, see Joe (1999). This is the copula *AKS*, the copula *Associated to the Kimeldorf and Sampson* copula, also known as the *Survival Clayton* copula. The *AKS* copula possesses $\lambda_U = 2^{-1/\delta}$, which goes to zero as $\delta \to 0$, and goes to perfect dependence 1, when $\delta \to \infty$.

The Clayton copula was also obtained by Juri and Wüthrich (2002) as the conditional limit copula of Archimedean copulas. In few words, if C is a copula, for any $(u,v) \in (0,1]^2$, the conditional distribution of (U,V) given $U \le u$, $V \le v$ is given by $\frac{C(x,y)}{C(u,v)}$, for $0 \le x \le u$ and $0 \le y \le v$. This is the so called lower tail dependence copula (LTDC). They show that if C is Archimedean, then the LTDC is still Archimedean. They also show the only absolutely continuous invariant by truncature copula is the Clayton copula. It follows that the limit as $u, v \to 0$ of the LTDC derived from an Archimedean copula with differentiable generator is the Clayton copula.

This is a limit result. For the real data set used, considering the trade-off between the applicability of asymptotic results (large thresholds, few data points) and good fit for the data (larger samples), we try the following copula families possessing tail dependence, including one non-exchangeable copula.

Galambos copula. This is an extreme value copula (family B7 in Joe (1999)) given by $C(u,v;\delta) = uv\exp\{(\tilde{u}^{-\delta} + \tilde{v}^{-\delta})^{-1/\delta}\}$, where $0 \le \delta < \infty$, and where $\tilde{a} = -\log(a)$. It is an exchangeable copula with coefficient of upper tail dependence equal to $2 - 2^{1/\delta}$, for $\delta > 1$. When $\delta = 0$, it corresponds to the product copula, i.e., the copula of independent marginals.

Joe-Clayton copula. It is given by $C(u,v;\delta,\theta) = 1 - [1 - ([1-(1-u)^{\theta}]^{-\delta} + [1-(1-v)^{\theta}]^{-\delta} - 1)^{-1/\delta}]^{1/\theta}$, where $\theta \ge 1$, $\delta \ge 0$. This is an exchangeable copula possessing both (not equal in general) coefficients of tail dependence. Upper tail dependence is given by $\lambda_U = 2 - 2^{1/\theta}$ independent of δ, and lower given by $\lambda_L = 2^{-1/\delta}$, independent of θ. When $\lambda_U = 0$ ($\theta = 1$) it reduces to the Clayton copula. When $\delta \le 1$, concordance increases with θ (Joe, 1999). When either $\theta \longrightarrow \infty$ or $\delta \longrightarrow \infty$, it approaches the perfect positive dependence copula and $\lambda_U \longrightarrow 1$ (family BB7 in Joe (1999)).

Joe copula. When the parameter δ in the Joe-Clayton copula is very close to its lower bound zero, we could rather fit the one-parameter Joe copula. It is given by $C(u,v,\theta) = 1 - \left(\bar{u}^{\theta} + \bar{v}^{\theta} - \bar{u}^{\theta}\bar{v}^{\theta}\right)^{1/\theta}$, where $\bar{u} = 1-u$ and $\bar{v} = 1-v$, $\theta \ge 1$.

Asymmetric Logistic Model copula. The *ALM* copula is given by $C(u,v,\delta,p_1,p_2) = \exp\left[-(p_1^{\delta}\tilde{u}^{\delta} + p_2^{\delta}\tilde{v}^{\delta})^{1/\delta} - (1-p_1)\tilde{u} - (1-p_2)\tilde{v}\right]$, for $(p_1,p_2) \in [0,1]^2$ and $\delta \ge 1$. This is a non exchangeable copula, obtained as a mixture of the Gumbel and the product copulas (see Genest, Ghoudi and Rivest (1998)). Its upper tail dependence coefficient is given by $\lambda_U = p_1 + p_2 - (p_1^{\delta} + p_2^{\delta})^{1/\delta}$.

3 Fractionally Integrated GARCH models

In this section we provide a brief review of fractional integration within the volatility context.

Among the so called *stylized facts* that characterize a return series, the behavior of the autocorrelation function (ACF) of the data and squared data deserves close attention. For the return series the sample ACF is typically negligible at almost all lags, except for the first and second ones (it decays exponentially). However, the sample ACF of the absolute values or their squares are all positive, decays slowly and tends to stabilize for large lags (hyperbolic decay rate). Figure 1 illustrates and shows at the top panel the sample ACF of the returns of the Bangkok S.E.T. Index from Thailand. The lower panel shows the sample ACF of their squares. This empirical fact is usually interpreted as evidence of long memory in volatility.

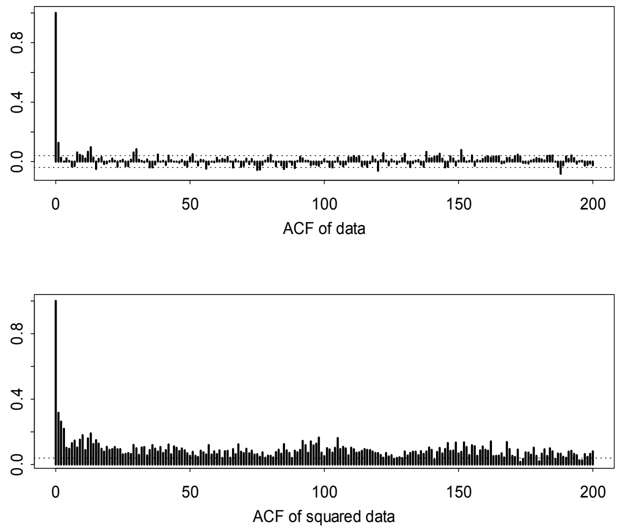

Figure 1: *Sample ACF of the returns (top panel) and squared returns (bottom panel) of the Bangkok S.E.T. Index from Thailand.*

The first long memory time series model proposed (for the mean) was the Fractionally Integrated ARMA model, the ARFIMA model, introduced by Granger (1980). An ARFIMA(p, d, q) process is a general class of processes for the mean which ranges from the unit root ARIMA$(p, d = 1, q)$ process, up to integrated processes of order 0. Robinson (1995) extended the ARFIMA framework to model long memory in volatility, giving rise to the long memory Autoregressive Conditionally Heteroskedastic (ARCH) model. Perhaps the most theoretically discussed and empirically tested (Bollerslev and Mikkelsen (1999), Bollerslev and Wright (2000), Caporin (2002), Mikosch and Stărică (2003), among others) long range dependence class of models consists of the Fractionally Integrated Generalized ARCH models, FIGARCH models, introduced by Baillie, Bollerslev, and Mikkelsen (1996), and Bollerslev and Mikkelsen (1996). Other important alternative models are the Fractionally Integrated Stochastic Volatility models of Breidt et al (1998), and the Two Component model of Ding and Granger (1996).

Let $\{r_t\}_{t=1}^{T}$ be a time series of asset returns. To capture the varying conditional variance of r_t it is assumed that

$$r_t = C + \varepsilon_t \tag{2}$$

where C is a constant and

$$\varepsilon_t | \mathcal{F}_{t-1} = \sigma_t z_t, \tag{3}$$

where z_t is an *i.i.d.* sequence of random variables with zero mean and unit variance, and $\mathcal{F}_t$ represents the information set up to time t. According to Baillie, Bollerslev, and Mikkelsen (1996) and Bollerslev and Mikkelsen (1996), a FIGARCH(r, d, s) model for the conditional variance σ_t^2 satisfies

$$\varepsilon_t^2 (1 - \phi(\mathcal{L}))(1 - \mathcal{L})^d = w + (1 - \beta(\mathcal{L}))(\varepsilon_t^2 - \sigma_t^2) \tag{4}$$

where $\omega > 0$ is a real constant, the *fractional integration parameter* $d \in [0, 1]$, $\mathcal{L}$ is the lag operator, $\phi(\mathcal{L}) = \alpha(\mathcal{L}) + \beta(\mathcal{L})$, and $\beta(\mathcal{L}) = \sum_{j=1}^{s} \beta_j \mathcal{L}^j$. The *fractional difference operator* $(1 - \mathcal{L})^d$ can be expanded in a binomial series to produce an infinite polynomial in $\mathcal{L}$:

$$(1 - \mathcal{L})^d = 1 - \sum_{k=1}^{\infty} \delta_{d,k} \, \mathcal{L}^k = 1 - \delta_d(\mathcal{L}) \ , \tag{5}$$

where the coefficients $\delta_{d,k} = d \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(1-d)}$ in (5) are such that

$$\delta_{d,k} = \delta_{d,k-1} \left(\frac{k - 1 - d}{k} \right), \tag{6}$$

for all $k \geq 1$, where $\delta_{d,0} \equiv 1$.

The FIGARCH(r, d, s) process has the infinite ARCH representation:

$$\sigma_t^2 = \omega \, (1 - \beta(\mathcal{L}))^{-1} + \lambda(\mathcal{L}) \varepsilon_t^2, \tag{7}$$

where the polynomial $\lambda(\mathcal{L})$ is given by

$$\lambda(\mathcal{L}) = \sum_{k=0}^{\infty} \lambda_k \mathcal{L}^k = 1 - (1 - \beta(\mathcal{L}))^{-1} \phi(\mathcal{L})(1 - \mathcal{L})^d. \tag{8}$$

FIGARCH(r,d,s) processes must meet some parameters restrictions to ensure positivity of the conditional variance σ_t^2. In the case of a FIGARCH$(1,d,1)$ process one must have $\beta_1 - d \leq \phi_1 \leq \frac{2-d}{3}$; $d(\phi_1 - \frac{1-d}{2}) \leq \beta_1(d+\alpha_1)$; and $\phi_1 = \alpha_1 + \beta_1$.

Even though the series σ_t^2 is non-observable, its persistence properties are propagated to the observable series r_t^2. Since the second moment of the unconditional distribution of r_t is infinite, the FIGARCH process is not weakly stationary. Discussions about stationarity property of FIGARCH processes may be found in Mikosch and Starika (2003), among others.

To assure the positiveness of the conditional variance, Bollerslev and Mikkelsen (1996) proposed the Fractionally Integrated Exponential GARCH (FIEGARCH) model:

$$\phi(\mathcal{L})(1-\mathcal{L})^d \ln\sigma_t^2 = w + \sum_{j=1}^{r}(\beta_j|\frac{\varepsilon_{t-j}}{\sigma_{t-j}}| + \gamma_j \frac{\varepsilon_{t-j}}{\sigma_{t-j}}) \ , \tag{9}$$

where $\gamma_j \neq 0$ indicates the existence of leverage effects. By including the leverage term we allow the conditional variance to depend both on sign and magnitude of expected returns. This asymmetric model is an attempt to model another stylized fact about asset returns, the effect of *bad news:* risky stocks respond differently to positive high gains and low negative falls. The larger the leverage parameter value, the larger the risk.

We also consider the very interesting (FI)GARCH-in-mean model which extends (2) to

$$r_t = C + \pi g(\sigma_t^2) + \varepsilon_t,$$

where $g(\cdot)$ can be an arbitrary function of the volatility, we use $g(\sigma_t^2) = \sigma_t^2$. This model captures the effect of volatility on expected returns. One of the rationales behind this model is the fact that a price fall reduces the value of an equity and then increases the debt-to-equity ratio, raising volatility.

4 Empirical Analysis

In this section we empirically investigate the dependence structure of log-returns co-exceedances. The analysis is performed in two steps: first we use the daily log-returns. Then we repeat the analysis using the residuals from FIGARCH fits.

The series of log-returns were collected from the Datastream database. Specifically, the data consist of the closing daily levels of the: General Index (Argentina), IBOVESPA (Brazil), IGPA (Chile), IPC (Mexico), Bombay Sensitivity Index (India), Jakarta Stock Exchange Composite (Indonesia), Seoul Composite (Korea), Kuala Lumpur Composite (Malaysia), Manila Composite (Philippines), Singapore Straits Industrial (Singapore), Taipei Weighted Price Index (Taiwan), and Bangkok S.E.T. Index (Thailand). Taiwan is the largest emerging market, with a total market capitalization of US$ 379 billion, followed by Korea (US$ 298 billion) and India (US$ 252 billion).

The sample spans the period from January 1, 1994 through January 31, 2005. The returns are calculated as the difference between consecutive logarithm daily prices, resulting in a total of $T = 2891$ observations. For all series of log-returns we did not reject the null hypothesis of stationarity.

Consistent with several previous reports on the stylized facts of return series, the series present approximately zero mean, high kurtosis[3], show volatility clusters in the time series plots, show short range dependence on just few lags and evidence of long run dependence in the autocorrelogram of the squared data (as illustrated in Figure 1). The Ljung-Box statistic of order 20 computed for the squared returns is significant for all series.

To set the notation, let $((r_{1,1}, r_{1,2}), ..., (r_{T,1}, r_{T,2}))$ be observations of (R_1, R_2) (which are either raw log-returns or filtered returns), and define a pair of threshold values (q_{1,p_1}, q_{2,p_2}) obtained as the empirical quantiles in each margin i, $i = 1, 2$. That is, the lower (upper) thresholds q_{i,p_i} are such that $Pr\{R_i < q_{i,p_i}\} = p_i$ (similarly, $Pr\{R_i > q_{i,p_i}\} = p_i$). The probabilities p_i for both margins and tails may be all different.

For fixed (p_1, p_2), the joint excesses over the threshold values (q_{1,p_1}, q_{2,p_2}) are the observed pairs $((x_{1,1}, x_{1,2}), ..., (x_{n,1}, x_{n,2}))$ of the random vector (X_1, X_2), where $(X_1, X_2) = (R_1 - q_{1,p_1}\ ,\ R_2 - q_{2,p_2})\mathbf{1}_{[(R_1 > q_{1,p_1})\text{and}(R_2 > q_{2,p_2})]}$.

We fit by maximum likelihood method the generalized Pareto distribution (GPD) to the n observations of X_i, $i = 1, 2$. An important issue is the trade-off between bias and inefficiency of the GPD parameter estimates (for example, Coles (2001), Longin and Solnik (2001)). We do not address this issue here, but we indeed do some sensitivity analysis and experiment with 5 values for p_i, we try $p_i = 0.250, 0.225, 0.200, 0.175, 0.150$, for $i = 1, 2$, thus trying 25 combinations. The value (q_{1,p_1}, q_{2,p_2}) is chosen after examining, for both margins, the GPD parameters standard errors and the result of a goodness of fit test (the Kolmogorov test). Our procedure allows for different threshold values for each series and each tail, thus adapting for market scale and shape. In this work we observed that the fraction n/T of observed pairs was approximately $0.05 - 0.09$, much smaller than each individual p_i. Note that the way the joint data is collected breaks out the (possible) serial dependence for the exceedances.

The Uniform$(0, 1)$ data are obtained by plugging the GPD parameters estimates in the GPD distribution function, and the five selected copula families are fitted by maximum likelihood. This two-steps fully parametric estimation procedure is usually called inference functions for margins (IFM). Joe (1999) argues that we can expect the IFM method to be quite efficient because it is fully based on maximum likelihood estimation, see Joe (1999) and Xu (1996).

[3] For the sake of conciseness, we do not report basic descriptive statistics, the tests for heteroskedasticity and for serial dependence on the data. Many empirical papers have already did such exploratory analysis.

Selection of best copula fit follows by comparing the log-likelihood value, the Akaike Information Criterion (AIC) [4], and the (discrete) L_2 distance between the fitted copulas and the empirical copula, see Ané and Kharoubi (2003). To test goodness of fit we used a bivariate extention of the usual Pearson test, described in Genest and Rivest (1993). For each pair, the two uniform data sets are ordered and divided into parts, forming a table, and the usual chi-squared goodness-of-fit test statistic is defined. Finally, we test independence using the standard likelihood ratio test. Parameters estimates standard errors are approximated using the observed information matrix evaluated at the maximum likelihood estimates, and tail dependence coefficient standard errors are computed using the delta method.

As a final remark we should note that in our empirical analysis, for a given data set, frequently all copula fits did not reject the null hypothesis of goodness of fit, with high and close p-values. Moreover, all provided very close log-likelihood values. In those cases we selected the copula presenting the smaller L_2 distance to the empirical copula. For example, this happened several times involving the AKS and the Galambos copulas. Fitting copulas to data may be very trick, and good procedure for help choosing the right copula is still missing.

Analysis of raw daily returns. Table 1 shows in the left panel a summary of the results for the dependent (40 out of 66) joint negative exceedances. First column names the pairs of markets and gives the number n of joint observations. They are ordered according to the value of their lower tail dependence coefficient λ_L, given in the third column. The first three positions are occupied by the Latin American pairs. Most of the symmetric fits were based on the AKS copula. There are 12 asymmetric cases.

Results for the positive co-exceedances are given in the left panel of Table 2. We first note the asymmetry between bear and bull markets. All dependent pairs in Table 1, present in Table 2 a smaller asymptotic dependence, that is, $\lambda_U < \lambda_L$. There are only 36 dependent pairs in the right upper tail, being 16 of them based on the asymmetric copula, and also 16 based on the AKS copula.

The Asian markets show stronger dependence during bull markets. Among the 10 first positions, 8 are occupied by the Asian markets (in the case of bear markets they occupy 3 out of 10). Stronger linkages are observed for pairs involving either Singapore or Phillipines, being this true also for the joint negative extreme events.

[4] Let LL represent the log likelihood computed at the maximum likelihood estimates, and k the number of parameters in the model. The AIC$= -2LL + 2k$.

Table 1: *Copula parameters and λ_L estimates (standard errors) for dependent negative (raw and filtered) co-exceedances. Pairs are ranked according to the strength of their lower tail dependence coefficient.*

	Raw Data		Filtered Data	
Pairs(n)	COPULA - Estimates (s.e.)	λ_L(s.e.)	COPULA - Estimates (s.e.)	λ_L(s.e.)
Arge-Braz(337)	JOE-1.59(0.08)	0.46(0.04)	JOE-1.49(0.08)	0.41(0.04)
Arge-Mexi(310)	JOE-1.50(0.16)	0.41(0.06)	JOE-1.37(0.08)	0.34(0.05)
Braz-Mexi(266)	AKS-0.78(0.08)	0.41(0.04)	JOE-1.51(0.10)	0.42(0.05)
Phil-Sing(289)	AKS-0.70(0.10)	0.37(0.05)	AKS-0.46(0.10)	0.22(0.07)
Arge-Chil(311)	JOE-1.41(0.10)	0.36(0.05)	ALM-1.82(0.45)-0.30(0.13)-0.36(0.09)	0.18(0.06)
Arge-Mala(197)	ALM-1.35(0.15)-0.98(0.28)-0.22(0.13)	0.36(0.09)	Independent	0.00
Chil-Mexi(294)	JOE-1.38(0.08)	0.35(0.05)	ALM-1.84(0.13)-0.24(0.07)-0.66(0.11)	0.19(0.06)
Braz-Chil(303)	AKS-0.64(0.09)	0.34(0.05)	JOE-1.23(0.08)	0.24(0.07)
Sing-Thai(298)	ALM-1.40(0.03)-0.98(0.06)-0.94(0.07)	0.34(0.04)	ALM-1.51(0.23)-0.68(0.12)-0.90(0.07)	0.32(0.04)
Mala-Sing(295)	ALM-1.40(0.08)-0.76(0.06)-0.98(0.08)	0.31(0.04)	ALM-1.50(0.15)-0.34(0.04)-0.98(0.17)	0.21(0.06)
Indo-Sing(280)	AKS-0.55(0.08)	0.28(0.05)	AKS-0.48(0.09)	0.23(0.06)
Mala-Thai(278)	ALM-1.50(0.09)-0.50(0.03)-0.92(0.08)	0.27(0.07)	GAL-0.43(0.06)	0.20(0.04)
Kore-Sing(244)	ALM-2.20(0.14)-0.40(0.03)-0.40(0.05)	0.25(0.08)	Independent	0.00
Indo-Mala(298)	ALM-1.25(0.12)-0.98(0.04)-0.94(0.07)	0.25(0.05)	ALM-5.9(0.50)-0.18(0.07)-0.14(0.06)	0.13(0.06)
Mala-Phil(260)	GAL-0.49(0.06)	0.24(0.04)	Independent	0.00
Indi-Thai(223)	ALM-2.55(0.11)-0.54(0.10)-0.26(0.05)	0.23(0.04)	Independent	0.00
Chil-Sing(261)	ALM-1.25(0.09)-0.72(0.12)-0.98(0.08)	0.22(0.06)	Independent	0.00
Chil-Mala(234)	ALM-1.34(0.08)-0.98(0.16)-0.46(0.09)	0.21(0.06)	Independent	0.00
Phil-Thai(243)	AKS-0.44(0.11)	0.21(0.08)	JC-1.15(0.10)-0.13(0.10)	0.17(0.09)
Chil-Phil(231)	GAL-0.44(0.08)	0.21(0.05)	Independent	0.00
Kore-Thai(252)	AKS-0.44(0.09)	0.20(0.07)	Independent	0.00
Indo-Phil(284)	AKS-0.42(0.09)	0.19(0.06)	AKS-0.28(0.09)	0.09(0.06)
Mala-Taiw(206)	GAL-0.40(0.08)	0.18(0.05)	Independent	0.00
Indi-Sing(261)	AKS-0.4(0.08)	0.18(0.06)	Independent	0.00
Chil-Kore(177)	ALM-1.18(0.16)-0.86(0.21)-0.98(0.06)	0.18(0.07)	Independent	0.00
Mexi-Sing(218)	ALM-1.51(0.11)-0.28(0.09)-0.9(0.04)	0.18(0.07)	ALM-1.31(0.11)-0.98(0.02)-0.26(0.05)	0.13(0.02)
Mexi-Phil(218)	ALM-1.96(0.07)-0.16(0.05)-0.98(0.08)	0.15(0.04)	Independent	0.00
Indi-Kore(241)	AKS-0.36(0.07)	0.14(0.05)	Independent	0.00
Chil-Bang(216)	GAL-0.34(0.12)	0.13(0.06)	Independent	0.00
Indi-Taiw(149)	AKS-0.34(0.13)	0.13(0.09)	Independent	0.00
Taiw-Thai(156)	AKS-0.34(0.13)	0.13(0.09)	AKS-0.19(0.10)	0.03(0.06)
Indo-Thai(238)	AKS-0.32(0.11)	0.12(0.08)	ALM-1.31(0.09)-0.40(0.05)-0.98(0.08)	0.18(0.04)
Braz-Phil(214)	AKS-0.30(0.08)	0.10(0.06)	Independent	0.00
Arge-Phil(184)	AKS-0.30(0.08)	0.10(0.06)	Independent	0.00
Mexi-Mala(226)	AKS-0.28(0.08)	0.08(0.06)	AKS-0.20(0.08)	0.03(0.04)
Kore-Mala(200)	ALM-3.7(0.17)-0.90(0.09)-0.08(0.04)	0.08(0.03)	Independent	0.00
Arge-Indo(182)	AKS-0.24(0.09)	0.06(0.03)	Independent	0.00
Arge-Bang(182)	AKS-0.24(0.09)	0.06(0.06)	Independent	0.00
Braz-Sing(255)	AKS-0.20(0.09)	0.03(0.05)	Independent	0.00
Braz-Mala(220)	AKS-0.19(0.08)	0.03(0.04)	Independent	0.00

Analysis of FIGARCH filtered returns. We assume a more general expression for (2):

$$r_t = \delta r_{t-1} + \pi\sigma_t^2 + \varepsilon_t + \theta\varepsilon_{t-1}$$

$$\epsilon_t|\mathcal{F}_{t-1} = \sigma_t z_t,$$

where δ is the autoregressive term, θ is the moving average term, z_t is an *i.i.d.* sequence with distribution $N(0,1)$, with σ_t^2 being specified according to (4) or (9) with $r, s = 0, 1$, and where we include the GARCH-in-mean term[5] π to assess the impact of contemporaneous relationship between return and volatility on the volatility process.

[5]We would like to use FIGARCH-in-mean, but the SPlus code for that is still not avaliable

Table 2: *Copula parameters and λ_U estimates (standard errors) for dependent positive (raw and filtered) co-exceedances. Pairs are ranked according to the strength of their upper tail dependence coefficient.*

	Raw Data		Filtered Data	
Pairs(n)	COPULA - Estimates (s.e.)	λ_U(s.e.)	COPULA - Estimates (s.e.)	λ_U(s.e.)
Sing-Thai(316)	JOE-1.49(0.06)	0.41(0.03)	Independent	0.00
Phil-Thai(237)	AKS-0.72(0.12)	0.38(0.07)	Independent	0.00
Arge-Braz(296)	JOE-1.40(0.09)	0.36(0.05)	Independent	0.00
Indo-Phil(251)	ALM-1.88(0.18)-0.70(0.05)-0.62(0.05)	0.36(0.04)	Independent	0.00
Indo-Sing(282)	AKS-0.66(0.10)	0.35(0.05)	ALM-1.48(0.14)-0.50(0.03)-0.50(0.09)	0.20(0.04)
Kore-Sing(189)	ALM-1.94(0.04)-0.68(0.02)-0.52(0.06)	0.34(0.04)	ALM-5.90(0.18)-0.44(0.11)-0.12(0.06)	0.12(0.05)
Phil-Sing(256)	AKS-0.60(0.11)	0.32(0.06)	Independent	0.00
Indo-Thai(269)	ALM-2.14(0.07)-0.38(0.01)-0.68(0.06)	0.29(0.04)	ALM-1.84(0.20)-0.12(0.02)-0.98(0.08)	0.11(0.03)
Arge-Mexi(274)	AKS-0.50(0.09)	0.25(0.06)	AKS-0.26(0.08)	0.07(0.06)
Indi-Sing(184)	ALM-1.25(0.19)-0.98(0.02)-0.21(0.03)	0.25(0.02)	Independent	0.00
Mala-Sing(346)	AKS-0.46(0.09)	0.23(0.06)	ALM-1.41(0.13)-0.44(0.04)-0.78(0.08)	0.21(0.04)
Kore-Thai(258)	ALM-1.52(0.19)-0.42(0.08)-0.76(0.07)	0.23(0.02)	Independent	0.00
Indo-Kore(243)	ALM-3.00(0.14)-0.22(0.02)-0.54(0.07)	0.21(0.04)	Independent	0.00
Indi-Kore(260)	ALM-1.51(0.24)-0.34(0.08)-0.82(0.08)	0.20(0.05)	Independent	0.00
Arge-Chil(287)	ALM-1.92(0.28)-0.24(0.06)-0.74(0.08)	0.20(0.04)	Independent	0.00
Kore-Phil(233)	ALM-2.55(0.26)-0.30(0.05)-0.28(0.03)	0.20(0.03)	Independent	0.00
Arge-Indi(180)	ALM-1.64(0.18)-0.28(0.03)-0.66(0.06)	0.19(0.04)	AKS-0.20(0.11)	0.03(0.06)
Chil-Mala(159)	ALM-1.34(0.09)-0.98(0.07)-0.40(0.04)	0.19(0.03)	Independent	0.00
Braz-Mexi(297)	AKS-0.41(0.07)	0.18(0.05)	ALM-1.66(0.31)-0.16(0.03)-0.54(0.07)	0.12(0.06)
Mala-Thai(282)	AKS-0.40(0.08)	0.18(0.06)	Independent	0.00
Braz-Sing(193)	ALM-1.68(0.26)-0.42(0.07)-0.30(0.07)	0.17(0.04)	Independent	0.00
Mexi-Taiw(178)	ALM-2.40(0.25)-0.18(0.03)-0.68(0.05)	0.17(0.05)	Independent	0.00
Chil-Mexi(273)	AKS-0.38(0.08)	0.16(0.06)	Independent	0.00
Arge-Kore(142)	GAL-0.37(0.17)	0.16(0.08)	Independent	0.00
Kore-Taiw(222)	ALM-1.98(0.22)-0.14(0.03)-0.72(0.05)	0.13(0.02)	Independent	0.00
Mala-Phil(211)	AKS-0.34(0.12)	0.13(0.08)	AKS-0.22(0.08)	0.04(0.05)
Kore-Mala(248)	ALM-5.10(0.28)-0.28(0.07)-0.12(0.03)	0.12(0.03)	Independent	0.00
Indi-Thai(218)	AKS-0.32(0.09)	0.12(0.06)	Independent	0.00
Arge-Sing(218)	AKS-0.28(0.09)	0.09(0.06)	Independent	0.00
Sing-Taiw(209)	AKS-0.27(0.09)	0.08(0.07)	Independent	0.00
Chil-Thai(161)	AKS-0.26(0.10)	0.07(0.06)	Independent	0.00
Chil-Indo(181)	AKS-0.24(0.07)	0.06(0.05)	Independent	0.00
Mexi-Kore(208)	AKS-0.24(0.09)	0.06(0.06)	Independent	0.00
Mexi-Mala(204)	AKS-0.24(0.09)	0.06(0.06)	Independent	0.00
Mexi-Sing(246)	AKS-0.24(0.08)	0.05(0.05)	Independent	0.00
Phil-Taiw(226)	ALM-5.9(021)-0.22(0.06)-0.04(0.02)	0.04(0.02)	Independent	0.00

We used the AIC to discriminate between models. Maximum likelihood estimation may be tricky, as one often gets a local maximum. Values provided by SPlus functions not always meet model constrains. A summary of the results is given in Table 3. Note most of the markets exhibit significant leverage effect. All estimates of γ_1 are negative, indicating the large effect of bad news. We observe that all markets (except those modeled by the GARCH-in-mean process) present significant estimate of (strong) long memory.

Table 3: *FIGARCH fits.*

Country	$\delta(s.e.)$	$\theta(s.e.)$	$\pi(s.e.)$	$w(s.e.)$	$\alpha_1(s.e.)$	$\beta_1(s.e.)$	$\gamma_1(s.e.)$	$d(s.e.)$
Argent.	0.099(0.021)		0.009(0.008)	0.168(0.019)	0.101(0.007)	0.861(0.009)	-0.259(0.028)	
Brazil		0.078(0.020)	0.024(0.007)	0.157(0.022)	0.108(0.010)	0.859(0.011)	-0.237(0.033)	
Mexico		0.152(0.019)		-0.130(0.015)	0.182(0.020)	0.496(0.089)	-0.133(0.015)	0.540(0.031)
Chile	0.310(0.018)			-0.276(0.020)	0.357(0.026)		-0.035(0.012)	0.691(0.026)
India	0.515(0.079)	-0.361(0.087)		-0.253(0.017)	0.342(0.024)	0.010(0.078)	-0.083(0.012)	0.655(0.030)
Indon.	0.206(0.020)			0.146(0.019)		0.117(0.027)		0.314(0.023)
Korea	0.081(0.019)			-0.083(0.017)	0.112(0.022)	0.446(0.138)	-0.045(0.009)	0.751(0.037)
Malays.	0.145(0.017)			-0.149(0.016)	0.203(0.023)	0.183(0.113)	-0.081(0.011)	0.754(0.013)
Philip.	0.167(0.017)		-0.013(0.013)	0.048(0.007)	0.053(0.003)	0.918(0.005)	-0.409(0.038)	
Singap.	0.118(0.019)			-0.137(0.015)	0.182(0.020)	0.416(0.087)	-0.069(0.010)	0.698(0.019)
Taiwan	0.923(0.047)	-0.897(0.055)		-0.086(0.010)	0.127(0.014)	0.666(0.067)	-0.103(0.012)	0.423(0.043)
Thaila.	0.494(0.118)	-0.384(0.128)		0.230(0.039)		0.193(0.030)		0.296(0.027)

All described steps of the statistical analysis performed on the raw log-returns are now applied to the free of volatility clusters residuals. A summary of the results is provided at the right panel of Table 1, in the case of negative co-exceedances, and in the right panel of Table 2 in the case of positive co-exceedances.

Overall, the results strongly indicate that most of the observed dependence may be credited to volatility. No pair found independent under raw data modeling, was found dependent after filtering. On the contrary, among the 40 dependent pairs during bear markets, 22 are independent (55%) if volatility is filtered. Those still dependent pairs show now smaller degree of dependence, as measured by the value of the lower tail dependence coefficient. In the right upper tail the results are even more impressive: the four pairs possessing strongest tail dependence are now independent, and just 8 out of 36 pairs (22%) possess a non-zero, though small, λ_U. For example, the raw log-returns from Korea and Thailand provided $\lambda_L = 0.20$ and $\lambda_U = 0.23$, but the co-exceedances became independent after filtering. Another interesting finding is that after whitening the data, Korea and India become independent from *all* other emerging markets during bear markets. This behavior is also observed for Phillipines, Taiwan, Chile, and India during bull markets.

The t-tests carried on the differences between the *before* and *after* filtering tail dependence coefficients (λ_L and λ_U), provided zero p-values. So, in overall, the strength of tail dependence existing among pairs of emerging markets statistically decrease when volatility dynamic is filtered.

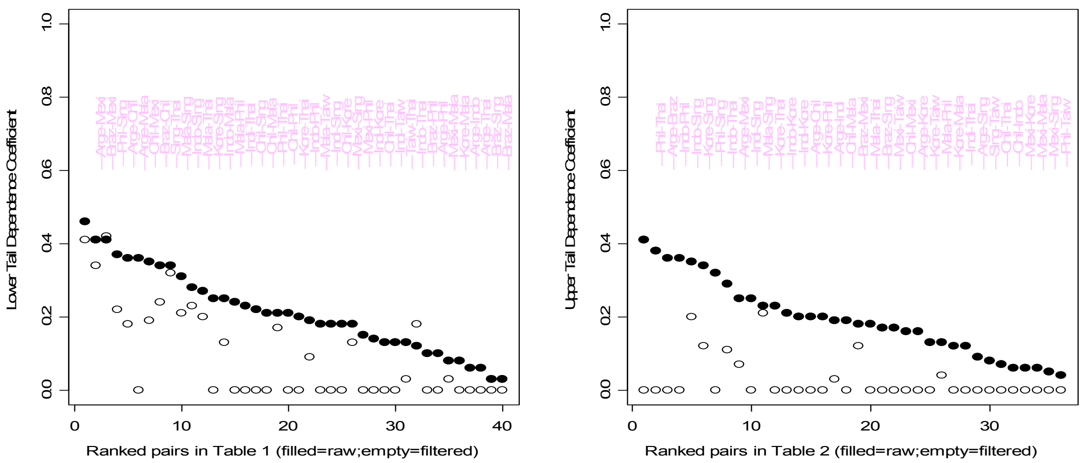

Figure 2: *Tail dependence coefficients computed for raw excesses (filled balls) and filtered excesses (empty balls), for pairs ranked in Table 1 and Table 2.*

Figure 2 graphically shows the results given in Table 1 and Table 2, and plots the values of the tail dependence coefficients. On the left (right) panel we show results for the left lower tail (upper right tail). The sequence of filled balls represent the value of the tail dependence coefficient for the ranked pairs of markets fitted using raw returns. The empty balls represent the value of the tail dependence coefficient for the corresponding filtered residuals. We note that in only two cases (Phillipines-Singapore and Indonesia-Thailand) we observed an (small) increase in the λ_L value. The results for λ_U are very impressive, being the less dramatic the one related to the pair Malaysia-Singapore.

The observed asymmetry between the lower left and the upper right tails may be tested by carrying on tests based on 2×2 contigency tables. We used the results from the filtered data and defined tail dependence as *weak* if $\lambda_L < 0.10$ (or $\lambda_U < 0.10$), and *not weak* otherwise. The p-value of 0.10 did not reject the null hypothesis of independence between the two categories defined on the two tails.

As we have commented in the Introduction, non-exchangeability in the joint excesses implies that crisis transmission is not symmetric. However, identically distributed margins may possess an asymmetric dependence structure. How much $G(x_1, x_2)$ differs from $G(x_2, x_1)$ is, of course, of great interest in the financial world. Any normalized measure of the difference $G(x_1, x_2) - G(x_2, x_1)$ may be used to measure the degree of non-exchangeability between X_1 and X_2. Nelsen (2005) proposed to compute the maximum of the absolute value of the differences $G(x, y) - G(x, y)$. For identically distributed margins this is equivalent to compute $\mu = 3 * \max |C(u, v) - C(v, u)|$, for all $u, v \in [0, 1]^2$, where C is the fitted (asymmetric) copula. Using the excess data we empirically estimate this quantity.

Singapore-Thailand, for which the λ_L estimate did not statistically change after filtering, is an interesting example where the intrinsic structure seems to be asymmetric, but the high volatility and the long memory in volatility seems to lead to symmetric propagation of crisis. We accepted the null hypothesis of equality of distributions for the raw and filtered excesses, with both p-values above 0.90. However, the measure μ estimated using the fitted copulas provided for the raw excesses the value 0.0029, and for the filtered excesses the value of 0.0200. This behavior was also found for the other pairs for which the asymmetric *ALM* copula was the best fit. Their empirical estimate of μ are, respectively, before and after filtering: Mexico-Singapore, $\mu = 0.0200, 0.0436$; Malaysia-Singapore, $\mu = 0.0159, 0.0469$; Indonesia-Malaysia, $\mu = 0.0018, 0.0145$.

As a final exercise with the purpose of illustrating one application of the findings of this paper, we considered the evolution through time of the accumulated gains of two equally weighted portfolios. The first portfolio is composed by Singapore and Thailand. This pair showed the strongest dependence at extreme levels during bull markets, $\lambda_U = 0.41$,

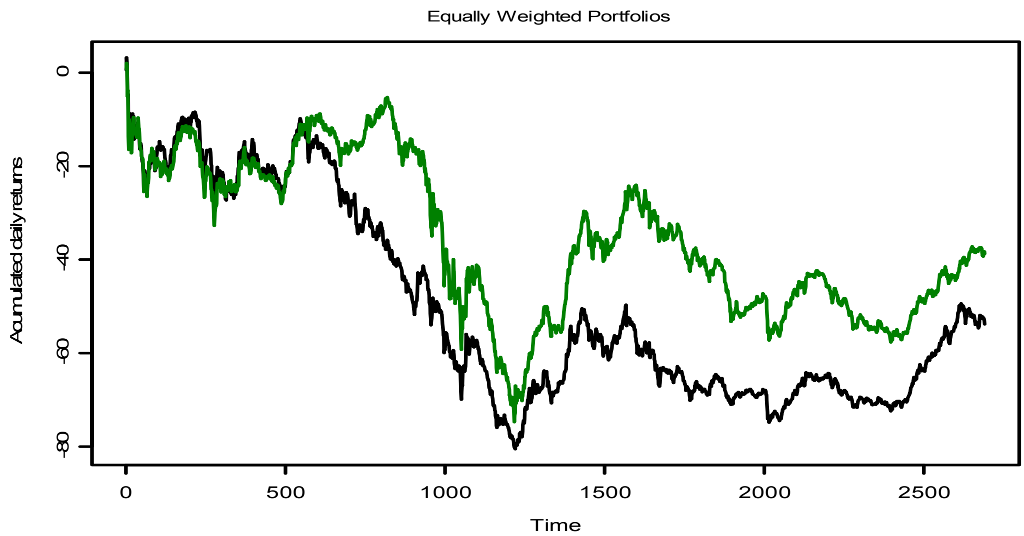

Figure 3: *The evolution of the accumulated gains from both portfolios throughout the span of the data. In black, the Singapore-Thailand portfolio, and in green the Malaysia-Singapore portfolio.*

and the impressive result after filtering (independence), $\lambda_U = 0.00$. At extreme joint losses the tail dependence coefficients are, before and after treatment, $\lambda_L = 0.34$ and $\lambda_L = 0.32$. The second portfolio corresponds to the pair Malaysia and Singapore which shows, respectively, for the raw and filtered data, $\lambda_U = 0.23$, and $\lambda_U = 21$. At extreme joint losses the tail dependence coefficients are, before and after treatment, $\lambda_L = 0.31$ and $\lambda_L = 21$. Note that the correlation coefficient estimated for the two portfolios are very close, respectively 0.43 and 0.41 (after cleaning the estimates are 0.37 and 0.41). As indicated by their lower tail dependence coefficients, one may expect similar performance from both portfolios during crisis. Thus one may expect to get better performance from the first portfolio which promises to yield higher returns during booms. However, this is not what Figure 3 reveals. This figure shows the evolution of the accumulated gains from both portfolios throughout the span of the data. In black we have the first portfolio (Singapore-Thailand), and in green the second (Malaysia-Singapore) portfolio.

5 Conclusions

In this paper we carried on a comprehensive study investigating effects of long and short range memory on dependence structures of emerging markets co-exceedances. We fitted copulas to joint excess log-returns and computed measures of tail dependence. Then we filtered the data using FIGARCH processes, and fitted the same selected copulas to the joint excess residuals. We observed that all markets (except those modeled by the GARCH-in-mean process) presented significant estimate of (strong) long memory. The observed changes on the dependence structure provided valuable information on how volatility dynamics affects interdependencies.

All dependent pairs presented asymmetry between bear and bull markets, typically $\lambda_U < \lambda_L$, this being true for raw and filtered data. Overall, the results strongly indicate that most of the observed dependence may be credited to volatility. No pair found independent under raw data modeling, was found dependent after filtering. On the contrary, among the 40 dependent pairs during bear markets, 22 are independent (55%) if volatility is filtered. Those still dependent pairs show now smaller lower tail dependence coefficient. In the right upper tail the results are even more impressive: the four pairs possessing strongest tail dependence were found independent after filtering, and just 8 out of 36 pairs (22%) possessed a non-zero, though small, λ_U.

In summary, we found that volatility masked the true dependence structure (found in the filtered excesses) in many ways. For example, symmetric propagation of crisis as well as the observed degree of interdependence could be an effect of short and long memory in volatility. We provided examples where non-exchangeability was found for identically distributed random variables, and long memory in volatility was responsible for changes in dependence structure, increasing extremal dependence.

As a final exercise with the purpose of illustrating one application of the findings of this paper, we considered the evolution through time of the accumulated gains of two equally weighted portfolios. One of the portfolios with strong upper tail dependence did not yield high returns as expected. Many other applications may follow this analysis, and we leave this for future work.

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